WITH A FOREWORD BY DARA O BRIAIN

VISIONS OF MATHEMATICS

EDITED BY SAM PARC

OXFORD

CHAPTER 45

Leapfrogging into the future: How child's play is at the heart of weather and climate models

PAUL WILLIAMS

To most people, leapfrog is a traditional children's game. Depending on your age, you probably have fond memories of playing it when you were at school, or perhaps you still play it today. To play, one participant bends over, and another participant runs up from behind and vaults over the first participant's stooped back. Games of this sort have been played since at least the 16th century (see Fig. 45.1).

According to the Oxford English Dictionary, the first recorded use of the word 'leapfrog' dates from 1600, appearing in William Shakespeare's *Henry V*:

If I could win thee at leapfrog,
Or with vawting with my armour on my backe,
Into my saddle,
Without brag be it spoken,
Ide make compare with any.

Leapfrog is played around the world but different countries name it after different leaping animals. It is called 'leapsheep' (*saute-mouton*) in French, 'leaphorse' (*umatobi*) in Japanese, and 'goat' (*capra*) in Romanian.

What has a sixteenth-century children's game got to do with modern mathematics? To mathematical scientists like me, leapfrog is not just a children's game, but also a mathematical technique that is named after the vaulting children. The technique is at the heart of modern weather and climate prediction.

At around the time of the formation of the Institute of Mathematics and its Applications, atmospheric scientists were busy developing the first ever computer models of the weather. These models encapsulate our understanding of the physical laws that govern the atmosphere and ocean, represented as nonlinear mathematical equations. Over time the models have evolved into sophisticated computer codes, containing millions of lines of instructions and requiring hundreds of trillions of calculations a second. Only the world's fastest supercomputers are capable of

Fig 45.1 Detail from *Children's Games* (1560) by the Flemish Renaissance artist Pieter Bruegel the Elder.

performing these calculations. Today, the models are used routinely by national meteorological offices to forecast the weather and to predict the climate.

Children do not have the monopoly on playing leapfrog: weather and climate models do it too. The role of the leapfrog in models is to march the weather forward in time, to allow predictions about the future to be made. In the same way that a child in the playground leapfrogs over another child to get from behind to in front, the models leapfrog over the present to get from the past to the future. In mathematical terms, the time derivative at the present step, which represents the instantaneous rate of change of the atmospheric and oceanic variables, is used to extrapolate forward linearly from the past step to the future step.

The models must proceed in the above manner because, although time in the real world flows continuously from one moment to the next, time in computer simulations is divided up into discrete chunks. In mathematical terms, the differential equations governing the evolution of the atmospheric and oceanic fluid flow are discretised in time, to turn them into algebraic finite-difference equations. The task of forecasting tomorrow's weather is broken up into hundreds of mini-forecasts, each of which advances the prediction by just a few minutes.

Anyone who has ever played leapfrog in the playground will know that it is all too easy to become unstable and fall over. Leapfrogging over a single participant is usually not a problem, but leapfrogging over many participants in quick succession is riskier. The same is true of the mathematical leapfrog in computer models. Each leapfrog step takes the model further into the future but also increases the instability that is inherent in leapfrogging. Eventually, the instability becomes too large and the model is said to crash.

A brilliant Canadian atmospheric scientist called André Robert (1929–1993) fixed this problem 45 years ago, by devising a sort of mathematical glue that holds the model together. The glue is now called the Robert filter and the stickiness stops the instability from growing. How it does this is best described by analogy with the children's game. The filter basically creates subtle forces, which push downwards on the leapfrogger when he or she leaps too high, and which push upwards when he or she starts falling to the ground. The invisible hand of the Robert filter keeps the leapfrogger on a stable trajectory. The filter has been used in most weather and climate models for nearly five decades and it continues to be used widely today.

Unfortunately, although the subtle forces of the Robert filter stabilise the leapfrog technique and prevent the model from crashing, they also introduce errors into the model. These errors are a problem because weather and climate models must be as accurate as possible, given the importance of meteorological events to society and the economy. For example, research published in the *Bulletin of the American Meteorological Society* has calculated that the value of weather forecasts in the USA is $31.5bn annually. The US National Oceanic and Atmospheric Administration has estimated that the cost of underpredicting or overpredicting electricity demand due to poor weather forecasts is several hundred million dollars annually. The US Air Transport Association has calculated that air traffic delays caused by weather cost about $4.2bn annually, of which $1.3bn is potentially avoidable by improved weather forecasts.

In addition to the economic costs of errors in models, there are also societal costs. Many atmospheric and oceanic phenomena represent critical perils for society because of their vast destructive power. Examples include hurricanes, typhoons, mid-latitude wind storms, tsunamis, and El Niño events. These phenomena can lead to various problems, such as flooding, landslides, droughts, heatwaves, and wildfires. Improving the accuracy of weather models is important because it helps to minimise the societal impact of these extreme events. Improving the accuracy of climate models is equally important because their predictions are used in international political negotiations.

For all the above reasons, the errors caused by Robert's 45-year-old invisible hand needed to be fixed. What was needed was a pair of hands, each pushing in opposite directions at slightly different times. Although the force applied by each hand causes an error, the forces applied by the pair of hands are equal and opposite. They cancel each other out, resulting in no net force. This cancellation greatly reduces the errors caused by the Robert filter, and increases the accuracy of the model. In a sense, a different kind of glue is used to prevent the leapfrog instability and hold the model together. Moreover, the method is easy to include in an existing model, requiring the addition of only a couple of lines of computer code.

Since the modified Robert filter was developed, it has been found to increase the accuracy of weather forecasts and ocean simulations significantly. In one atmospheric model, predictions of the weather five days ahead with the modified filter were found to be as accurate as predictions of the weather four days ahead with the original filter. Therefore, the modified filter has added a crucial extra day's worth of predictive skill to tropical medium-range weather forecasts. The modified filter is currently being included in various other atmosphere and ocean models around the world, including models of storms, clouds, climate, and tsunamis. It is somewhat humbling that this mathematical and scientific advance had its roots partly in a simple children's game that I used to play when I was at school. Sometimes, scientific progress really is child's play!

FURTHER READING

[1] Jacob Aron (2010). *Mathematics matters: Predicting climate change*. IMA/EPSRC leaflet, also published in *Mathematics Today*, December, pp. 302–303.

[2] Damian Carrington (2011). How better time travel will improve climate modelling. *The Guardian*, 4 February 2011. <http://www.guardian.co.uk/environment/damian-carrington-blog/2011/feb/04/time-travel-climate-modelling> (site accessed January 2013).
[3] Paul Williams (2009). A proposed modification to the Robert–Asselin time filter. *Monthly Weather Review*, vol. 137, pp. 2538–2546.

Source

This article was the winning entry in a competition run in conjunction with *Mathematics Today*, the magazine of the UK's Institute of Mathematics and its Applications.