**Workshop on the Mathematics of Weather and Climate Prediction**
**Met Office, Exeter, 30 March – 3 April 2009**

# Report from breakout group 1: How can work in nonlinear PDEs most benefit climate prediction?

**Paul Williams, University of Reading (Chair)**
**Marek Wlasak, Met Office (Rapporteur)**

Andy Majda discussed a methodology for forecasting low frequency teleconnection patterns, such as the Arctic Oscillation (AO) and North Atlantic Oscillation (NAO). By applying a fluctuation-dissipation theorem (FDT), the climate response to small external forcing can be estimated from well-chosen statistics of the present climate.

FDTs centre on identifying the appropriate response function, using an ergodic assumption and assuming that the evolution is in statistical equilibrium. The technique may be generalised to calculate accurate variance and mean estimates. Sensitivity information can also be obtained such that the forcing can be readily determined from a given response. We may also use FDTs to infer the regional perturbations that contain the most information.

FDTs require strong mixing. They have been shown to work in the Lorenz (1996) model with 40 modes and in other simple climate models. They are also expected to work in more complex climate models.

The validity of the response function is limited by the fidelity of the climate model from which it is generated. Climate models are numerical discretisations with finite dimensional attractors that do not perfectly coincide with the true attractor. Further details of the FDT method are described by Gritsun, Branstator & Majda (2008) and Abramov & Majda (2009).

Now that the framework has been established, the next steps are to determine a reduced subspace (or key variables) that can be considered skilful and to consider the full three-dimensional problem.

The Chair posed four questions to provoke debate and discussion among the group:

## 1) What can be done about the fact that GCM climate simulations are not converged as resolution increases?

Some of the group expressed that we cannot expect convergence to occur except in a statistical sense in some key functionals, such as global-mean annual-mean surface temperature.

Linked to this was concern about conservation properties in climate models. For example, many atmosphere GCMs lose mass at the top of the atmosphere. Also, many ocean GCMs do not conserve oceanic mass, potentially leading to spurious sea-level signals if care is not taken. In future, coupled atmosphere-ocean GCMs will need to conserve water in all its forms, including sea-ice and glaciers.

## 2) Should climate GCMs be deterministic or stochastic?

Deterministic closure schemes in GCMs uniquely and unjustifiably slave the unresolved processes to the resolved flow. Stochastic closure schemes are an attractive alternative and have proven useful in weather prediction. A recent special issue of *Philosophical Transactions of the Royal Society* was devoted to the topic of stochastic physics and climate modelling (Palmer & Williams 2008).

The group generally endorsed the introduction of stochasticity to climate GCMs. There is a need to improve stochastic parameterisations, to allow climate models to have better physical-dynamical coupling and to better represent processes such as the Madden-Julian Oscillation (MJO). It is well known that in order to get accurate mean information, we need to get the fluctuations right.

The issue of multi-model climate ensembles was discussed. One use for these ensembles is to identify where the biggest discrepancies occur and trust only the output where differing models agree.

## 3) Should we worry about structural instability in climate GCMs?

Some in the community have raised the potential problem of structural instability in climate models, i.e. small changes in the equations giving large changes in the attractor. Simple models (e.g. Lorenz 1996) have been shown to be formally structurally unstable. It is unclear whether the same is true of climate GCMs.

However, formal structural instability is not necessarily a problem. Abramov & Majda (2007) show that in the Lorenz (1996) model, despite structural instability, an accurate response function can still be obtained using a FDT.

## 4) Is there a meaningful climate 'slow manifold' upon which meteorological motions are absent?

The slow manifold of fluid dynamics is a hypothetical sub-manifold of the full phase space, upon which fast inertia-gravity waves are absent. By projecting the full equations onto the slow manifold, to obtain balanced equations, predictions may be made about the 'slow' component of the flow without explicit consideration of the 'fast' component. By analogy, is it possible to make predictions of climate without explicit consideration of weather?

It was quickly identified that 'slow manifold' is not the best label in the climate context. The group expressed doubt about the existence of such an object: explicit simulation of the fast, weather components was thought to be unavoidable. What can be done, however, is to identify key subspaces of dynamical interest.

Currently, the observed empirical relationship between sea surface temperature and the North Atlantic Oscillation has better predictive skill than seasonal GCM forecasts. Clearly, some aspects of the climate system are predictable without explicit consideration of the fast, weather components, and there is a need to improve the predictive skill of GCMs for these phenomena.

## References

Abramov, R. V. and A. J. Majda (2007) Blended response algorithms for linear fluctuation-dissipation for complex nonlinear dynamical systems. *Nonlinearity* **20**, 2793-2821.

Abramov, R. V. and A. J. Majda (2009) A new algorithm for low-frequency climate response. *Journal of the Atmospheric Sciences* **66**, 286-309.

Gritsun, A., G. Branstator and A. Majda (2008) Climate response of linear and quadratic functionals using the fluctuation-dissipation theorem, *Journal of the Atmospheric Sciences* **65**, 2824-2841.

Lorenz, E. N. (2006) Predictability – a problem partly solved. Chapter 3 of *Predictability of weather and climate* (Tim Palmer and Renate Hagedorn, Eds), pp 40-58, Cambridge University Press.

Palmer, T. N. and P. D. Williams (2008) Introduction: Stochastic Physics and Climate Modelling. *Philosophical Transactions of the Royal Society* A, **366**(1875), 2421-2427.