Maximum Entropy Production Principle: Where is the positive feedback?

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Outline

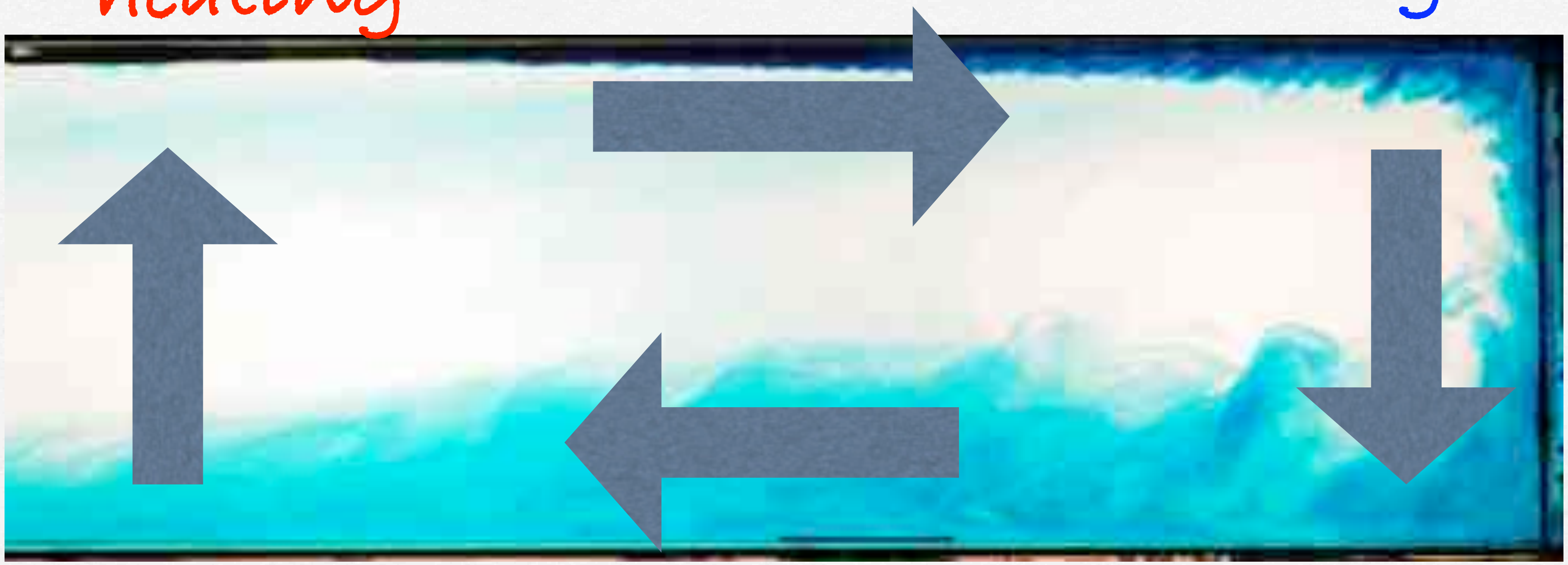
- The context: Horizontal convection
- Formulation of the MEP principle
- Importance of stirring: Zeldovich's result
- Basic ideas on turbulent diapycnal mixing
- Power input due to surface buoyancy fluxes
- The positive feedback

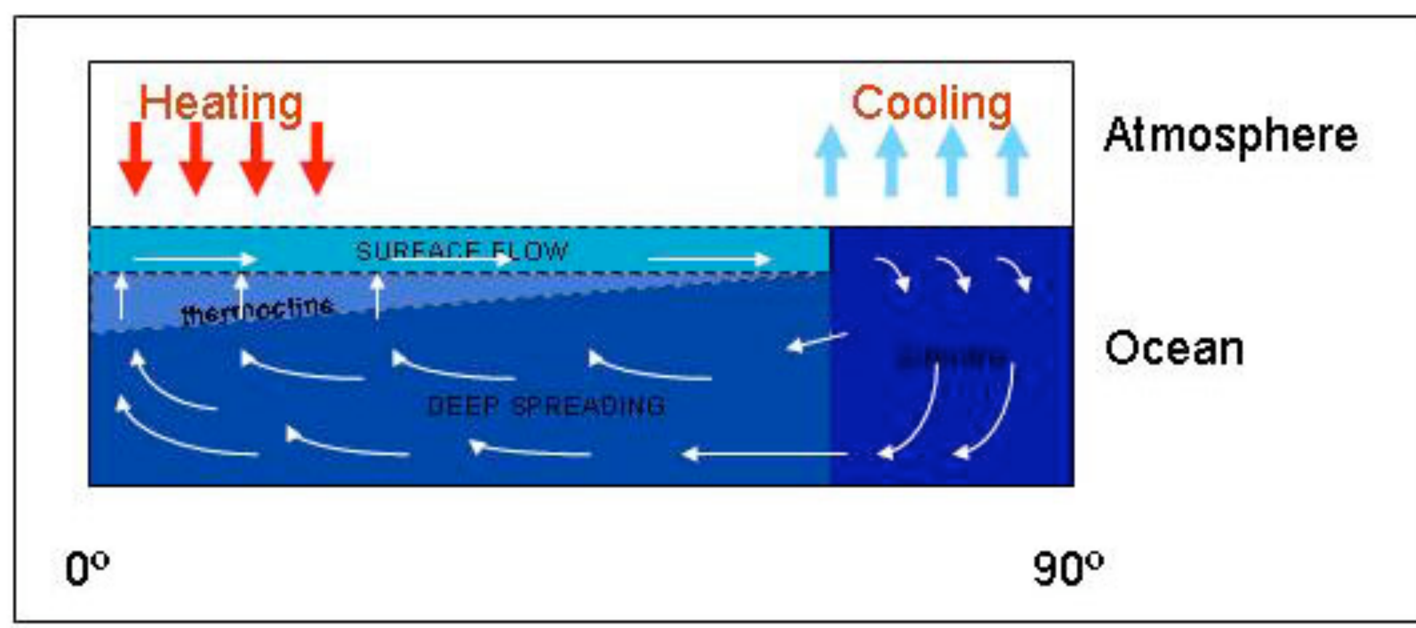
Horizontal Convection

- buoyancy-driven circulation forced by stabilizing and destabilizing buoyancy fluxes applied at the same level (as in the oceans)
- Physical ingredients
  - High-latitude cooling-> dense plumes
  - Turbulent molecular diffusion to avoid the oceans to fill up with cold water

Stabilizing heating

Destabilizing Cooling





Entropy balance Equation

$$\frac{d\Sigma}{dt} = \int_V \frac{\rho \dot{Q}}{T} dV + \int_V \frac{\rho \varepsilon}{T} dV$$

$$\rho \dot{Q} = \nabla \left(\kappa \rho C_p \nabla T \right)$$

$$-\int_S \frac{\kappa \rho C_p \nabla T \cdot \mathbf{n} dS}{T} = \int_V \frac{\rho \kappa C_p \|\nabla T\|^2}{T^2} dV + \int_V \frac{\rho \varepsilon}{T} dV$$

Production

Dissipation by molecular diffusion

Viscous dissipation

MEPP assumption

Maximization of Entropy production:

$$-\int_V \frac{\kappa \rho C_p \nabla T \cdot \mathbf{n} dS}{T} = -\left[\frac{Q_{in}}{T_{in}} + \frac{Q_{out}}{T_{out}}\right] = \left(\frac{T_{in} - T_{out}}{T_{out}}\right)\frac{Q_{in}}{T_{in}}$$

Equivalent to maximizing Entropy destruction

$$\int_V \frac{\kappa \rho C_p \|\nabla T\|^2}{T^2} dV + \int_V \frac{\rho \varepsilon}{T} dV \approx \frac{\rho \kappa C_p}{T_0^2} \int_V \|\nabla T\|^2 \, dV$$

Puzzling points

- What is the relevant optimization problem?
- Is the MEPP a property of the Navier-Stokes equations, or a new principle of Nature?
- The controversy about statistical versus thermodynamic entropy

Approach

- Reaching a steady state requires:
  - A positive feedback to make irreversible entropy production grow
  - A negative feedback to limit the rate of irreversible entropy production
  - What are these? Can we identify both a positive and negative feedback?

Boussinesq HC, linear eos

$$\frac{D\mathbf{v}}{Dt} + 2\mathbf{\Omega} \times \mathbf{v} + \frac{1}{\rho_0}\nabla P = b\mathbf{z} + \nu\nabla^2\mathbf{v}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{Db}{Dt} = \kappa\nabla^2 b$$

Dimensionless parameters

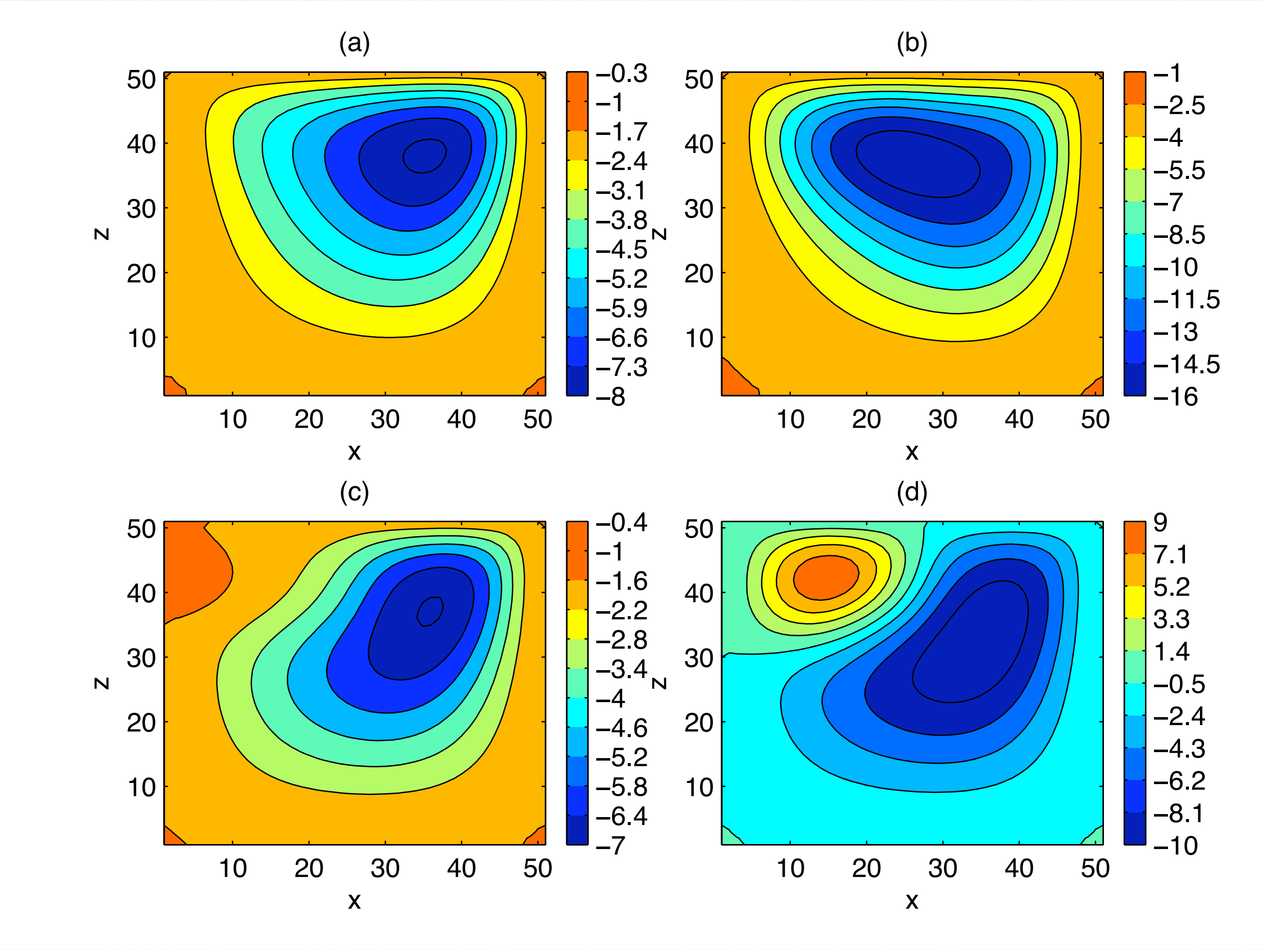
$$R_h = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$

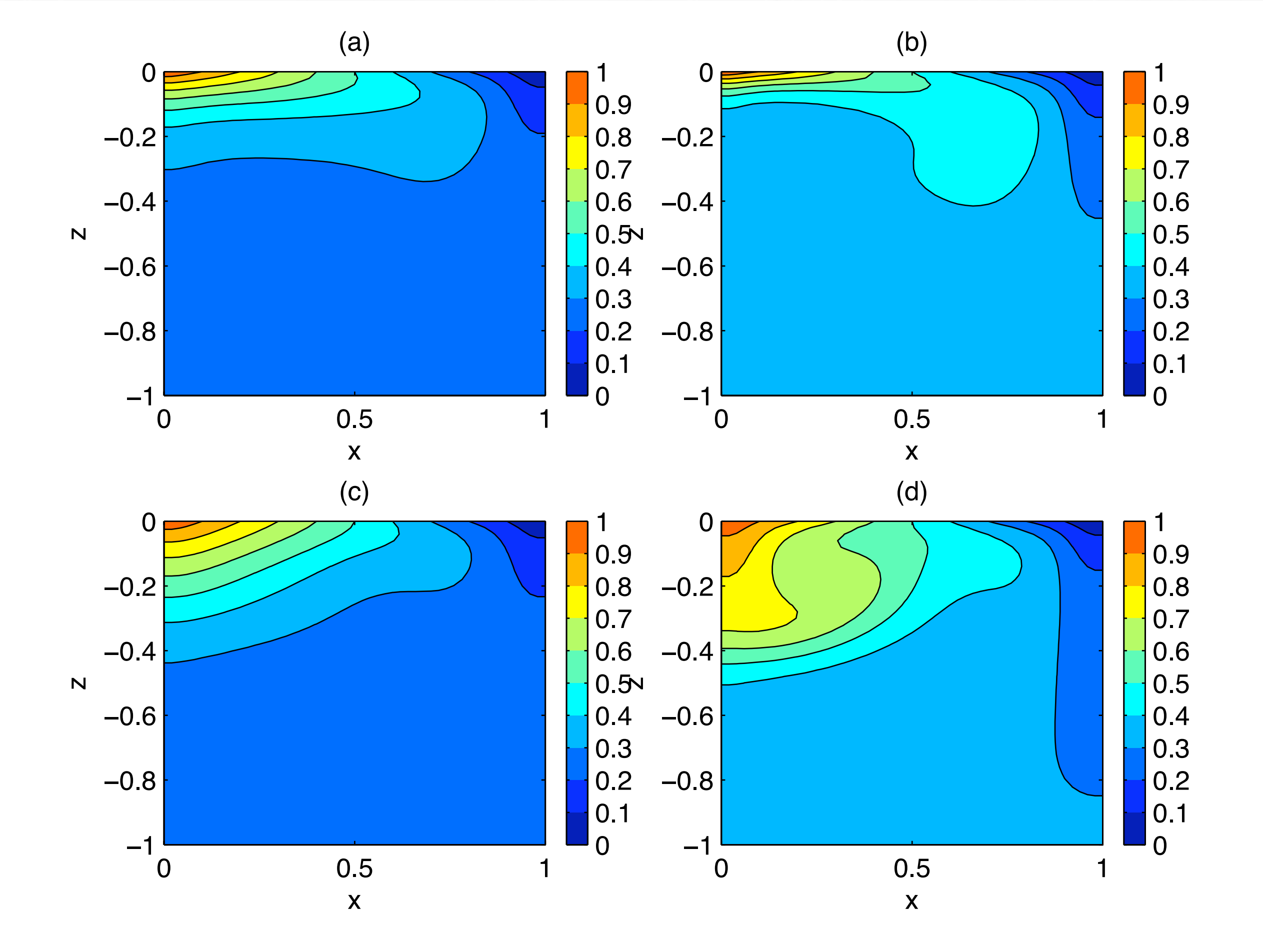
Horizontal Rayleigh number

$$P_r = \frac{\nu}{\kappa}$$

Prandtl number

Numerical Examples





Importance of stirring: Zeldovich (1937) result

$$\Phi = \frac{\int_V \kappa \|\nabla T\|^2 dV}{\int_V \kappa \|\nabla T_c\|^2} dV > 1$$

T and Tc solutions of the following problems, with same boundary conditions

$$\kappa \nabla^2 T_c = 0$$

$$\kappa \nabla^2 T = \mathbf{v} \cdot \nabla T$$

→

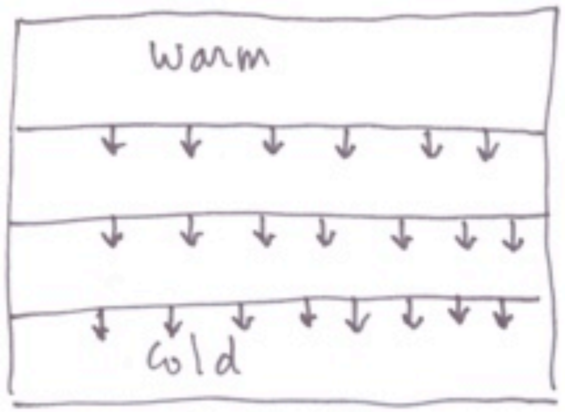
Stirring required for MEP principle!

Stirring versus Mixing

- STIRRING: Adiabatic lifting up of dense parcels and pushing down of light parcels → Stirring requires power!
  - Does not affect pdf of entropy
- MIXING: Destruction of tracer variance by molecular diffusion
  - Affects pdf of entropy

How Stirring enhances Mixing

Laminar case

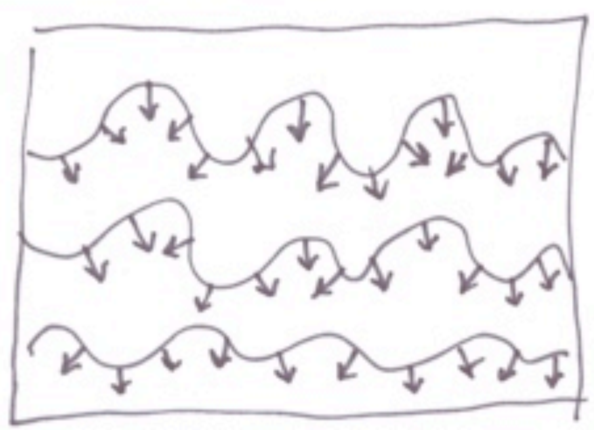


Total Heat Flux =

$$-\kappa_T \, A_{laminar} \, \rho \, C_p \, \frac{\partial T}{\partial z}$$

$$= F_{laminar}$$

Turbulent case



Total heat Flux =

$$-\kappa_T \, A_{turbulent} \, \rho \, C_p \left\langle \frac{\partial T}{\partial n} \right\rangle$$

$$= F_{turbulent}$$

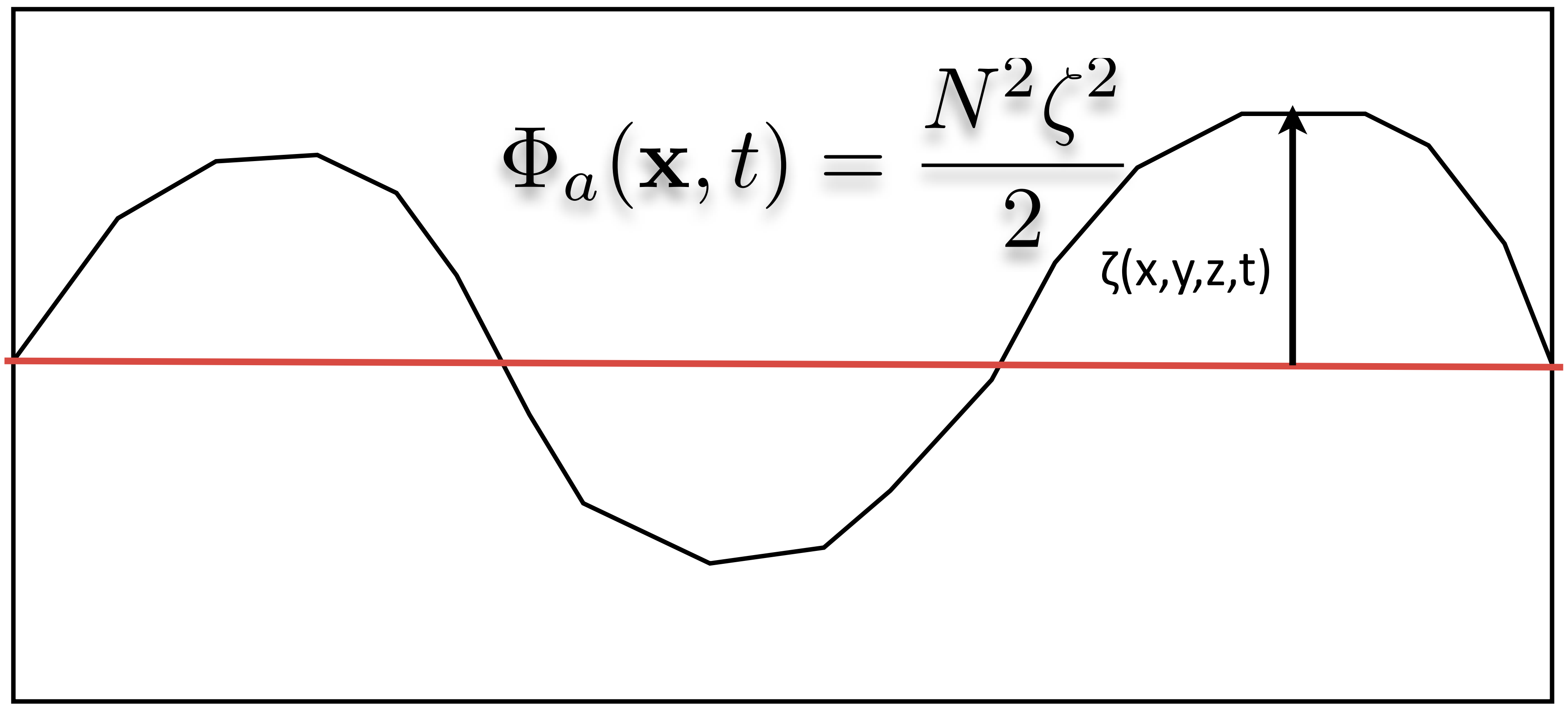
One shows that: $\left\langle \frac{\partial T}{\partial n} \right\rangle \simeq \frac{A_{turbulent}}{A_{laminar}} \times \frac{\partial T}{\partial z}$

$$\Rightarrow \quad F_{turbulent} = \left(\frac{A_{turbulent}}{A_{laminar}} \right)^2 F_{laminar}$$

Bounds on horizontal convection

- Bounds on epsilon
- Bounds on chi

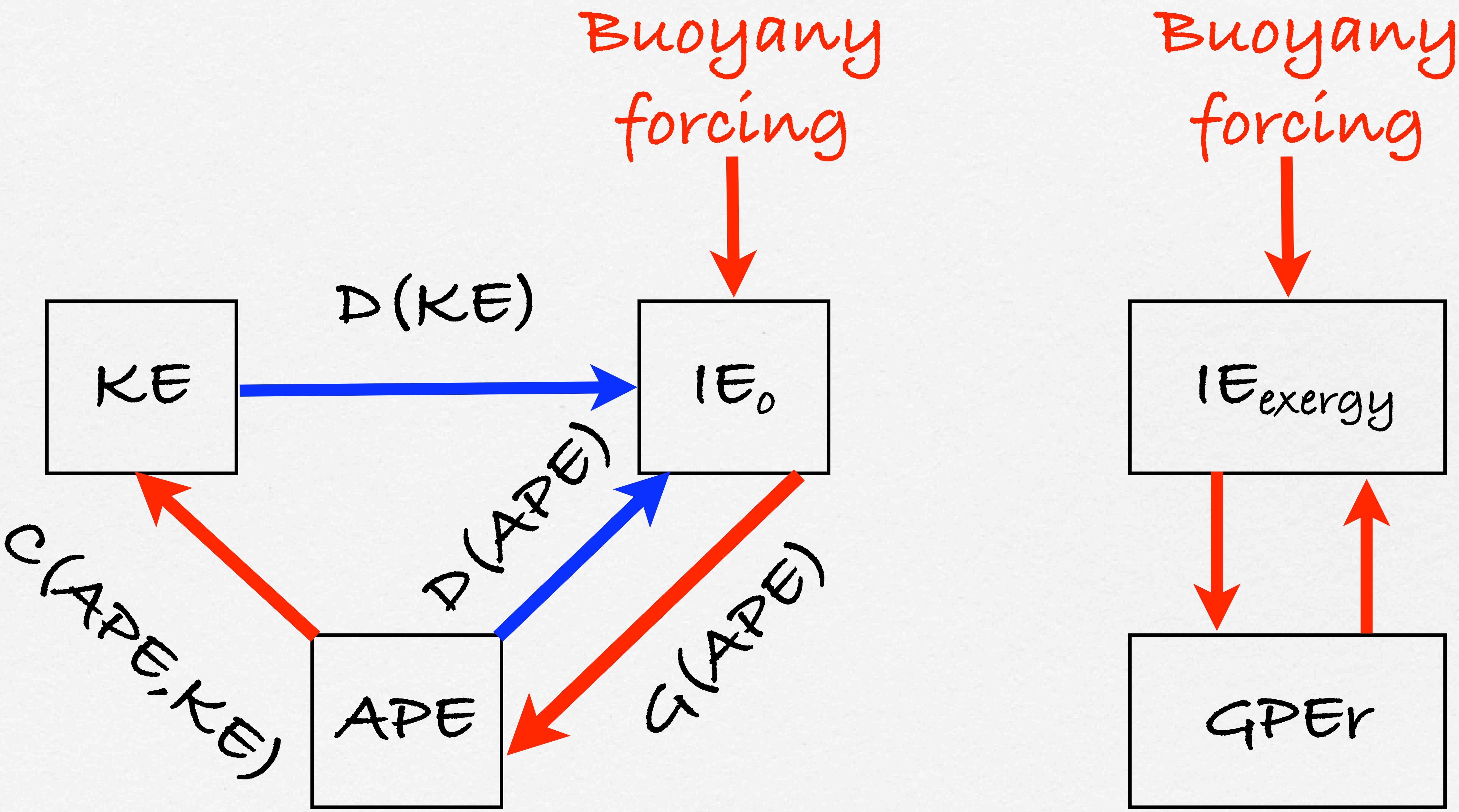
Available Potential Energy (APE) and APE density



$$\Phi_a(\mathbf{x}, t) = \frac{N^2 \zeta^2}{2}$$

$$APE(t) = \iiint_V \Phi_a(x,t)\,dm$$

Energetics Issues



Mechanical Energy (APE+KE) balance:

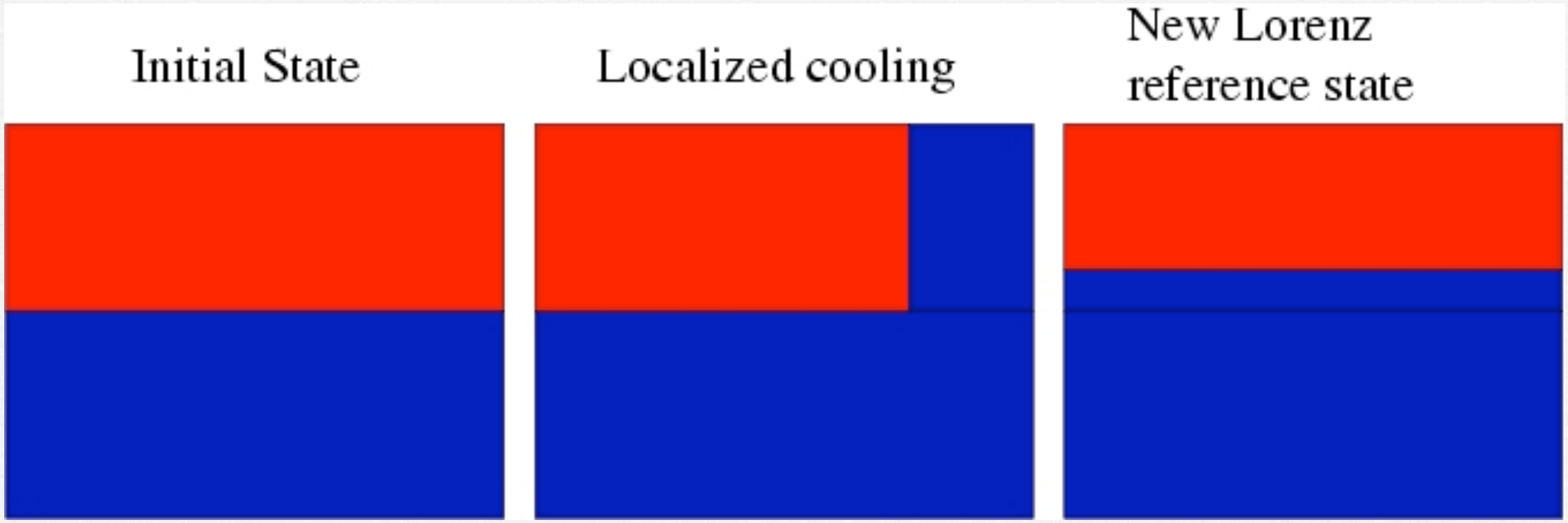
$$G(APE) = D(KE) + D(APE)$$

Production by buoyancy

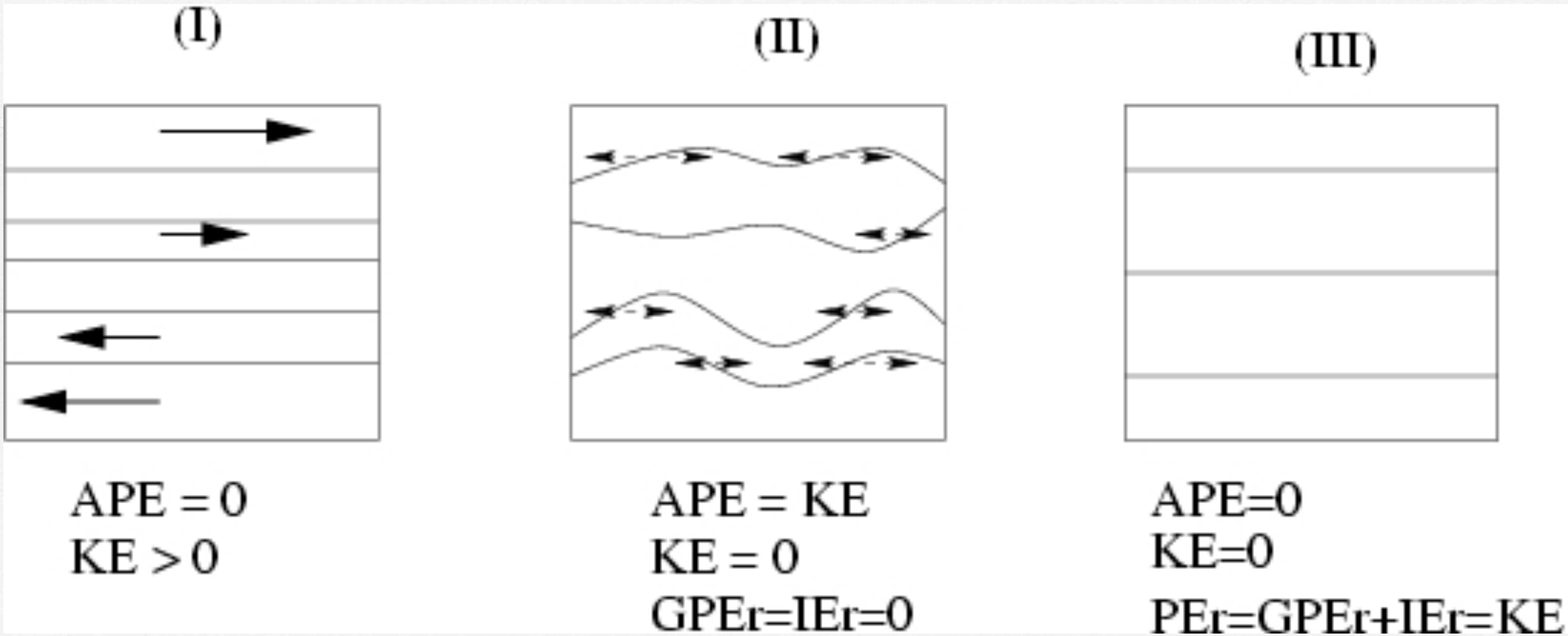
Viscous dissipation

Diffusive dissipation

APE production by cooling



Diffusive APE dissipation



The nature of G(APE)

$$G(APE) = \int_S \frac{\alpha g z_r}{C_p} Q_{surf} dS \approx \frac{\alpha g}{C_p} \left[h_{cooling} - h_{heating}\right] Q_{cooling}$$

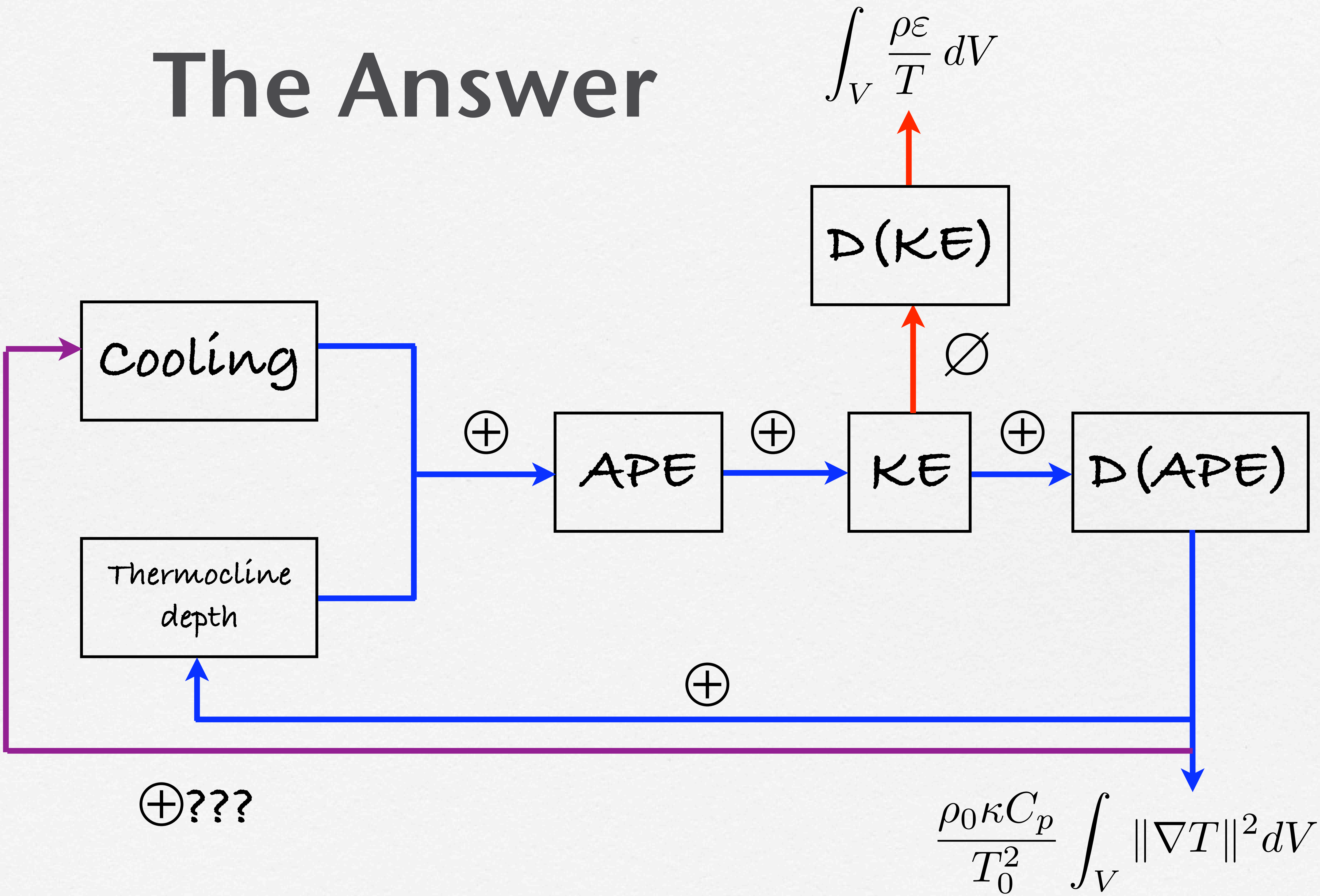
$h_{heating} \approx 0$

$h_{cooling}$ = penetration depth of dense plumes

$\approx$ Measure of thermocline depth

Key Result: The power input due to buoyancy forcing depends on the total amount of cooling and on the thermocline structure!

The Answer



$\int_V \frac{\rho\varepsilon}{T}\, dV$
D(KE)
∅
Cooling
⊕
APE
⊕
KE
⊕
D(APE)
Thermocline depth
⊕
⊕???
$\frac{\rho_0 \kappa C_p}{T_0^2} \int_V \|\nabla T\|^2 dV$

Postulate

- MEPP seems to require the maximization of the so-called mixing efficiency $\mathbf{\Gamma}$, often thought to be close to $\mathbf{\Gamma}$=0.2

$$\Gamma = \frac{D(APE)}{D(KE)}$$

Numerical Results (200 x 200)

	G(KE)	G(APE)	D(APE)	Γ	Ψ
I	0	6.97	5.54	2.42	7.16
II	4.92	11.2	10.2	1.58	10.89
III	-0.45	5.91	4.35	2.20	6.68
IV	19.2	10.4	9.18	0.44	11.3

I = Thermally-direct buoyancy-driven circulation
II = "Direct'' Mechanical Forcing
III = Weak "Indirect" Forcing
IV = Strong "Indirect" Forcing

Dependency on Ra (Pr=10)
Buoyancy-driven horizontal convection

$$R_a = \frac{g\alpha \Delta T H^3}{\nu\kappa}$$

$R_a = 10^{20}$ in the Oceans

R_a	10^5	10^6	10^7	...	10^{20}
Γ	2.28	3.37	5.01	...	?

Numerical Resolution: 150 x 150

Summary

- MEPP requires maximizing stirring and dissipation
- Maximizing stirring requires maximizing power input and minimizing dissipation → Conflict
- Way out: Minimizing viscous dissipation (maximize stirring) and maximizing diffusive dissipation (maximize G(APE))

Summary (cont'd)

- This is equivalent to maximizing the so-called mixing efficiency
- It seems possible for horizontal convection to satisfy MEPP
- Is that useful? (i.e., to make predictions of overturning strength, to predict behavior as function of Rh, and so on...)