

1 **The implications of an idealised large-scale circulation for mechanical work**  
2 **done by tropical convection**

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## ABSTRACT

11 A thermodynamic analysis is presented of an overturning circulation simu-  
12 lated by two cloud resolving models, coupled by a weak temperature gradient  
13 parametrisation. Taken together, they represent two separated regions over  
14 different sea surface temperatures, and the coupling represents an idealised  
15 large-scale circulation such as the Walker circulation. It is demonstrated that a  
16 thermodynamic budget linking net heat input to the generation of mechanical  
17 energy can be partitioned into contributions from the large-scale interaction  
18 between the two regions, as represented by the weak temperature gradient  
19 approximation, and from convective motions in the active warm region and  
20 the suppressed cool region. Model results imply that such thermodynamic  
21 diagnostics for the aggregate system are barely affected by the strength of  
22 the coupling, even its introduction, or by the SST contrast between the re-  
23 gions. This indicates that the weak temperature gradient parametrisation does  
24 not introduce anomalous thermodynamic behaviour. We find that the vertical  
25 kinetic energy associated with the large-scale circulation is more than three  
26 orders of magnitude smaller than the typical vertical kinetic energy in each  
27 region. However, even with very weak coupling circulations, the contrast be-  
28 tween the thermodynamic budget terms for the suppressed and active regions  
29 is strong and is relatively insensitive to the degree of the coupling. Addition-  
30 ally, scaling arguments are developed for the relative values of the terms in  
31 the mechanical energy budget.

## 32 1. Introduction

33 The representation of the interactions between large-scale tropical circulations and local  
34 convective processes is an issue of fundamental importance to the simulation of tropical circula-  
35 tions. Computational constraints mean that models that explicitly simulate both convection and  
36 large-scale motion over domains of appropriate size can rarely be used. Hence a strategy, often  
37 adopted in the development of convection parametrisations for the tropics for GCMs, is to carry  
38 out studies using cloud resolving models (CRMs) that explicitly model convection in conjunction  
39 with a parametrisation of the large-scale dynamics that is influenced by local conditions. The  
40 weak temperature gradient approximation (Sobel and Bretherton 2000; Raymond and Zeng 2005)  
41 is one such parametrisation.

42  
43 The atmospheric circulation can be analysed from the perspective of a generalized Carnot heat  
44 engine, that converts a temperature difference between two reservoirs into mechanical energy  
45 (e.g. Emanuel (1986) for the case of tropical cyclones). It has been shown by Pauluis (2011) that  
46 the impact of the hydrological cycle is to reduce the efficiency of such a conversion relative to  
47 the generalized Carnot maximum. This paper addresses convective and large-scale circulations  
48 as heat engines, unpacking how efficient they are at transforming potential work into kinetic  
49 energy dissipation and water lifting, in the face of the thermodynamic penalties associated with  
50 irreversible phase transitions in a moist atmosphere. Hitherto, similar analyses have not been  
51 applied in a context where the influence of large-scale circulations on local convection is modeled  
52 separately. Such an approach has the potential not only to clarify the consequences of different  
53 large-scale parametrisation approaches and to identify constraints on them but also to shed light  
54 on the nature of the interaction between the large-scale and the local dynamics. In particular, we

55 examine whether the mechanical energy budget is well-behaved when two convecting regions are  
56 coupled by a parametrised large-scale circulation.

57

58 The Weak Temperature Gradient (WTG) approximation was first suggested by Sobel and  
59 Bretherton (2000) and subsequently modified by introducing a short relaxation time (e.g.  
60 Raymond and Zeng 2005). Alternative methods such as the damped gravity wave (DGW)  
61 approach (Kuang 2008; Romps 2012), a spectral variant of the WTG (Herman and Raymond  
62 2014) and others have been proposed. In this paper we focus on the weak temperature gradient  
63 approximation, but work on the DGW approach is ongoing. Preliminary analysis suggests that  
64 the DGW approach would give similar results to those presented in this paper.

65

66 A recent project has compared the results produced by different implementations of some of  
67 these methods (Daleu et al. 2015b, 2016), suggesting that the WTG approach is more prone to  
68 multiple equilibria than the DGW approach, and that the latter produces smoother large-scale  
69 vertical velocities. One of the advantages of the WTG approach is that its simplicity permits an  
70 analytical approach, and for that reason it is used here.

71

72 A common formulation of the WTG approximation assumes that a large-scale vertical wind is  
73 prescribed so as to contribute to the elimination of horizontal temperature contrasts. This paper  
74 adopts a typical definition, where the prescribed vertical large-scale wind  $w_{LS}$  is:

$$w_{LS} \frac{\partial \theta_r}{\partial z} = \frac{\theta'}{\tau} \quad , \quad (1)$$

75 where  $\theta_r$  represents a reference profile for potential temperature,  $\theta'$  a local potential temperature  
76 anomaly,  $\tau$  a relaxation timescale, and  $z$  the height above sea level.  $w_{LS}$  is usually interpolated

77 to zero at the surface from the predicted value at the top of the planetary boundary layer and a  
78 minimum value for the static stability imposed. The time scale,  $\tau$ , represents the strength of the  
79 coupling between regions and typical values of 2-3 hours can be justified as the spatial scale of  
80 the system being modeled ( $\sim 500$  km) divided by the speed of the fastest internal gravity waves  
81 ( $\sim 50$  m/s).

82

83 This circulation can be superimposed on the normal convective motions representing an ide-  
84 alised large-scale circulation that provides a linkage between two regions. Most studies have used  
85 a reference column approach, whereby the large-scale circulation is determined by temperature  
86 differences between the area modeled and an assumed environmental profile; more recently Daleu  
87 et al. (2012, 2015a) extended this approach to two coupled regions, which enables a more explicit  
88 representation of the consequences of the large-scale coupling mechanism. The reference column  
89 approach implies an infinite reservoir of energy and entropy, whereas by using coupled regions it  
90 is possible to analyse the mechanical energy budget of a closed system, separately attributing the  
91 contribution of the large-scale circulation described above and of the residual convective motions  
92 in the two regions.

93

94 The approach to the diagnosis of the mechanical work done by convection adopted in this paper  
95 broadly follows the isentropic perspective introduced by Pauluis and Mrowiec (2013) and Pauluis  
96 (2016). The conditional average of the vertical mass flux of air parcels,  $\langle \rho w \rangle$ , where  $\rho$  is the  
97 density of dry air and  $w$  is vertical velocity, is calculated in  $(z, \theta_e)$  space,  $\theta_e$  being the equivalent  
98 potential temperature. This is illustrated in Figure A1a. This conditionally averaged vertical mass

99 flux can be used to derive a streamfunction  $\Psi$  (Figure A1b):

$$\Psi(z, \theta_e) = \int_0^{\theta_e} \langle \rho w \rangle(z, \theta'_e) d\theta'_e \quad . \quad (2)$$

100 This describes mass flows in  $(z, \theta_e)$  space and can be regarded as a projection onto thermody-  
 101 namic variables of the overturning circulation. It may be treated as a streamfunction providing  
 102 divergent flows in that space do not exist. Likewise, conditional averages in  $(z, \theta_e)$  space of  
 103 variables such as temperature ( $T$ ), vapour mixing ratio ( $r_v$ ), liquid water mixing ratio ( $r_l$ ), ice  
 104 mixing ratio ( $r_i$ ), entropy ( $S$ ) and buoyancy ( $B$ ) can be obtained. See Pauluis and Mrowiec (2013)  
 105 for a further discussion of the merits of this approach.

106  
 107 Pauluis (2016) derives an approximate expression which relates components of work produced  
 108 by a closed contour of the streamfunction which can be expressed as:

$$\underbrace{\oint T dS}_{W_{TdS}} - \underbrace{\sum_{w=v,l,i} \oint (-\mathcal{G}_w) dr_w}_{\Delta \mathcal{G}} \approx \underbrace{\oint B dz}_{W_b} + g \underbrace{\oint r_T dz}_{W_p} \quad , \quad (3)$$

109 where  $r_T = r_v + r_l + r_i$  is the total water mixing ratio,  $g$  represents the acceleration due to gravity  
 110 and  $\mathcal{G}_w$  the specific Gibbs free energy for the various phases of water represented in the model.  
 111  $W_{TdS}$ , the first item on the left hand side of this equation, can be interpreted as the work that  
 112 would be done by a generalized Carnot engine with the same path in  $(T, S)$  coordinates. The  
 113 second term,  $\Delta \mathcal{G}$ , referred to as the “Gibbs penalty”, is the lost part of that work that is needed to  
 114 evaporate liquid water at below saturation into the system in order to maintain the mean humidity  
 115 profile against the hydrological cycle and is discussed in detail in Pauluis and Held (2002a,b)  
 116 and Pauluis (2011). The first term on the right hand side,  $W_b$ , is the generation of kinetic energy  
 117 by resolved motions in the model and the second,  $W_p$ , the increase in geopotential energy of the  
 118 water content lifted by convection. Ultimately, the buoyancy term  $W_b$  generates kinetic motion

119 of air which is dissipated through turbulent processes mostly near the surface and the moisture  
120 elevator term  $W_p$  generates potential energy in hydrometeors which is subsequently dissipated  
121 through friction during precipitation (Pauluis et al. 2000). The Gibbs penalty term does not  
122 correspond to external energy transiting through the system, but serves to limit the work available  
123 for conversion into mechanical energy. In this sense, the Gibbs penalty is the part of the entropy  
124 budget that needs to be expended to maintain the composition changes in the hydrological cycle,  
125 and that therefore cannot be used to produce mechanical work.

126

127 The terms in this equation can be represented by thermodynamic diagrams of streamfunction  
128 contours in the appropriate spaces, for example that for  $W_{TdS}$  in  $(S, T)$  space. A weighted total of  
129 integrals along equally spaced contours of  $\Psi$  is used to calculate values of the terms in Eq. 3 for  
130 the entire system. Figure A2 shows contours for the most significant terms in the case of a single  
131 region in radiative-convective equilibrium for the model configuration described below.

132

133 This paper applies the diagnostic approach outlined above to unpack the generation of me-  
134 chanical work by a modeled overturning circulation superimposed on convective processes. In  
135 particular it analyses the relationship between the strength of WTG coupling and the strength  
136 of the circulation, partitioned into contributions from the large-scale circulation and convective  
137 circulations in relatively cool/dry and warm/moist regions, in order to deepen understanding of  
138 how the large-scale circulation can suppress or enhance convection. The impact of varying the  
139 SST difference between the regions is also analysed.

140

141 The paper is organised as follows. Section 2 explains how an isentropic analysis can be applied  
142 to two coupled regions and also suggests a refinement that improves the accuracy of the decom-

position for an anelastic model. Section 3 briefly describes the CRM used and presents numerical results for the mechanical energy budget, partitioned into a large-scale circulation and convective localised flows for varying strengths of coupling as well as further results for varying SST differences between the two regions. Section 4 develops an analytical expression for the generation of kinetic energy by the WTG coupling, which is seen to be consistent with results obtained from the CRM. Scaling arguments are also developed for comparison with other results obtained by the CRM. The implications of this study are discussed and some conclusions are drawn in Section 5.

## 2. Isentropic diagnostic framework

### *a. General approach*

As in Pauluis (2016), entropy is expressed as specific entropy per unit mass of dry air:

$$s = s_d + r_v s_v + r_l s_l + r_i s_i \quad , \quad (4)$$

where  $s_d$  is the specific entropy of dry air, and  $s_w$  for  $w = v, l, i$  are the specific entropies of the three phases of water. In order to achieve a one-to-one relationship between equivalent potential temperature and this specific entropy, the former is defined as:

$$\theta_e = T_0 \ln \frac{s}{c_p} \quad , \quad (5)$$

where  $T_0$  is the temperature at the triple point of water and  $c_p$  is the heat capacity of dry air at constant pressure.

Pauluis (2016) uses a weighted average of equally spaced contours of the streamfunction over the region where  $\Psi < 0$  to calculate the components of overall work throughput of the system as expressed in Eq. 3. Following that approach, these integrals are calculated here along contours of the streamfunction in  $(z, \theta_e)$  space, using the conditionally averaged values of the variables required for the calculation (such as  $T, S$  for  $W_{TdS}$  etc.) of the points defining the trajectory, rather



163 than seeking first to determine a streamfunction in  $(T, S)$  space and then to calculate contour inte-  
 164 grals in that space. The same results can also be calculated using appropriate surface integrals of  
 165 the streamfunction, e.g. the first term on the left hand side of Eq. 3 becomes  $\iint \Psi dSdT$  over the  
 166 same domain. Although the results shown in this paper are calculated using contours, the surface  
 167 integral approach is important in justifying the partitioning approach described below.

### 168 *b. Extension to two region case*

169 This paper applies the decomposition of work done discussed above to a two-region system,  
 170 where the regions are coupled by a large-scale circulation consisting of vertical winds specified by  
 171 the WTG approximation. The cooler region is identified by a subscript 1 and represents a fraction  
 172  $1 - \varepsilon$  of the total area, and the warmer region (subscript 2) a fraction  $\varepsilon$  (all the numerical results  
 173 shown in this paper are based on  $\varepsilon = 0.5$ ). When a temperature difference between the surfaces  
 174 underlying the two regions gives rise to a large-scale circulation, there will be mass transfers  
 175 between the two regions and the contours of the integral of the vertical mass flux (as in Eq. 2) in  
 176 each region will not be closed, making it impossible to estimate the components of Eq. 3 within  
 177 each region using either contour or surface integrals (the latter requiring a domain bounded by a  
 178 constant value of the streamfunction).

179  
 180 One could, of course, merge the two regions prior to performing any analysis, but this introduces  
 181 significant error as the dynamics are generally influenced by local rather than merged values of  
 182 fields (e.g. buoyancy for  $W_b = \oint B dz$ ), and such a method will not permit a partitioning of the  
 183 components of work done to localised features in each region and to the large scale circulation.

185 Instead, one can identify and separate out the vertical large-scale velocities  $w_{LS,1}(z), w_{LS,2}(z)$   
 186 (obtained from the values of  $\Psi(z, \theta_e)$  as  $\theta_e \rightarrow \infty$  and the dry air mass distribution in each region).  
 187 The circulations calculated using Eq. 2 for convective local velocities  $w_i^* = w_i - w_{LS,i}$  ( $i = 1, 2$ )  
 188 in each region will now be closed, as will that in a notional third “region” which consists of the  
 189 total of the large-scale circulation within the two regions, weighted by their relative areas, since  
 190  $(1 - \varepsilon)w_{LS,1} + \varepsilon w_{LS,2} = 0$ . These streamfunctions are additive by construction:

$$\Psi_1 + \Psi_2 = \Psi_1^* + \Psi_2^* + \Psi_{LS} \quad , \quad (6)$$

191 where  $\Psi_{1,2}$  are the observed circulations in each region,  $\Psi_{1,2}^*$  represent the adjusted local circula-  
 192 tions and  $\Psi_{LS}$  is the large-scale circulation. The respective surface integrals will also be additive,  
 193 providing that a domain is selected that is bounded by a contour for which all the  $\Psi = 0$  and that  
 194  $\Psi \leq 0$  throughout that domain (which should generally be possible for a sufficiently smooth sys-  
 195 tem). This additivity is important because it means that the partitioning of work in Eq. 3 can also  
 196 be extended to the components of the coupled two-region system. For example:

$$W_{TdS} = \iint \Psi_1^* dSdT + \iint \Psi_2^* dSdT + \iint \Psi_{LS} dSdT \quad (7)$$

197 Since the surface integrals are additive, a calculation of these quantities using contour integrals  
 198 will also be additive. An example of this partitioning can be seen in Figure A3.

199  
 200 As has been mentioned, these integrals are calculated for the entire system using weighted tra-  
 201 jectories along contours of the streamfunction in  $(\theta_e, z)$  space, and with averaged values for the  
 202 other quantities required for the calculation. The values are calculated separately for each region  
 203 while for the large-scale circulation a mean value is taken, weighted by the contribution of each  
 204 region to that point in the combined thermodynamic space. As a result of this, the calculation of  
 205 the large-scale terms is more approximate.

### 206 *c. Anelastic case*

207 The derivation of Eq. 3 uses the hydrostatic approximation for the reference profile, where the  
208 total pressure of all gases in the atmosphere (i.e. dry air and water vapour) is related to height. In  
209 the case of an anelastic model, such as the Met Office’s Large Eddy Model (LEM) which is used  
210 to derive the numerical results in this paper, it is the partial pressure of dry air that is assumed to  
211 be in hydrostatic equilibrium in the reference profile, and as will be seen the balance of Eq. 3 can  
212 be improved by a small correction which is conveniently included in the water vapour component  
213 of the Gibbs penalty term and is derived in Appendix A.

## 214 **3. Model description and results**

### 215 *a. Model setup*

216 The numerical simulations in this section have been produced using a configuration of version  
217 2.4 of the Met Office’s Large Eddy Model (LEM) in cloud-resolving mode amended to inves-  
218 tigate the effect of coupling two regions via the WTG in an idealised context. The underlying  
219 LEM is described in Gray et al. (2001) and more complete details of the configuration used  
220 here can be found in Daleu et al. (2012). The microphysics are represented by a five-category  
221 prognostic scheme, with mixing ratios for cloud water, rain, ice, graupel and snow and with  
222 number concentrations for ice, graupel and snow. Each region is modelled in a two-dimensional  
223 configuration with a width of 128km (resolution of 500m) and a height of 20km (60 vertical  
224 levels). A fixed tropospheric cooling profile is used rather than interactive radiation; In order  
225 to isolate the effect of the coupling strength, a fixed tropospheric cooling profile is used rather  
226 than interactive radiation. The cooling rate is fixed at 1.5 K/day below 220 hPa, and decreases  
227 linearly with pressure to 0 K/day at 120 hPa, corresponding to integrated atmospheric cooling

close to  $150\text{Wm}^{-2}$ , depending on the precise value of the surface pressure. The sub-grid model is parametrised using a modified first order Smagorinsky-Lilly approach. For consistency with previous studies with this model a mixing length of 250m was used, but the impact of this choice on the conclusions of this paper was tested and found to be minor.

The WTG velocity is specified by the region-mean potential temperature difference between the two regions and is used to perform advection of both temperature and moisture between the two regions. The two regions are of equal size and the temperature difference at the surface between them,  $\Delta T_s$ , unless otherwise specified, is set to 2K (302.7K and 304.7K for each region). The LEM is run to equilibrium for a 120 day duration (although no diurnal cycle is modeled), with the first 20 days discarded from the analysis to avoid any transient features, and results are obtained every 30 minutes, giving 4,800 samples of the required variables.

The LEM includes a considerable number of approximations, particularly with respect to the thermodynamics of the system. For example, the specific heat capacities of liquid water and water vapour are taken to be equal (and effectively zero) and the specific enthalpy of water vapour  $L_v$  does not depend on temperature; these are not consistent with the more exact approach underlying Eq. 3 that is implemented in the diagnostics. Although these approximations are not ideal, the application of a more sophisticated diagnostic framework to motions and variable values derived from a simplified model still provides insight into the full thermodynamic processes that are approximated in the LEM. This choice of model configuration involves a number of compromises that facilitate multiple long simulations over a more detailed representation of the features being modelled, consistent with the idealised nature of the experiments described.

251 *b. Results for single region*

252 The model was first run for an uncoupled single region over 302.7K sea-surface temperature.  
253 The resulting thermodynamic diagrams (Figure A2) have already been mentioned. Values for the  
254 work terms can be seen in Table A1, in which the effect of the modification described in Appendix  
255 1 is also identified. For comparison, the reference case values calculated in Pauluis (2016, Table  
256 1) are also shown.

257  
258 It will be seen that the values obtained here are in general some 30% higher than those in Pauluis  
259 (2016). Comparison of the  $W_p$  thermodynamic diagrams (Figure A2b in this paper and 4c in  
260 Pauluis (2016)) indicates that the atmosphere modeled here has a significantly higher total water  
261 mixing ratio, partially explained by the choice of model parameters to represent conditions in the  
262 Western Pacific – for example the SST for these simulations is 2.2K higher than in Pauluis (2016).  
263 Likewise, the  $W_{TdS}$  diagram (Figure A2d in this paper and 4a in Pauluis (2016)) shows higher  
264 values for entropy, and slightly higher temperatures. The relative values of the mechanical work  
265 components are consistent between the two models, and consistent with scaling arguments which  
266 are developed below.

267 *c. Dependency of components of work done on strength of coupling*

268 In order to investigate the impact of the coupling on the generation of mechanical work in the  
269 two regions, simulations were performed with values of the WTG timescale  $\tau$  between 1 hour  
270 and 50 hours; results for two uncoupled regions are also shown for comparison ( $\tau = \infty$ ). It will  
271 be seen that values of the components of work done for the total system remain broadly constant  
272 (Figure A4a), although the introduction of coupling produces a marginal increase in  $W_b$  and  $W_{TdS}$

which then decreases as the coupling increases.

Values for the local circulations in each of the regions (Figures A4c and A4d) on the other hand exhibit a significant difference from the overall mean. This difference is already marked at very weak coupling and increases as the value of  $\tau$  decreases, although the values for the cooler region remain almost constant for  $\tau < 10$  hrs, possibly reflecting the fact that the region approaches a humidity minimum (Figure A5). Column integrated water vapour in that region decreases significantly between  $\tau = 50$  hours and  $\tau = 10$  hours and then shows less variation, indeed a slight increase for  $\tau < 5$  hours. As the cooler region dries, the relative importance of the Gibbs penalty component will increase. Weaker convection is associated with a disproportionate decrease in the buoyancy component, as discussed in Pauluis (2016). Figure A6 shows the vertical mass flux for the two regions for  $\tau = 50$  hrs which confirms that convection in the cooler region is markedly suppressed even for such weak coupling.

The large-scale circulation (Figure A4b) shows a different pattern; this includes both regions, which maintain a temperature difference, and hence  $W_b$  plays a more prominent role, reducing as the coupling increases and the temperature difference decreases. This decrease is partly compensated by a slight increase in  $W_p$  as a stronger circulation lifts more moisture. The Gibbs penalty increases with the coupling up to a maximum at around  $\tau = 5 - 10$  hrs before decreasing; this can be attributed to increasing coupling first leading to drying of the cooler region and hence to a decrease in the relative humidity at which energy enters the system (see the scaling arguments below for how this impacts  $\Delta\mathcal{G}$  non-linearly). As discussed above, as  $\tau$  decreases below 5 hours the cooler region marginally moistens which contributes to the increase in  $\Delta\mathcal{G}$ . The mechanical work terms for components of the large-scale circulation for strong coupling

are similar in magnitude to those for the localised circulation in the cooler region; both are substantially smaller than those for the warmer region. As previously mentioned, the balance in the large-scale circulation is more approximate than for the other components, and in fact is not improved by inclusion of the correction term described in Appendix A.

#### *d. Components of kinetic energy*

The relationship between the strength of the coupling and components of kinetic energy in the total system is shown in Figure A7. The coupling has an insignificant effect on the kinetic energy in the aggregate system, but a strong influence on the vertical kinetic energy associated with the large-scale circulation, which increases with coupling strength. The increase in vertical kinetic energy for smaller  $\tau$  indicates that the effect of reducing  $\tau$  in increasing  $w$  in Eq. 1 is stronger than the effect that enhanced coupling has on reducing the temperature contrast  $\theta'$  between the regions.

A similar comparison between large-scale and convective horizontal components of kinetic energy would require further assumptions as to the distance between the two regions and their geometry. From a thermodynamic perspective, horizontal motion is of less relevance to the generation of mechanical energy, as it traverses only a small temperature contrast (as forced by the coupling). It is true to say that the horizontal near-surface flow will likely contribute substantially to the total frictional dissipation in the real system, but in our modelled system this dissipation is constrained to occur in the two regions.

319 This comparison between large-scale and convective vertical kinetic energies indicates how  
 320 weak the large-scale circulation is when compared with convective motions. Despite this, the  
 321 introduction of a large-scale circulation through WTG coupling has very noticeable effects on the  
 322 strength and nature of the convection within the two regions. In the absence of this large-scale  
 323 circulation the mechanical energy budgets in the two regions are very similar. However, even  
 324 with a very weak large-scale overturning circulation, convection in the warm region is markedly  
 325 enhanced and that in the cool region suppressed.

#### 326 *e. Dependence on SST difference between the regions*

327 A further set of numerical experiments was performed for strong coupling ( $\tau = 2$  hours) where  
 328 the temperature difference between the two regions was varied between 0K and 2K. In each exper-  
 329 iment, the regions were of equal area and the mean SST of the aggregate system was 303.7K. Plots  
 330 of the components of the mechanical budget are shown in Figure A9. As in the experiment with  
 331 varying coupling strength, the values of the four components of the mechanical work budget re-  
 332 main broadly constant for the system in aggregate, independent of the SST difference. The values  
 333 for the cool and warm regions coincide for  $\Delta T_s = 0$  and then diverge gradually. The components of  
 334 the budget for the large-scale circulation exhibit a quadratic dependency on  $\Delta T_s$  redfor low values  
 335 of  $\Delta T_s$  (see also the log plot in Figure A10) which is discussed below.

## 336 **4. Analytical expressions and scaling**

### 337 *a. Energy conversion under the Weak Temperature Gradient Approximation*

338 The simple form of the WTG means that it is possible to derive an analytical expression for  
 339 the rate of conversion of potential energy into kinetic energy that it implies. Following Romps  
 340 (2012) one can express the thermodynamic equation for a one-dimensional WTG system in height



341 coordinates as follows:

$$\frac{\partial B}{\partial t} = -N^2 w + Q \quad , \quad (8)$$

342 where  $B = \frac{g\theta'}{\theta}$  is the dry buoyancy ,  $N$  is the local Brunt-Väisälä frequency ( $N^2 = -\frac{g}{\theta} \frac{\partial \theta}{\partial z}$ ), and  
 343  $Q$  is the external heat input into the system. It is assumed that  $N$  does not have any dependency on  
 344 time although it can vary with height.

345 If the WTG approximation is used, the vertical velocity (above the boundary layer) can be ex-  
 346 pressed as:

$$w = \frac{\theta'}{\tau \frac{\partial \theta}{\partial z}} = \frac{B}{\tau N^2} \quad . \quad (9)$$

347  
 348 The Available Potential Energy (APE) introduced by Lorenz (1960),  $\bar{A}$ , of the system can be  
 349 described locally in terms of these variables by dry “availability”  $A = \frac{B^2}{2N^2}$ . It can be shown that  
 350 the density-weighted column integral of  $A$  is equal to  $\bar{A}$ . The time derivative of the local availability  
 351 is given by:

$$\frac{\partial A}{\partial t} = \frac{2B}{2N^2} \frac{\partial B}{\partial t} = \frac{BQ}{N^2} - Bw = \frac{BQ}{N^2} - \frac{2A}{\tau} \quad . \quad (10)$$

352 This can be interpreted as indicating that availability is increased by a differential heating term  
 353  $\frac{BQ}{N^2}$  which is balanced by a conversion term from potential to kinetic energy of:

$$C_{A \rightarrow KE} = \frac{2A}{\tau} \quad . \quad (11)$$

354 Thus for the system as a whole (ignoring the complication of the boundary layer interpolation)  
 355 the conversion to kinetic energy will be  $\frac{2\bar{A}}{\tau}$ . It will also be noted that Eq. 10 requires heat input to  
 356 correlate with positive buoyancy, which is consistent with the second law of thermodynamics. The  
 357 effect of the boundary layer interpolation is to weaken the coupling in the lowest layers, which can

358 be represented by increasing the value of  $\tau$  for those layers - the impact of this adjustment is minor.

359

360 Figure A8 compares the conversion term from Eq. 11 and the large-scale components of the  
 361 mechanical energy budget as in Figure A4b, which of course includes moist variables. The me-  
 362 chanical work done terms  $W_b$  scales as  $\frac{2\bar{A}}{\tau}$ , in fact the values coincide. As the APE has a quadratic  
 363 dependency on the temperature difference between the two regions (which will in turn depend lin-  
 364 early on the temperature difference at the surface), a quadratic dependency of the work terms on  
 365  $\Delta T_s$  may be anticipated. This is indeed seen in Figure A9b as presented in Section 3e - a log-log  
 366 plot of the same values is shown in Figure A10.

### 367 *b. Scaling for components of work done*

368 Further scalings can be developed for the relative values of the terms in the mechanical energy  
 369 budget. These do not depend on the assumption of a coupling mechanism. In an anelastic model  
 370 such as the LEM, the buoyancy (see Gray et al. (2004, Eq. 11 - 13)) is calculated as:

$$B = g \left( \frac{\theta'}{\theta_r} + \left( \frac{R_v}{R_d} - 1 \right) r_v - \sum_{w \neq v} r_w \right) , \quad (12)$$

371 where  $R_v$  and  $R_d$  are the specific gas constants for water vapour and dry air and  $\theta_r$  is the  
 372 reference-state potential temperature. Hence, if we assume that variations in specific humidity  
 373 dominate the buoyancy rather than any temperature differences as is suggested by these model  
 374 results, and also that  $\delta r_T \approx \delta r_v$ , only the second term in Eq 12 need be considered for scaling  
 375 purposes. Hence the buoyancy lifting term,  $W_b$  in Eq. 3, will scale as  $(\frac{R_v}{R_d} - 1)W_p = 0.6 \times W_p$ ,  
 376 with  $W_p$  being the moisture lifting term. The approximate scaling between  $W_b$  and  $W_p$  can be  
 377 readily verified from the numerical values shown in Table A1 and also holds for the warmer  
 378 region for the two-region system (Figure A4d) which dominates the aggregate system (Figure

379 A4a). In circumstances where the temperature differences are significant (and correlated with  
 380 moisture anomalies), such as for the large-scale circulation at low coupling strength one could  
 381 expect  $\frac{W_b}{W_p}$  to have a value greater than 0.6 as is observed (Figure A4b).

382

383 Pauluis (2011, Eq. 25) gives an expression for the Gibbs penalty. Assuming that condensation  
 384 occurs close to saturation, that vapour anomalies dominate as before and that the sensible heat  
 385 component is negligible, an approximate expression for this term can be written in terms of the  
 386 net rate of evaporation at the surface  $E = \frac{\Delta r_v}{\Delta t}$ :

$$\Delta \mathcal{G} \approx -R_v T_{\text{out}} E \ln \mathcal{H}_{\text{in}} \quad (13)$$

387 where  $\mathcal{H}$  is relative humidity and the subscripts  $_{\text{in}}, _{\text{out}}$  indicate conditional harmonic means over  
 388 the zones where heat enters and leaves the system. For example, if we write  $F_{\text{in}}$  for heat input and  
 389  $\mathcal{A}$  as the system's boundary then following Ambaum (2010, Eq. 10.50):

$$\frac{1}{T_{\text{in}}} = \left( \int_{\mathcal{A}} F_{\text{in}} d\mathcal{A} \right)^{-1} \int_{\mathcal{A}} \frac{F_{\text{in}}}{T} d\mathcal{A}. \quad (14)$$

390  $T_{\text{out}}$  is defined likewise using  $F_{\text{out}}$ , the heat output of the system, and  $\mathcal{H}_{\text{in}}$  is defined as in Pauluis  
 391 (2011, Eq. 16).

392 Likewise,  $W_{TdS}$  will be mainly driven by the latent heat input and scales with the generalized  
 393 Carnot efficiency:

$$W_{TdS} \approx \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} (L_v E + \Delta \mathcal{G}) \quad (15)$$

$$\approx \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} L_v E, \quad (16)$$

394 where  $L_v$  is the enthalpy of vaporisation of water. It will be seen from Eq. 13 that the Gibbs  
 395 penalty term can be neglected in scaling Eq. 15, given that  $\frac{R_v T_{\text{out}}}{L_v} \approx 0.05$ . Eliminating  $E$  between  
 396 Eqs 13 and 16:

$$\Delta\mathcal{G} \approx -\frac{R_v T_{\text{out}} T_{\text{in}}}{L_v(T_{\text{in}} - T_{\text{out}})} \ln \mathcal{H}_{\text{in}} W_{TdS} \quad . \quad (17)$$

Values of 295 K for  $T_{\text{in}}$  and 280 K for  $T_{\text{out}}$  can be obtained from the model (or even by inspection of Figure A2d which shows values for the RCE case, which will also be typical for the aggregate system). An average value for  $\mathcal{H}_{\text{in}}$  is more problematic to define, but using 60% is consistent with model results and the observed values of  $\Delta\mathcal{G} \approx 0.5W_{TdS}$ .

It is possible to develop Equation 17 further by making use of the Clausius-Clapeyron relation in the form  $\frac{de_s}{e_s} = \frac{L_v dT}{R_v T^2}$  :

$$\frac{\Delta\mathcal{G}}{W_{TdS}} \approx -\frac{R_v T_{\text{out}} T_{\text{in}}}{L_v(T_{\text{in}} - T_{\text{out}})} \ln \mathcal{H}_{\text{in}} \quad , \quad (18)$$

$$\approx \frac{\ln \mathcal{H}_{\text{in}}}{\ln \frac{e_{s,\text{out}}}{e_{s,\text{in}}}} \quad , \quad (19)$$

where  $e_{s,\text{in}}$  and  $e_{s,\text{out}}$  are the saturated vapour pressures of water at temperatures  $T_{\text{in}}$  and  $T_{\text{out}}$ . Defining  $r_{v,\text{in}}$  as the vapour mixing ratio corresponding to  $\mathcal{H}_{\text{in}}$  at temperature  $T_{\text{in}}$  and dry pressure  $p_{d,\text{in}}$ :

$$\frac{\Delta\mathcal{G}}{W_{TdS}} = \frac{\ln \frac{R_v r_{v,\text{in}} p_{d,\text{in}}}{R_d e_{s,\text{out}}} + \ln \frac{e_{s,\text{out}}}{e_{s,\text{in}}}}{\ln \frac{e_{s,\text{out}}}{e_{s,\text{in}}}} \quad , \quad (20)$$

We now introduce  $r_{v,\text{out}}$  via  $e_{s,\text{out}} = R_v/R_d p_{d,\text{out}} r_{v,\text{out}}$  where  $p_{d,\text{out}}$  is the partial pressure of dry air at temperature  $T_{\text{out}}$ . Substituting for  $e_{s,\text{out}}$  in the first logarithm in the numerator of Eq. 20 leads to:

$$\frac{\Delta\mathcal{G}}{W_{TdS}} = 1 - \frac{\ln p_{d,\text{out}}/p_{d,\text{in}}}{\ln e_{s,\text{out}}/e_{s,\text{in}}} - \frac{\ln r_{v,\text{out}}/r_{v,\text{in}}}{\ln e_{s,\text{out}}/e_{s,\text{in}}} \quad . \quad (21)$$

Assuming that  $r_{vs,out} \lesssim r_{v,in}$  (see Figure A2b to confirm that this is reasonable for all but the shallowest circulations):

$$\frac{\Delta \mathcal{G}}{W_{Tds}} \lesssim 1 - \frac{\ln \frac{p_{d,out}}{p_{d,in}}}{\ln \frac{e_{s,out}}{e_{s,in}}} \quad (22)$$

$$(23)$$

Making a Taylor expansion in  $\ln(1+x) \approx x$ :

$$\frac{\Delta \mathcal{G}}{W_{Tds}} \lesssim 1 - \frac{\frac{\Delta z}{H_{pd}^*}}{\frac{\Delta z}{H_{es}^*}} \quad (24)$$

$$\lesssim 1 - \frac{H_{es}^*}{H_{pd}^*} \quad (25)$$

$$\lesssim 1 - \frac{R_v g \bar{T}}{R_d L_v \Gamma} \quad , \quad (26)$$

where  $\Delta z$  is the height difference between the in and out states, and  $H_{es}^* = \frac{-R_v \bar{T}^2}{L_v \Gamma}$ ,  $H_{pd}^* = \frac{R_d \bar{T}}{g}$  are the scale heights for saturated vapour pressure and the partial pressure of dry air,  $\bar{T}$  is an average temperature over the cycle between  $T_{in}$  and  $T_{out}$  and  $\Gamma$  is the atmospheric lapse rate. Using  $\bar{T} = 290\text{K}$  and  $\Gamma = -5\text{km}^{-1}$  gives  $\frac{\Delta \mathcal{G}}{W_{Tds}} < 0.64$ . This ratio will decrease as  $\frac{r_{v,out}}{r_{v,in}}$  decreases, i.e. as the temperature difference between the “in” and “out” states increases.

418

Combining these various elements as in Eq. 3, one can expect the efficiency with which such a system in aggregate generates mechanical work  $W_b$  to be typically 20% of the generalized Carnot maximum  $W_{Tds}$ . The values shown in Table A1 and in Figure A4 are consistent with these scalings.

422

There is an imbalance in latent heat input between the regions, consistent with the transport of moisture from the cool region to the warm region. The Gibbs penalty component of the mechanical energy budget for the large-scale circulation is strongly correlated with this moisture transport between the regions (Fig. A11). The relationship between these two quantities can be explained

426

427 using Eq. 13, using the net moisture advected by the large-scale circulation:

$$\Delta\mathcal{G} \approx \frac{-R_v}{L_v} T_{\text{out}} \ln \mathcal{H}_{\text{in}} \times \text{latent heat transport} \quad . \quad (27)$$

428 Using  $T_{\text{out}} = 285\text{K}$  and  $\mathcal{H}_{\text{in}} = 60\%$  as before provides a good fit.

## 429 5. Summary and discussion

430 This paper shows how the mechanical energy budget for a convecting system proposed in  
 431 Pauluis (2016) can be applied in the case of two coupled regions, permitting attribution of each  
 432 of the components to either localised features within one of the two regions or to the large-scale  
 433 circulation. It also proposes a refinement to the basic approach in the case of an anelastic cloud  
 434 resolving model. Our work has not identified any energetic inconsistencies introduced by the use  
 435 of the WTG approximation as a proxy for large-scale circulation.

436  
 437 Numerical results were obtained using a CRM in which two regions are coupled using the  
 438 WTG approximation. They demonstrate that the terms in the mechanical energy budget of the  
 439 system considered as a whole are insensitive to the strength (indeed even the presence) of the  
 440 coupling, indicating that the coupling does not distort the energy balance of the whole system.  
 441 However, even very weak coupling has a marked effect on the nature of the convection within  
 442 each region and its associated energy budget terms. As the strength of the coupling increases the  
 443 convection in the cool region is further suppressed and that in the warmer region enhanced. It is  
 444 possible that some of this insensitivity is a consequence of using a fixed cooling profile rather  
 445 than an interactive radiative model. However, other broad properties of the system appear to be  
 446 independent of the coupling strength which suggests that an interactive radiative scheme would be  
 447 likely to produce similar results. Further numerical experiments show that the mechanical energy

448 budget for the entire system is equally insensitive to changes in the difference in SSTs between  
449 the columns.

450

451 The ratios between the component terms contributing to the total energy flows for the entire  
452 system and for each of the regions (where the effect of the large-scale velocity is isolated) are  
453 relatively constant as a function of coupling strength and are consistent with estimates obtained  
454 by theoretical scaling arguments. On the other hand, the values of the component terms for the  
455 large-scale circulation exhibit markedly different features in that the buoyancy contribution is  
456 more significant than that due to the lifting of precipitation while the Gibbs penalty reaches a  
457 maximum as the cooler region dries out, before then decreasing as the differences between the  
458 regions are eliminated with stronger coupling.

459

460 The buoyancy contribution to the large-scale circulation is consistent with that predicted from  
461 an analytical approach to the energetics implied by WTG in a dry setup which also predicts  
462 its quadratic dependency on the SST difference between the columns that is observed. We also  
463 present more general scaling arguments for the components of a mechanical energy budget, which  
464 could serve to indicate the impact of changing atmospheric conditions on the generation of kinetic  
465 energy.

466

467 The vertical kinetic energy associated with the large-scale circulation is found to be more than  
468 three orders of magnitude smaller than the vertical kinetic energy associated with the convection  
469 within each region. Despite this weak magnitude, the coupling has a strong influence on the  
470 suppression and enhancement of convection in each of the regions. The imposition of the WTG  
471 approximation may be interpreted as a representation of the requirement for low temperature

472 gradients in the tropics due to weak rotational effects. In this sense, the contrast in local convective  
473 intensity can be seen as the consequence of a large-scale dynamical constraint and not as a local  
474 result of contrast in SSTs.

475

476 The techniques introduced in this paper, suitably modified, can be applied to reanalysis products  
477 to estimate a mechanical work budget for large-scale circulations such as the Walker or Hadley  
478 circulations, which would provide further insight into whether coupling techniques such as the  
479 WTG provide a useful approach to reflecting such phenomena in studies of convection. Similarly,  
480 they could shed light on the mechanics of convective aggregation, which can be studied using the  
481 WTG approximation (Emanuel et al. 2014).

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485 and David Raymond, Timothy W. Cronin and one anonymous reviewer for their helpful comments.

486

## APPENDIX A

487

### Components of work done in an anelastic model

488 In this appendix we explain how the derivation of the components of work (Eq. 3) in Pauluis  
489 (2016) can be refined in the case of an anelastic model and we derive an approximate expression  
490 for a suggested correction term.

#### 491 *a. Hydrostatic balance in an anelastic context*

492 The original derivation of Eq. 3 involves the use of the hydrostatic approximation  $dp = -\rho_{\text{ref}} g dz$   
493 in evaluating  $-\alpha_d dp$ , where  $\rho_{\text{ref}}$  is the density of dry air in the reference profile and  $\alpha_d$  is the



specific volume of dry air. In an anelastic model such as the LEM, this need not hold in general,  
but it does hold for the reference profile, where it relates the partial pressure of dry air  $p_0$  to height:  
 $dp_0 = -\rho_{\text{ref}} g dz$ .

*b. Derivation of approximate correction term*

The original derivation of Eq. 3 can be restated as below:

$$\oint T ds = - \oint \alpha_d dp - \sum_{w=v,l,i} \oint g_w dr_w \quad , \quad (\text{A1})$$

where:

$$\alpha_d = \frac{R_d T + r_v R_v T}{p} \quad (\text{A2})$$

$$\approx \frac{R_d T_{\text{ref}}}{p} \left( 1 + \frac{T' + r_v \left( \frac{R_v}{R_d} \right) T_{\text{ref}} - r_T T_{\text{ref}}}{T_{\text{ref}}} + r_T \right) \quad (\text{A3})$$

$$= \frac{R_d T_{\text{ref}}}{p} \left( 1 + \frac{B}{g} + r_T \right) \quad , \quad (\text{A4})$$

using the definition of  $B$  in Eq. 12, and where  $R_d$  and  $R_v$  are the specific gas constants for dry air  
and water vapour respectively, and  $T_{\text{ref}}$  the temperature of the reference profile, ignoring expres-  
sions that are second order in the mixing ratio or buoyancy. Likewise, one can then approximate  
the contour integrals of the second and third terms on the right hand side of Eq. A4:

$$- \oint \frac{R_d T_{\text{ref}} B}{g p} dp \approx - \oint \frac{R_d T_{\text{ref}} B}{g p_0} dp_0 = \oint B dz \quad (\text{A5})$$

$$- \oint \frac{R_d T_{\text{ref}} r_T}{p} dp \approx \oint g r_T dz \quad . \quad (\text{A6})$$

The contour integral of the first term on the right hand side of A4 is zero for an anelastic model if  
the full hydrostatic approximation is used as in Pauluis (2016). Applying instead the hydrostatic  
approximation for dry air, it can be simplified with the assumption that the reference profile is

close to the equilibrium state and that variations in the pressure of dry air at a given height are in-  
 significant when compared with variations in water vapour pressure, (ie  $p \approx p_{\text{ref}} + e$ ) and ignoring  
 higher orders of  $\frac{e}{p_{\text{ref}}}$  to obtain an additional term:

$$-\oint \frac{R_d T_{\text{ref}}}{p} dp = -\oint \frac{R_d T_{\text{ref}}}{p_{\text{ref}} + e} d(p_{\text{ref}} + e) \quad (\text{A7})$$

$$\approx -\oint \frac{R_d T_{\text{ref}}}{p_{\text{ref}}} dp_{\text{ref}} + \oint \frac{e R_d T_{\text{ref}}}{p_{\text{ref}}^2} dp_{\text{ref}} - \oint \frac{R_d T_{\text{ref}}}{p_{\text{ref}}} de \quad . \quad (\text{A8})$$

Making use of the gas law:

$$-\oint \frac{R_d T_{\text{ref}}}{p} dp \approx -\oint \frac{1}{p_{\text{ref}}} dp_{\text{ref}} + \oint \frac{e R_d T_{\text{ref}}}{p_{\text{ref}}^2} dp_{\text{ref}} - \oint \frac{R_d T_{\text{ref}}}{p_{\text{ref}}} de \quad . \quad (\text{A9})$$

Using the hydrostatic approximation, and remembering that  $e = r_v \frac{R_v}{R_d} p_{\text{ref}}$ :

$$-\oint \frac{R_d T_{\text{ref}}}{p} dp \approx \oint g dz + \oint \frac{r_v R_v T_{\text{ref}}}{p_{\text{ref}}} dp_{\text{ref}} - \oint \frac{r_v R_v T_{\text{ref}}}{p_{\text{ref}}} dp_{\text{ref}} - \oint R_v T_{\text{ref}} \frac{dr_v}{dp_{\text{ref}}} dp_{\text{ref}} \quad . \quad (\text{A10})$$

The first of the terms on the right hand side is zero, and the second and third cancel, leaving the  
 final term, which can be expressed as an integral in  $r_v$ :

$$-\oint \frac{R_d T_{\text{ref}}}{p} dp \approx -\oint R_v T_{\text{ref}} dr_v \quad . \quad (\text{A11})$$

This expression can be integrated by parts and then expressed in terms of an integral in  $z$ :

$$-\oint \frac{R_d T_{\text{ref}}}{p} dp \approx +\oint R_v r_v dT_{\text{ref}} = -\oint R_v r_v \Gamma dz \quad , \quad (\text{A12})$$

where  $\Gamma$  is the lapse rate of the reference profile. Eq. A11 is the form of the equation that is used  
 to calculate this term numerically in the contour integrals, Eq. A12 provides a more useful form  
 in developing scaling arguments below.

The new term in the form shown in Eq. A11 bears a similarity to a term in the water vapour  
 component of the Gibbs penalty  $\oint (-\mathcal{G}_v) dr_v$ , depending as it does on the same state variables since:

$$\mathcal{G}_v = c_l \left( T - T_f - T \ln \frac{T}{T_f} \right) + R_v T \ln \mathcal{H} \quad (\text{A13})$$

where  $c_l$  is the specific heat capacity of liquid water at constant pressure,  $T_f$  its freezing temperature and  $\mathcal{H}$  the relative humidity. One can therefore introduce a modified specific Gibbs free energy for the vapour state:

$$\mathcal{G}_v^* = c_l \left( T - T_f - T \ln \frac{T}{T_f} \right) + R_v T (1 + \ln \mathcal{H}) \quad (\text{A14})$$

and (neglecting a term in  $T_{\text{ref}} - T$ ) instead of Eq. 3 one can write:

$$\oint T dS \approx \oint B dz + \oint r_T g dz + \sum_{w=v,l,i} \oint (-\mathcal{G}_w^*) dr_w \quad (\text{A15})$$

where  $\mathcal{G}_w^* = \mathcal{G}_w$  for  $w = l, i$ . (It could be argued that from a physical perspective this term is a correction to the mechanical work generated and should therefore instead be combined with  $W_b$ .)

### c. Numerical values and scalings

The contribution of this new term can be illustrated by comparing numerical values for the components of Eq A15 with the modified version produced by the LEM for the case of a single uncoupled convecting region in a state of equilibrium over a surface at a temperature of 302.7K (as discussed in section 3.b). As Table A1 shows, the effect of the correction term is to reduce the discrepancy in the equation from around 5% of the largest term to less than 1%. Similar impacts are observed in other configurations of the model, including in the coupled version.

The additional term expressed in the form in Eq. A12 will scale as approximately 20% of  $W_p$  in Eq. 3, for a moist adiabatic temperature profile where the lapse rate of the reference state  $\Gamma$  will be approximately half the dry adiabatic lapse rate  $\frac{g}{c_p}$ , where  $c_p$  is the specific heat capacity of dry air at constant pressure given that  $\frac{R_v}{c_p} = 0.46$ .

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594 TABLE 1. Numerical values for the terms of Equation 3 for the control case of RCE in a single region.  $\Delta\mathcal{G}$  is  
595 the unmodified Gibbs penalty component and  $\Delta$  the corresponding discrepancy in the equation;  $\Delta\mathcal{G}^*$  and  $\Delta^*$  are  
596 the same quantities including the adjustment described in Appendix A.

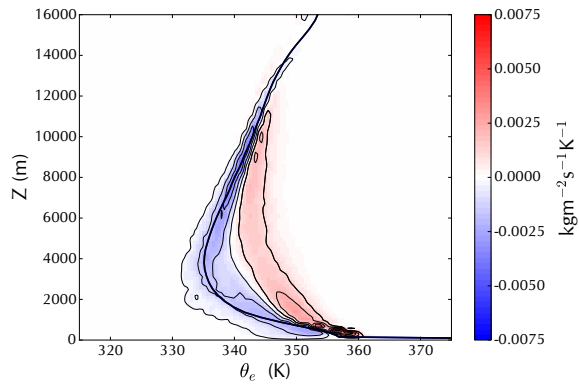
(Wm <sup>-2</sup> )	$W_{Tds}$	$W_b$	$W_p$	$\Delta\mathcal{G}$	$\Delta$	$\Delta\mathcal{G}^*$	$\Delta^*$
Base case	11.17	2.18	3.14	6.32	-0.47	5.81	0.04
Pauluis (2016)	8.3	1.5	2.5	4.6	-0.3	-	-



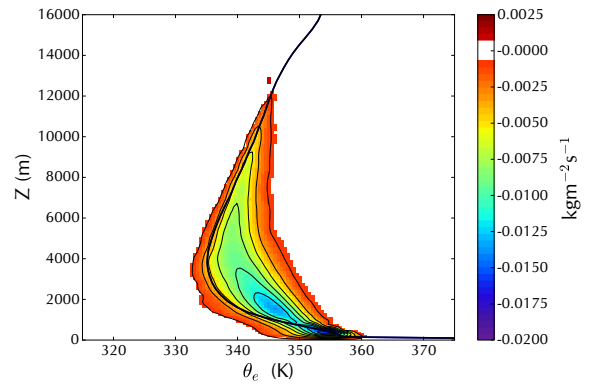
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618	<b>Fig. 5.</b>	Integrated column water vapour in the cool region (blue) and warm region (red) for varying values of WTG coupling relaxation timescale $\tau$ for an SST contrast of 2K. . . . . 39
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620	<b>Fig. 6.</b>	Isentropic distribution of the vertical mass flux $\langle \rho w \rangle$ for (a) the cool region and (b) the warm region with coupling $\tau = 50$ hours. The solid line represents the mean profile of the equivalent potential temperature $\theta_e$ . . . . . 40
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623	<b>Fig. 7.</b>	Components of kinetic energy - the blue line represents the horizontal component and the green line the vertical component for the aggregate system. The contribution of the large-scale motion to the vertical kinetic energy is shown in black. . . . . 41
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626	<b>Fig. 8.</b>	As Fig. 4 (b). In addition the conversion rate from potential to kinetic energy based on Eq. 11 is shown in violet. . . . . 42
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628	<b>Fig. 9.</b>	Components of Eq. 3 for (a) the total system, (b) the large-scale circulation and (c) (d) the convective circulations in the cooler and warmer regions for varying values of $\Delta T_s$ , the difference between the SSTs in the two regions. The blue line indicates $W_b$ , the buoyancy work done, the red $W_p$ , the moisture elevator term, the green $\Delta G^*$ , the Gibbs penalty, and the black line $W_{TDS}$ the Carnot maximum. In panel b) the red and blue lines largely coincide . . . . . 43
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633	<b>Fig. 10.</b>	Components of Eq. 3 for the large-scale circulation (as shown in Figure 9 ,but plotted on a log-log scale) for varying values of $\Delta T_s$ , the difference between the SSTs in the two regions. The blue line indicates $W_b$ , the buoyancy work done, the red $W_p$ , the moisture elevator term, the green $\Delta G^*$ , the Gibbs penalty, and the black line $W_{TDS}$ the Carnot maximum. . . . . 44
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637	<b>Fig. 11.</b>	Comparison between the Gibbs penalty component of the large-scale circulation (red, left hand axis) and latent heat transported between the regions (blue, right hand axis) for a) the
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639	numerical experiment where coupling strength is varied and b) the numerical experiment	
640	where SST difference is varied. . . . .	45



(a) Isentropic distribution of the vertical mass flux



(b) Isentropic stream function

FIG. 1. (a) Isentropic distribution and (b) streamfunction of the vertical mass flux  $\langle \rho w \rangle$  for a single uncoupled region at RCE over an SST of 302.7 K. The solid line represents the mean profile of the equivalent potential temperature  $\theta_e$ .

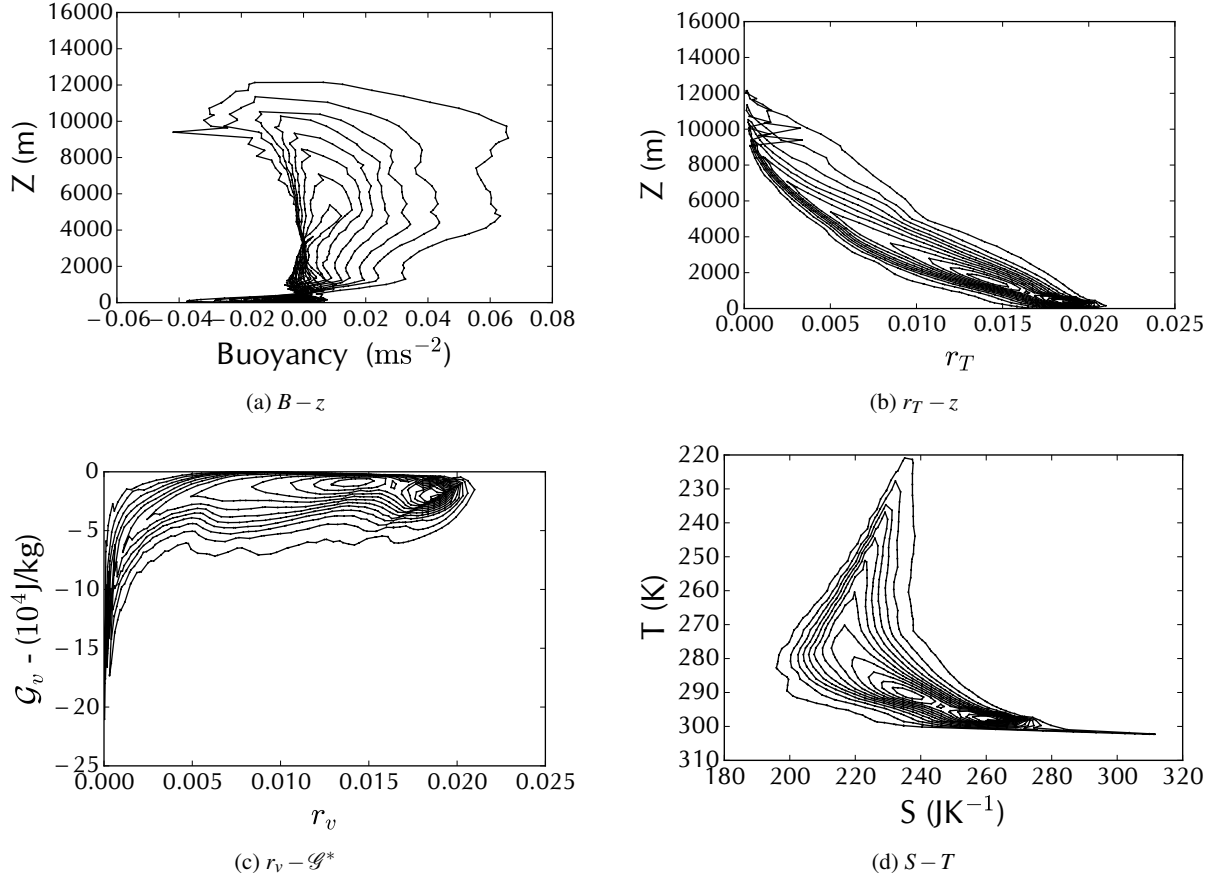


FIG. 2. Thermodynamic diagrams for the main terms in Eq. 3 for a single uncoupled region at RCE over a surface at temperature 302.7K showing 15 equally spaced contours of the streamfunction. (Numerical values elsewhere in this paper were calculated using 60 contours.)

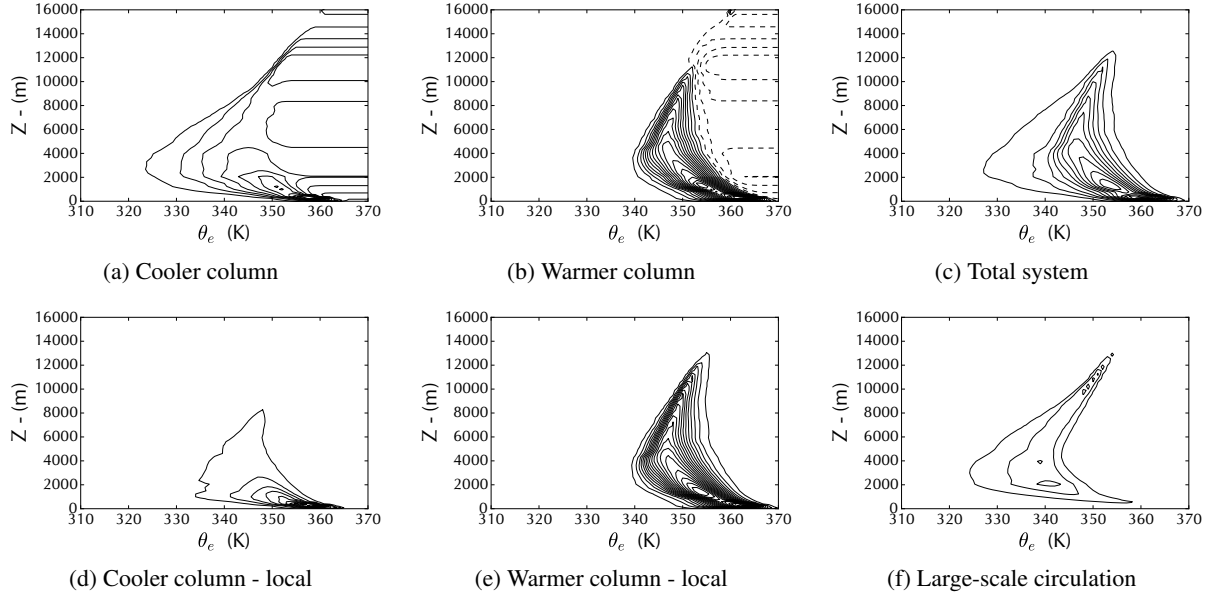
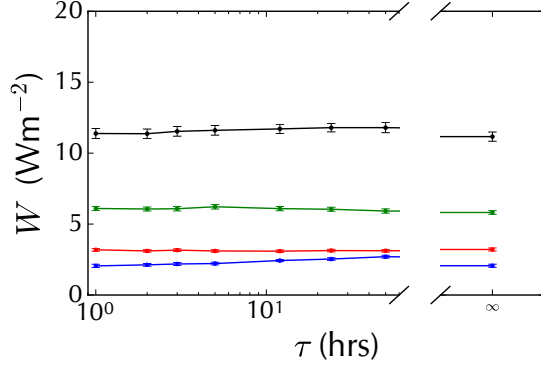
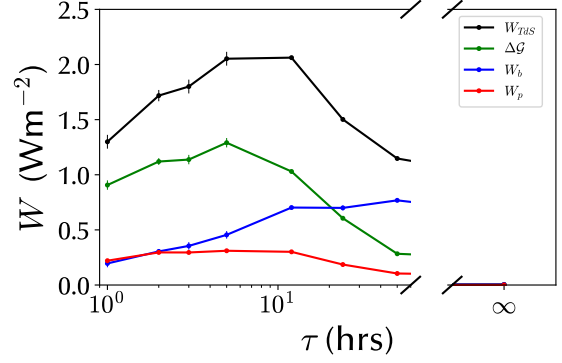


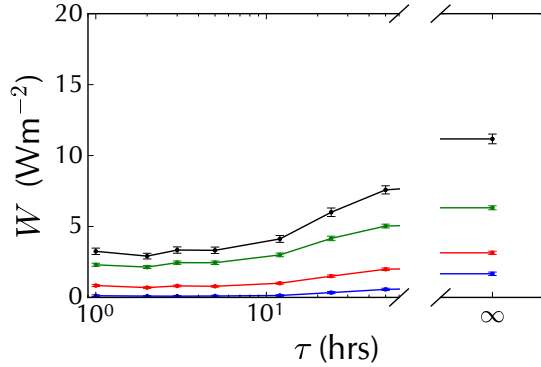
FIG. 3. a) (b) Contours of the integrated vertical mass flux for each of the two regions analysed in isolation. The regions are run to RCE and are coupled with  $\tau = 2$  hours. Note that the contours are not closed and so cannot be represented by a streamfunction. (c) shows the streamfunction of the total system. Once the large-scale circulation (f) is treated separately, the residual “convective” circulations in the two regions (d) and (e) exhibit closed contours. The same values for the contours are used in each of the plots; here solid lines indicate negative values and dashed lines positive values.



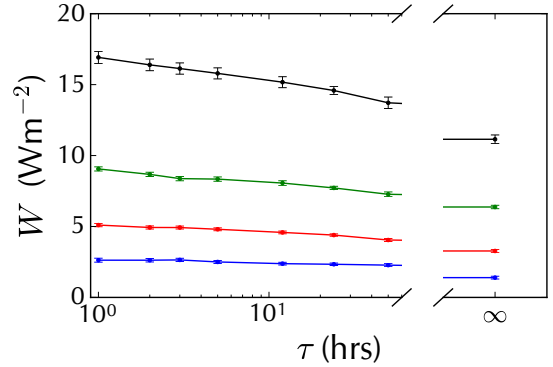
(a) Total system



(b) Large-scale circulation



(c) Cooler column



(d) Warmer column

FIG. 4. Components of Eq. 3 for (a) the total system, (b) the large-scale circulation and (c) (d) the convective circulations in the cooler and warmer regions for varying coupling strengths. A high value of the coupling timescale parameter  $\tau$  indicates weak coupling. The blue line indicates  $W_b$ , the buoyancy work done, the red  $W_p$ , the moisture elevator term, the green  $\Delta G^*$ , the Gibbs penalty, and the black line  $W_{Tds}$  the Carnot maximum. Error bars represent the standard error in the estimate of the parameter, obtained by subsampling.

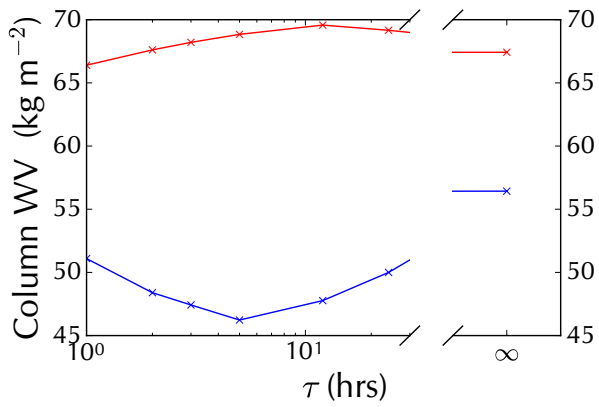
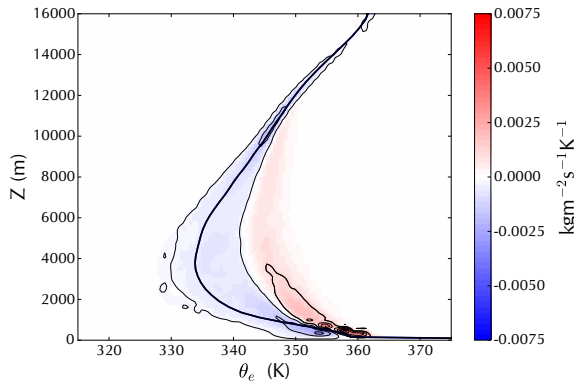
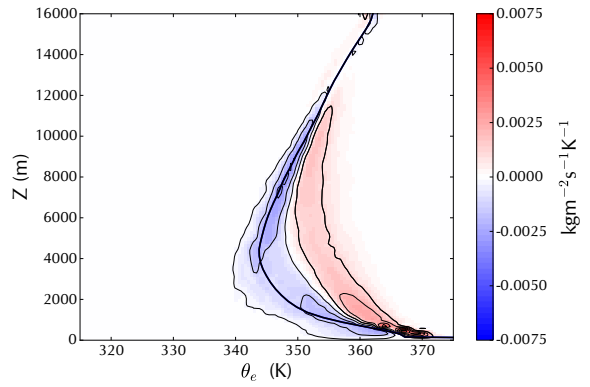


FIG. 5. Integrated column water vapour in the cool region (blue) and warm region (red) for varying values of WTG coupling relaxation timescale  $\tau$  for an SST contrast of 2K.



(a) Cooler column



(b) Warmer column

FIG. 6. Isentropic distribution of the vertical mass flux  $\langle \rho w \rangle$  for (a) the cool region and (b) the warm region with coupling  $\tau = 50$  hours. The solid line represents the mean profile of the equivalent potential temperature  $\theta_e$ .



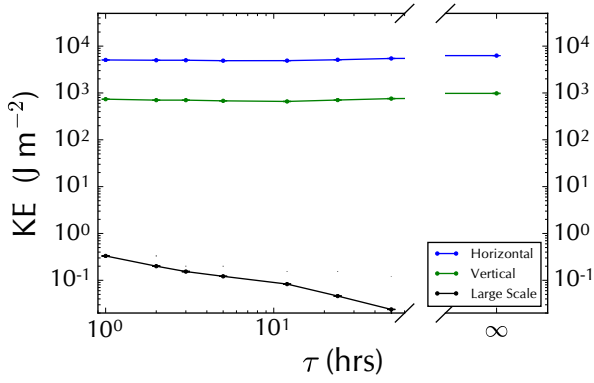


FIG. 7. Components of kinetic energy - the blue line represents the horizontal component and the green line the vertical component for the aggregate system. The contribution of the large-scale motion to the vertical kinetic energy is shown in black.

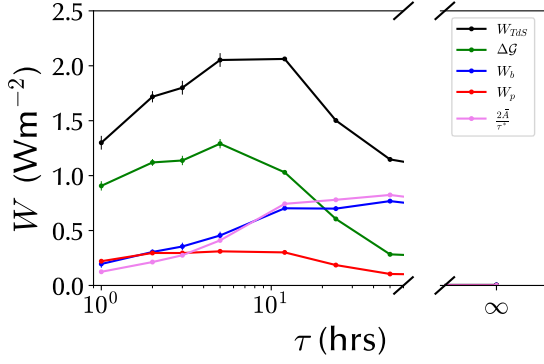
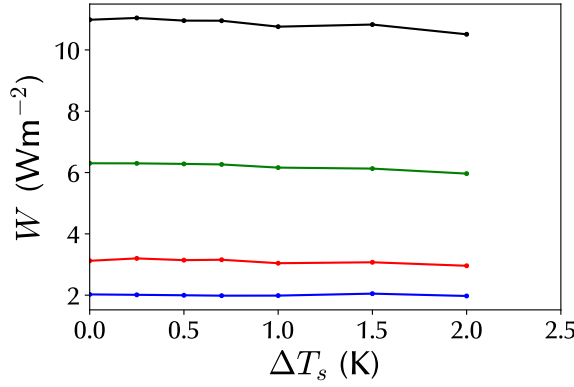
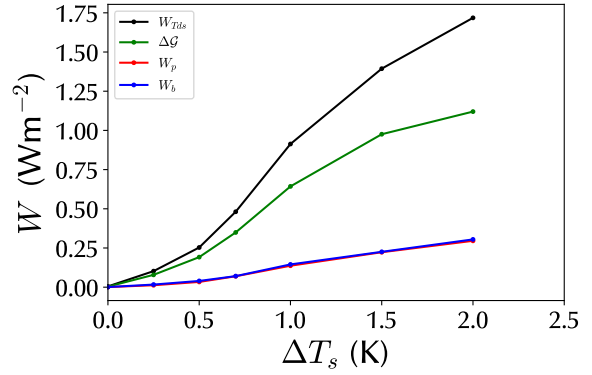


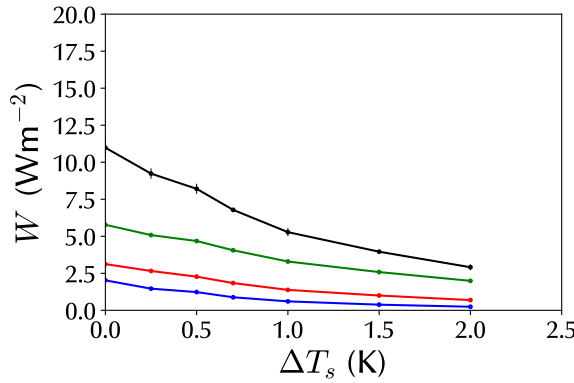
FIG. 8. As Fig. 4 (b). In addition the conversion rate from potential to kinetic energy based on Eq. 11 is shown in violet.



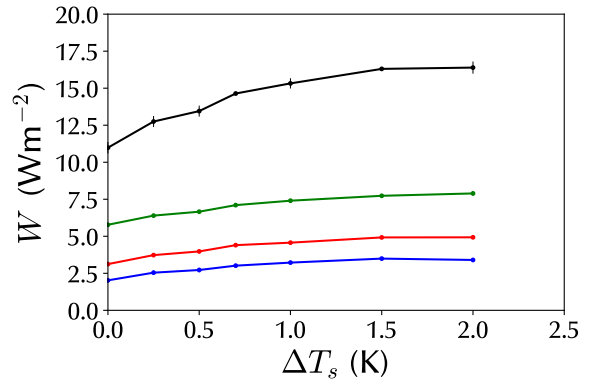
(a) Total system



(b) Large-scale circulation



(c) Cooler column



(d) Warmer column

FIG. 9. Components of Eq. 3 for (a) the total system, (b) the large-scale circulation and (c) (d) the convective circulations in the cooler and warmer regions for varying values of  $\Delta T_s$ , the difference between the SSTs in the two regions. The blue line indicates  $W_b$ , the buoyancy work done, the red  $W_p$ , the moisture elevator term, the green  $\Delta G^*$ , the Gibbs penalty, and the black line  $W_{TDS}$  the Carnot maximum. In panel b) the red and blue lines largely coincide

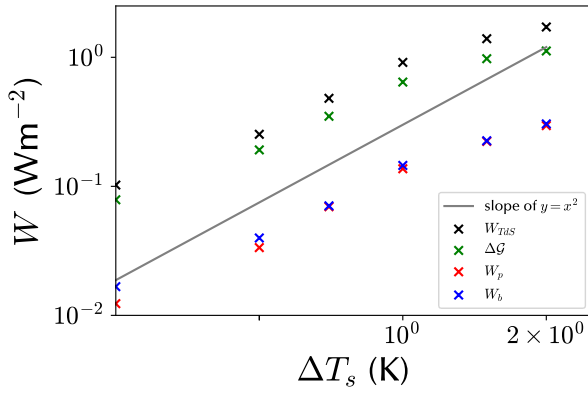
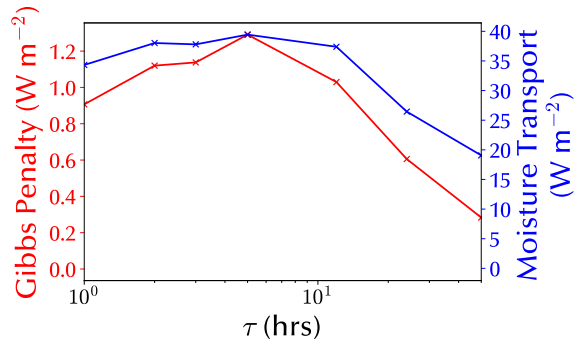
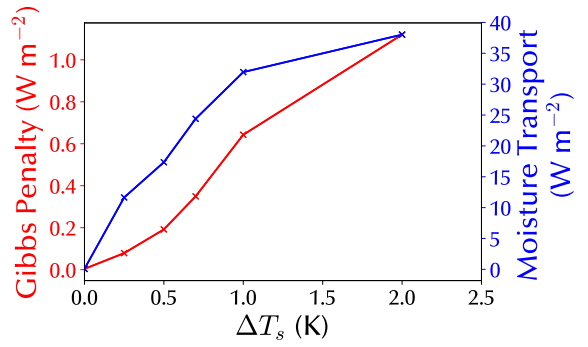


FIG. 10. Components of Eq. 3 for the large-scale circulation (as shown in Figure A9 ,but plotted on a log-log scale) for varying values of  $\Delta T_s$ , the difference between the SSTs in the two regions. The blue line indicates  $W_b$ , the buoyancy work done, the red  $W_p$ , the moisture elevator term, the green  $\Delta G^*$ , the Gibbs penalty, and the black line  $W_{Tds}$  the Carnot maximum.



(a) Coupling strength



(b) SST difference

FIG. 11. Comparison between the Gibbs penalty component of the large-scale circulation (red, left hand axis) and latent heat transported between the regions (blue, right hand axis) for a) the numerical experiment where coupling strength is varied and b) the numerical experiment where SST difference is varied.