Thermodynamic Efficiency and Entropy Production in the Climate System

Valerio Lucarini

University of Reading, Reading, UK

v.lucarini@reading.ac.uk

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Thermodynamics and Climate

- *Climate is non-equilibrium system, which generates entropy by irreversible processes and keeps a steady state by balancing the energy and entropy fluxes with the environment.*
- *Climate is FAR from equilibrium: FDT?*
- *MEPP? No, but…*
- *We have recently obtained some new results drawing a line connecting thermodynamic efficiency and entropy production*

Today

- *We present the theory: Lorenz Cycle + Carnot + Entropy Production (L., PRE, 2009)*
- *We focus on diagnostics describing the global thermodynamic properties of the climate system using PLASIM (U. Hamburg)*
  - *Onset and decay of snowball conditions due to variations in the solar constant*
    - *L., Fraedrich, Lunkeit, QJRMS (2010)*
  - *Impact of CO_2 changes & generalized sensitivities*
    - *L., Fraedrich, Lunkeit, ACPD (2010)*
  - *Thermodynamic Bounds from TOA budgets*
    - *L., Fraedrich, submitted GRL (2010)*

Energy Budget

- *Let the total energy of the climatic system be:*

$$E(\Omega) = \int_{\Omega} dV \rho e = \int_{\Omega} dV \rho (u + \phi + k),$$

- *where* $\boldsymbol{\rho}$ *is the local density, e is the total energy per unit mass, with* ***u****,* $\boldsymbol{\phi}$ *and* ***k*** *indicating the* ***internal, potential and kinetic energy*** *components*
- *Energy budget* $\dot{E}(\Omega) = \dot{P}(\Omega) + \dot{K}(\Omega)$

Detailed Balances

WORK

$$\dot{W} = C(P,K)$$

- *Kinetic energy budget*

$$\dot{K}(\Omega) = -\int_{\Omega} dV \varepsilon^2 + C(P,K) = -\dot{D} + C(P,K)$$

- *Potential Energy budget*

$$\dot{P}(\Omega) = \int_{\Omega} dV \rho \dot{Q} - \dot{W} \qquad \dot{Q} = 1/\rho \left(\varepsilon^2 - \vec{\nabla} \cdot \vec{H} \right)$$

- *Total Energy Budget*

$$\dot{E}(\Omega) = \int_{\Omega} dV \left(-\vec{\nabla} \cdot \vec{H} \right) = -\int_{\partial\Omega} dS \hat{n} \cdot \vec{H}$$

FLUXES

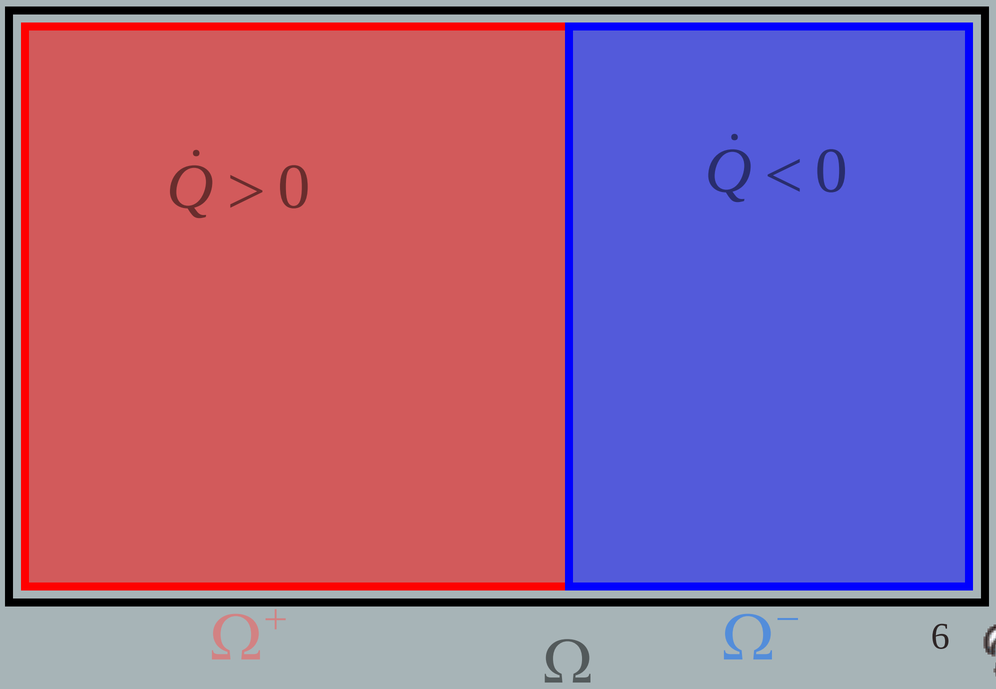
DISSIPATION (L&F, PRE 2009)

Johnson's idea (2000)

- *Partitioning the Domain*

$$\dot{P}(\Omega)+\dot{W} = \int_{\Omega^+} dV \rho \dot{Q}^+ + \int_{\Omega^-} dV \rho \dot{Q}^- = \dot{\Phi}^+ + \dot{\Phi}^-$$

- *Better than it seems!*



Long-Term averages

- *Stationarity:* $\overline{\dot{E}(\Omega)} = \overline{\dot{P}(\Omega)} = \overline{\dot{K}(\Omega)} = 0$

- *Work = Dissipation* $-\overline{\dot{K}(\Omega)} + \overline{\dot{W}} = \overline{\dot{W}} = \overline{\dot{D}} > 0$

- *Work = Input-Output* $\overline{\dot{P}(\Omega)} + \overline{\dot{W}} = \overline{\dot{W}} = \overline{\dot{\Phi}^{+}} + \overline{\dot{\Phi}^{-}} > 0$

- *Inequalities come from 2nd law*
- *A different view on Lorenz Energy cycle*

Entropy

- *Mixing neglected (small on global scale), LTE:* $\dot{Q} = \dot{s}T$
- *Entropy Balance of the system:*

$$\dot{S}(\Omega) = \int_{\Omega^+} dV \frac{\rho \dot{Q}^+}{T} + \int_{\Omega^-} dV \frac{\rho \dot{Q}^-}{T} = \int_{\Omega^+} dV \rho \dot{s}^+ + \int_{\Omega^-} dV \rho \dot{s}^- = \dot{\Sigma}^+ + \dot{\Sigma}^-$$

- *Long Term average:*

$$\overline{\dot{S}(\Omega)} = \overline{\dot{\Sigma}^+} + \overline{\dot{\Sigma}^-} = 0 \Rightarrow \overline{\dot{\Sigma}^+} = \left|\overline{\dot{\Sigma}^-}\right| = -\overline{\dot{\Sigma}^-} > 0$$

- *<u>Note</u>: if the system is stationary, its entropy does not grow* $\rightarrow$ *balance between generation and boundary fluxes*

Carnot Efficiency

- *Mean Value Theorem:*

$$\overline{\dot{\Phi}^{+}} = \overline{\dot{\Sigma}^{+}}\,\Theta^{+} \qquad\qquad \overline{\dot{\Phi}^{-}} = \overline{\dot{\Sigma}^{-}}\,\Theta^{-}$$

- *We have* $\Theta^{+} > \Theta^{-} > 0$

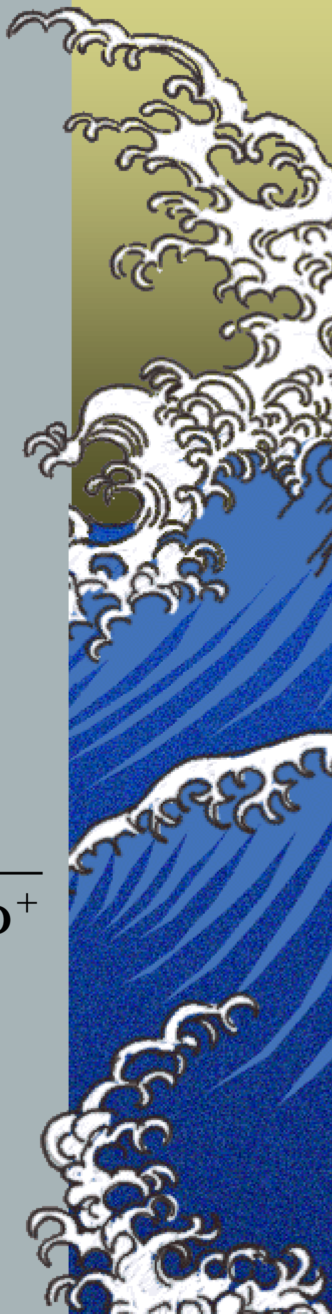
Hot *Cold* *reservoirs*

- *Work:*

$$\overline{\dot{W}} = \frac{\overline{\dot{\Phi}^{+}} + \overline{\dot{\Phi}^{-}}}{\overline{\dot{\Phi}^{+}}}\,\overline{\dot{\Phi}^{+}} = \frac{\Theta^{+} - \Theta^{-}}{\Theta^{+}}\,\overline{\dot{\Phi}^{+}}$$

- *Carnot Efficiency:*

$$\eta = \frac{\overline{\dot{\Phi}^{+}} + \overline{\dot{\Phi}^{-}}}{\overline{\dot{\Phi}^{+}}} = \frac{\Theta^{+} - \Theta^{-}}{\Theta^{+}}$$



Bounds on Entropy Production

- *Minimal Entropy Production:*

$$\overline{\dot{S}_{in}(\Omega)} \geq \overline{\dot{S}_{\min}(\Omega)} = \overline{\left(\frac{\int_{\Omega} dV \rho \dot{Q}}{\int_{\Omega} dV \rho T} \right)} \approx \frac{\overline{\dot{W}}}{\left(\Theta^{+} + \Theta^{-}\right)/2} \approx \eta \overline{\dot{\Sigma}^{+}}$$

$$\Delta\Theta / \left(\Theta^{+} + \Theta^{-}\right) << 1$$

- *Efficiency relates minimal entropy production and entropy fluctuations*
- *Min entropy production is due to dissipation:*

$$\overline{\dot{S}_{\min}(\Omega)} \approx \int_{\Omega} dV \overline{\left(\frac{\varepsilon^{2}}{T} \right)}$$

and the rest?

Entropy Production

- *Total entropy production: contributions of viscous dissipation plus heat transport:*

$$\overline{\dot{S}_{in}(\Omega)} = \int_{\Omega} dV\, \overline{\vec{H}\cdot\vec{\nabla}\left(\frac{1}{T}\right)} + \int_{\Omega} dV\, \frac{\overline{\varepsilon^2}}{T} \approx \int_{\Omega} dV\, \overline{\vec{H}\cdot\vec{\nabla}\left(\frac{1}{T}\right)} + \overline{\dot{S}_{\min}(\Omega)}$$

- *We can quantify the "excess" of entropy production, degree of irreversibility with α:*

$$\alpha = \int_{\Omega} dV\, \overline{\vec{H}\cdot\vec{\nabla}\left(\frac{1}{T}\right)} \Big/ \overline{\dot{S}_{\min}(\Omega)}$$

- *Heat Transport downgradient T field increases irreversibility*

MEPP re-examined

- *Let's look again at the Entropy production:*

$$\overline{\dot{S}_{in}(\Omega)} \approx \overline{\dot{S}_{\min}(\Omega)}(1+\alpha) \approx \eta\overline{\Sigma^{+}}(1+\alpha)$$

- *If heat transport down-gradient the temperature field is strong, η is small*
- *If the transport is weak, α is small.*

$$\Downarrow$$

- *MEPP requires a joint optimization of heat transport and of the production of mechanical work*

Can this be useful?

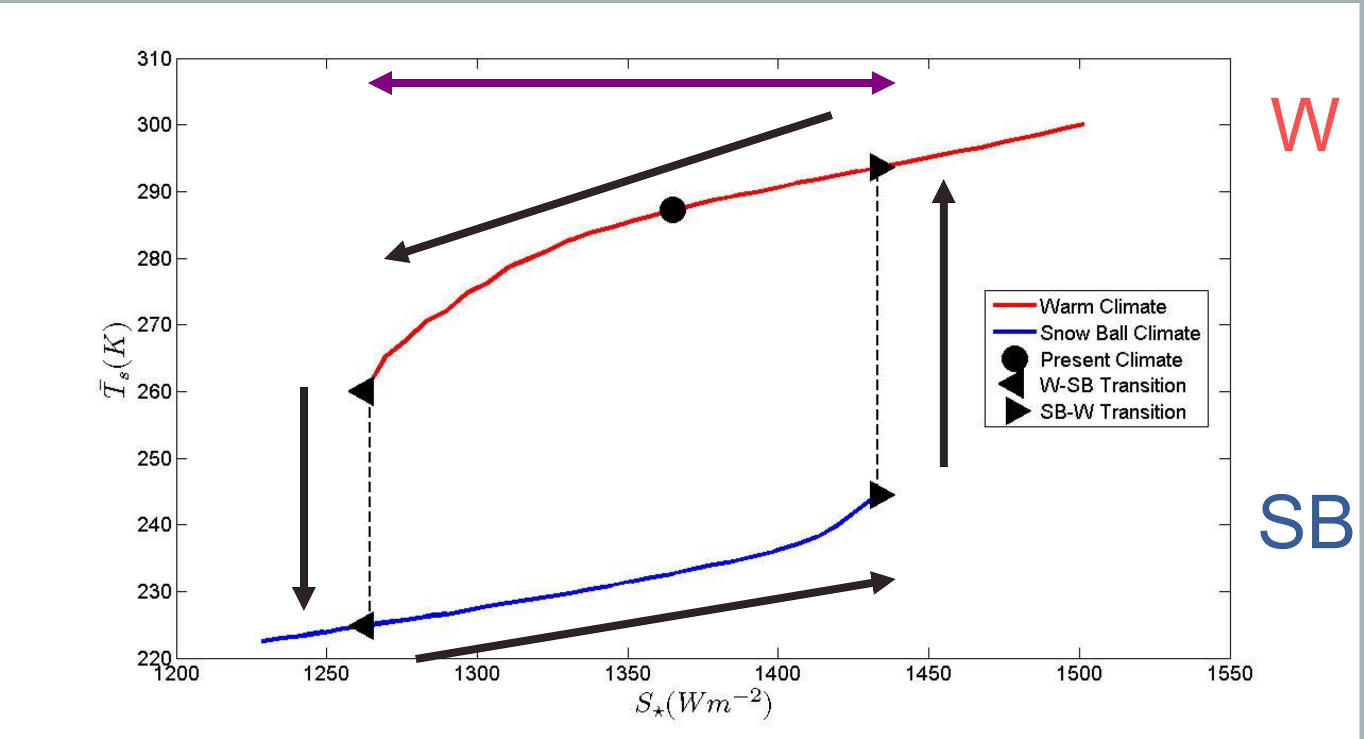
- *What we have shown provides a series of diagnostic tools for:*
  - *Defining thermodynamics of the climate system*
  - *Validating, intercomparing climate models*
  - *Analyzing impact of natural and anthropogenic forcing on climate*
    - *Dynamic Paleoclimatology à la Saltzman*
  - *Climate Feedbacks*
    - *radiation ⇔ dynamics*
  - *We have tested it together with Fraedrich & Lunkeit on classic climate experiments: Snowball Earth & CO_2 climate sensitivity*

PLASIM

- *Climate model developed at U. Hamburg (Fraedrich, Lunkeit, Blender, Kirk) from PUMA*
- *State-of-the art AGCM but T21*
- *50m mixed-layer swamp ocean with sea ice*
- *Reasonable present climate*
- *Good for long simulations, sensitivity tests; can be adapted to studying other planets…*
- *We test the theory just proposed, try to analyze macro-climatic variability using 1st and 2nd law diagnostics*

Hysteresis experiment

- *In 8000 years we make a swing of the solar constant S_* by ±10% starting from present climate*
  - *→ hysteresis experiment!*
- ***Global average surface temperature T_S***
  - *Wide (about 10%) range of S_* with bistable regime*
  - *ΔT_S is about 40-50 K*
  - d T_S/d S_* >0 *everywhere, almost linear*



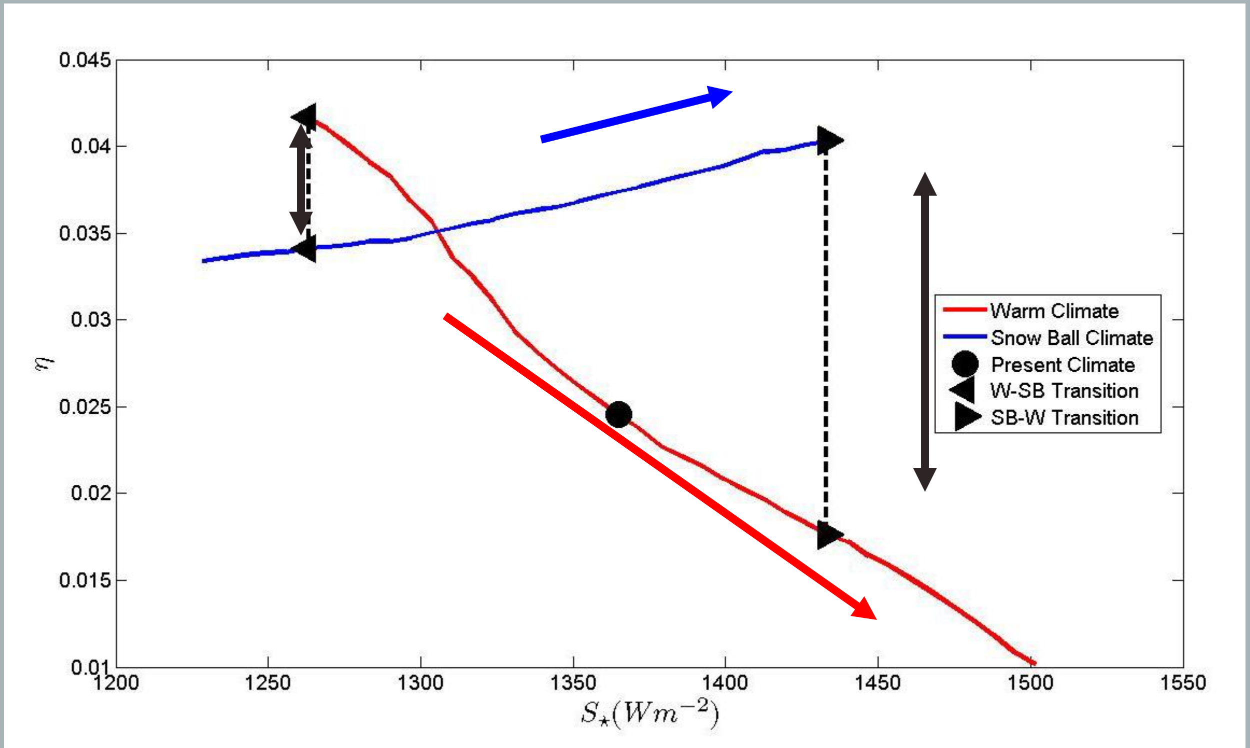
Thermodynamic Efficiency

- d η /d S_* >0 in SB regime
  - *Large T gradient due to large albedo gradient*
- d η /d S_* <0 in W regime
  - *System thermalized by efficient LH fluxes*
- *η decreases at transitions $\rightarrow$ System more stable*

η=0.04

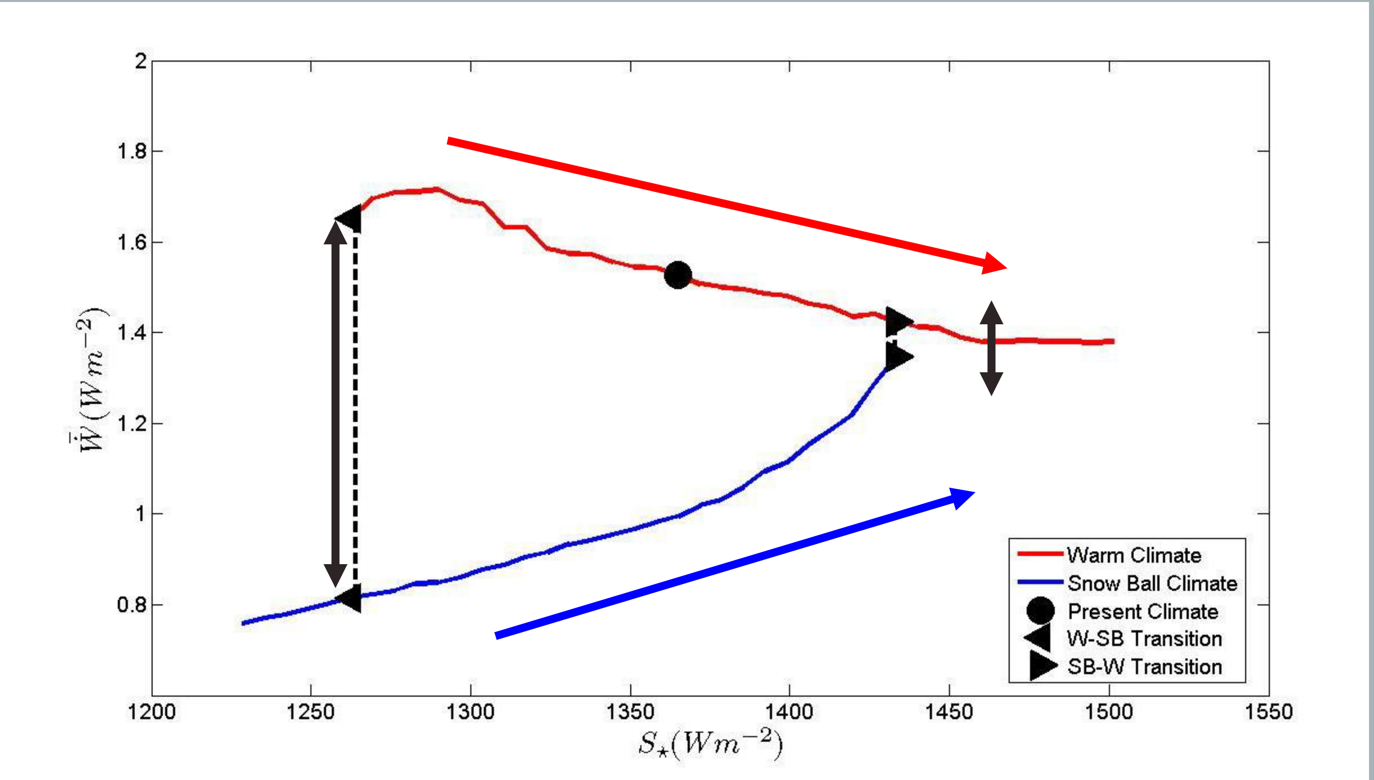
$\leftrightarrow$

$\Delta\theta$=10K



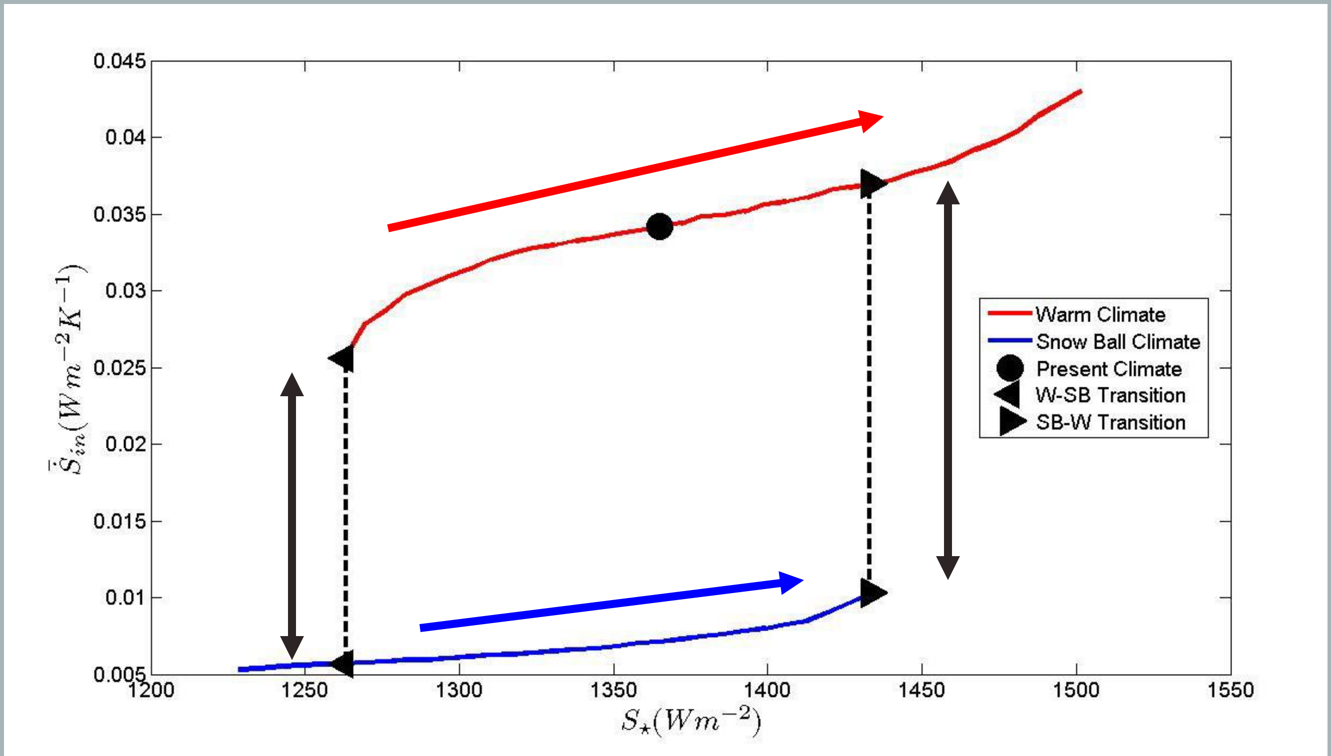
D=C(P,K) - Lorenz energy cycle

- Energy input increases with S_* in both regimes
- d W/d S_* *>0 in SB regime*
  - *Stronger circulation: more input & higher efficiency*
- d W/d S_* *≤ 0 in W regime*
  - *Weaker circulation: more input & much lower efficiency*



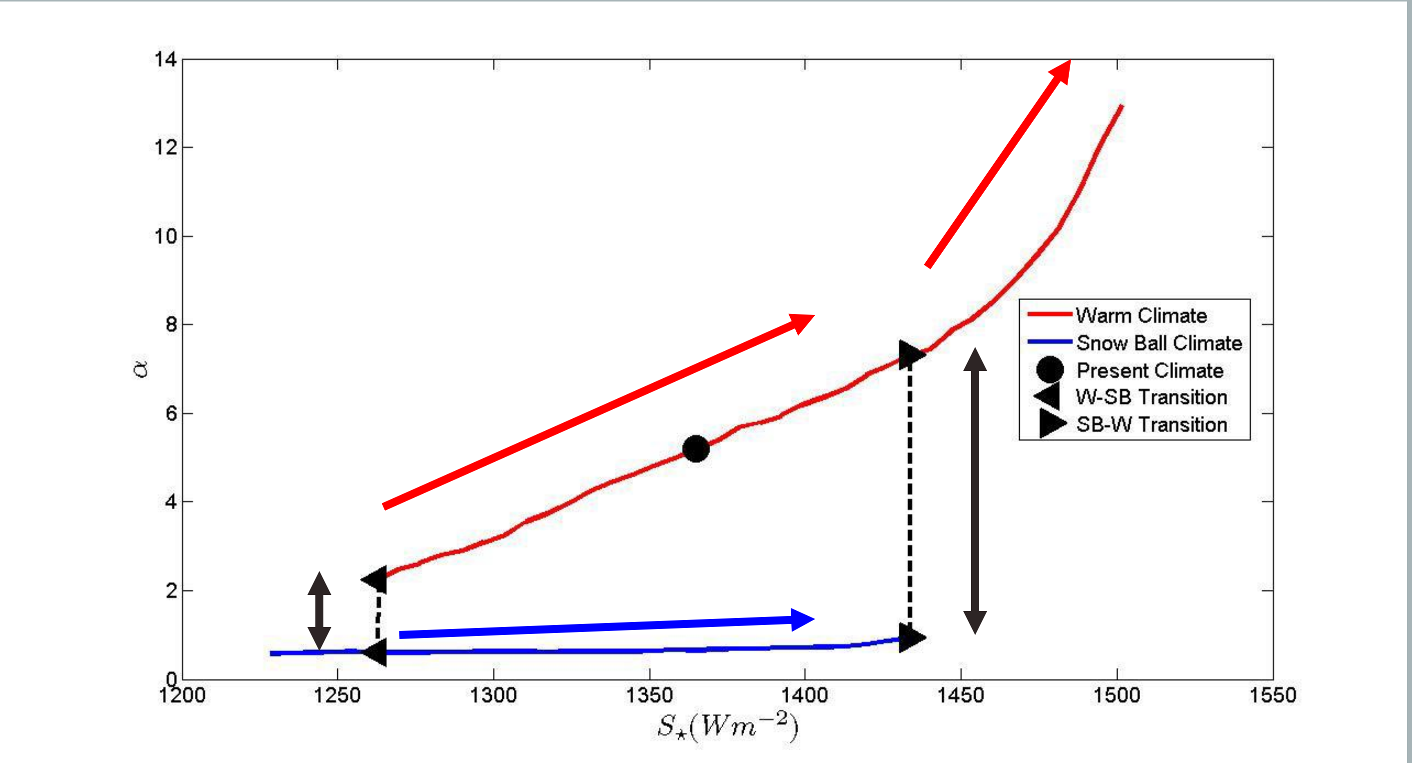
Entropy Production

- d S_{in}/d S_* *>0 in SB & W regime*
  - *Entropy production is "like" T_S… but better than T_S!*
  - *ΔS_{in} is about 400% → benchmark for SB vs W regime*
- S_{in} *is an excellent state variable*



Irreversibility

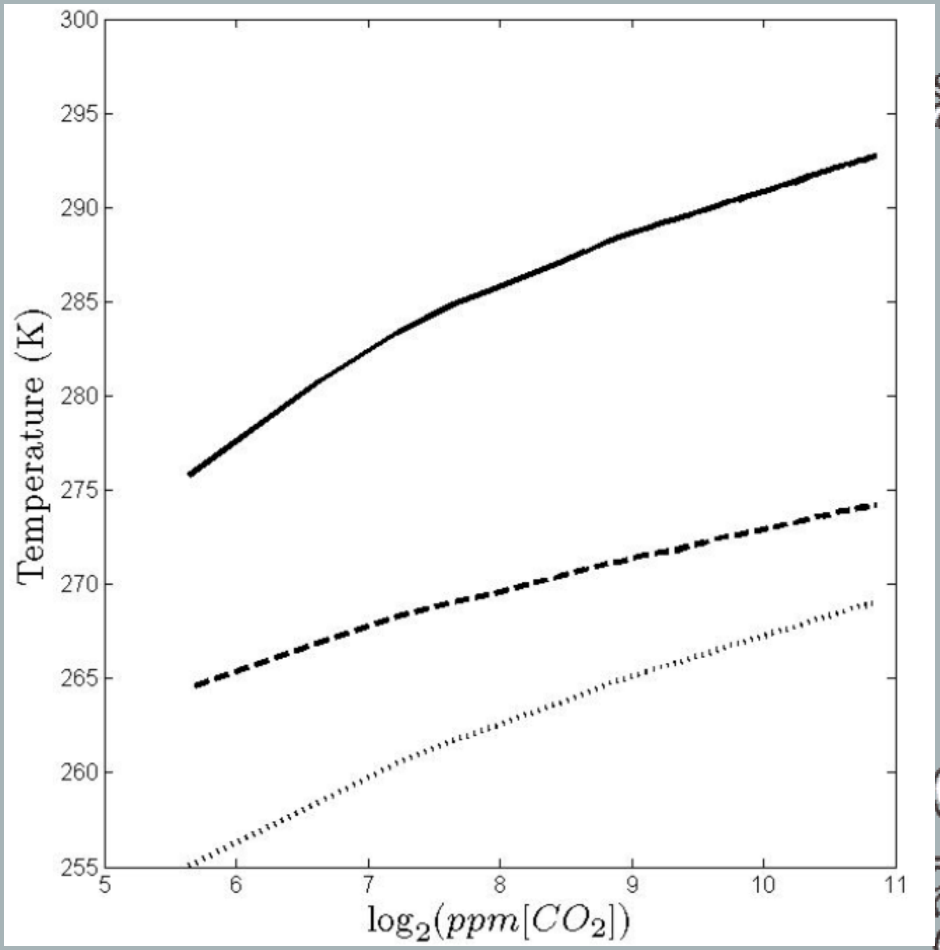
- d α/d S_* >0 *in W regime*
  - *System is VERY irreversible;* α explodes for high S_*
- d α/d S_* ~ *0 in SB regime*
  - α *is about 1*
- *Again: a qualitative difference between W and SB*

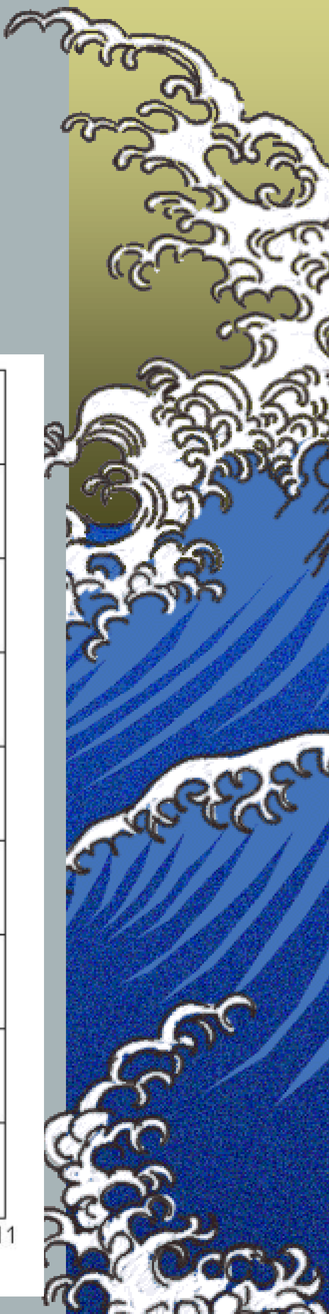


Climate sensitivity experiment

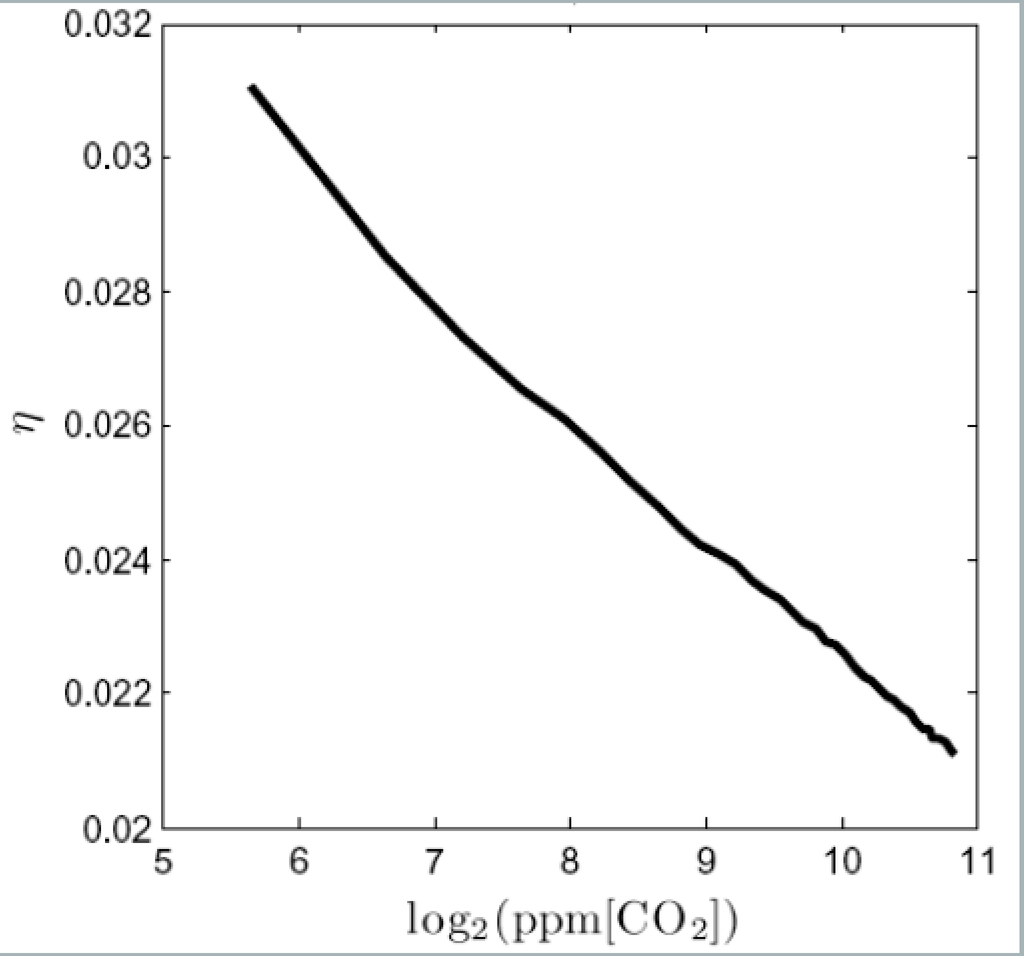
- *We explore the statistical properties of climate resulting from CO_2 concentrations ranging from 50 to 1750 ppm*
  - *→ no bistability!*
  - *→ We define a set of generalized thermodynamical sensitivities*

- ***Temperature variables***
  - *Surface temperature has the largest sensitivity*
  - *Cold bath becomes relatively warmer → linearity wrt log([CO_2])*
  - *System tends to become more isothermal with higher [CO_2]*



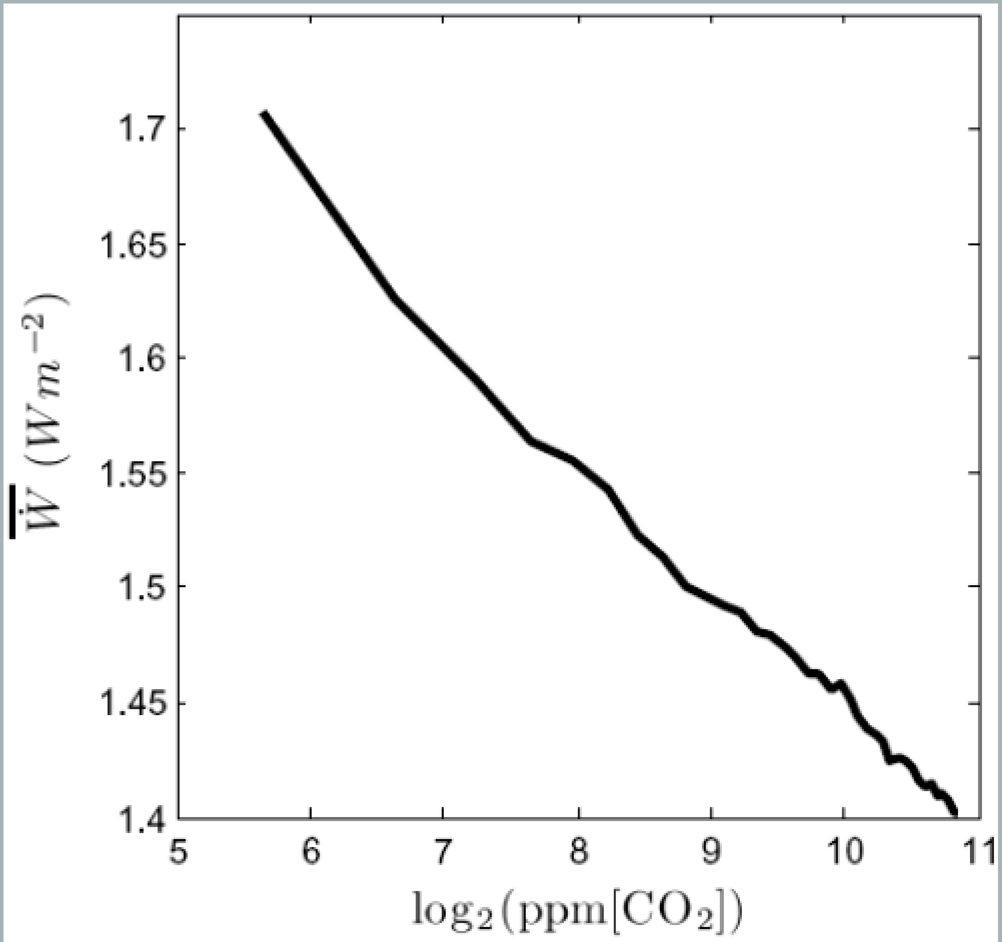


Efficiency



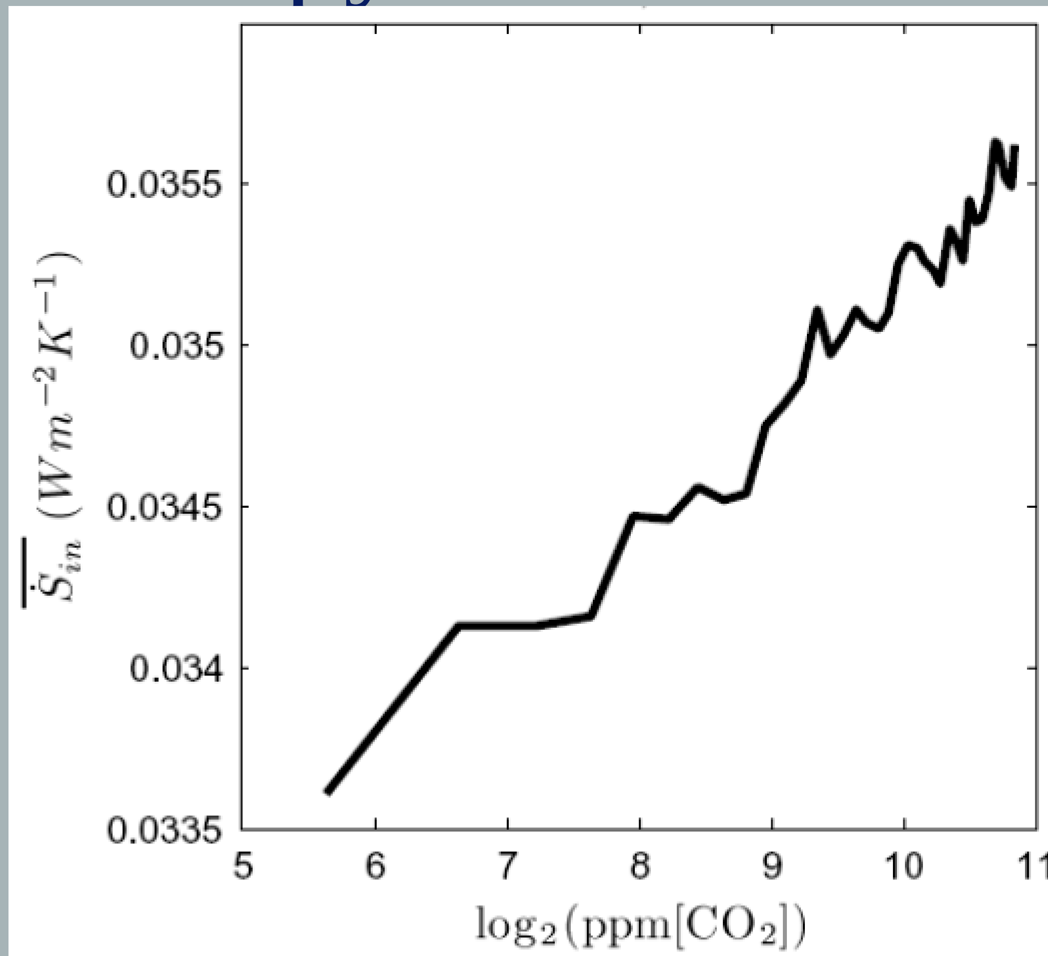
- *The system becomes more isothermal → The efficiency decreases with increasing CO_2 concentration*
- *Relative decrease is 6% per CO_2 doubling*
- *Latent heat fluxes play a crucial role*

Lorenz Energy Cycle



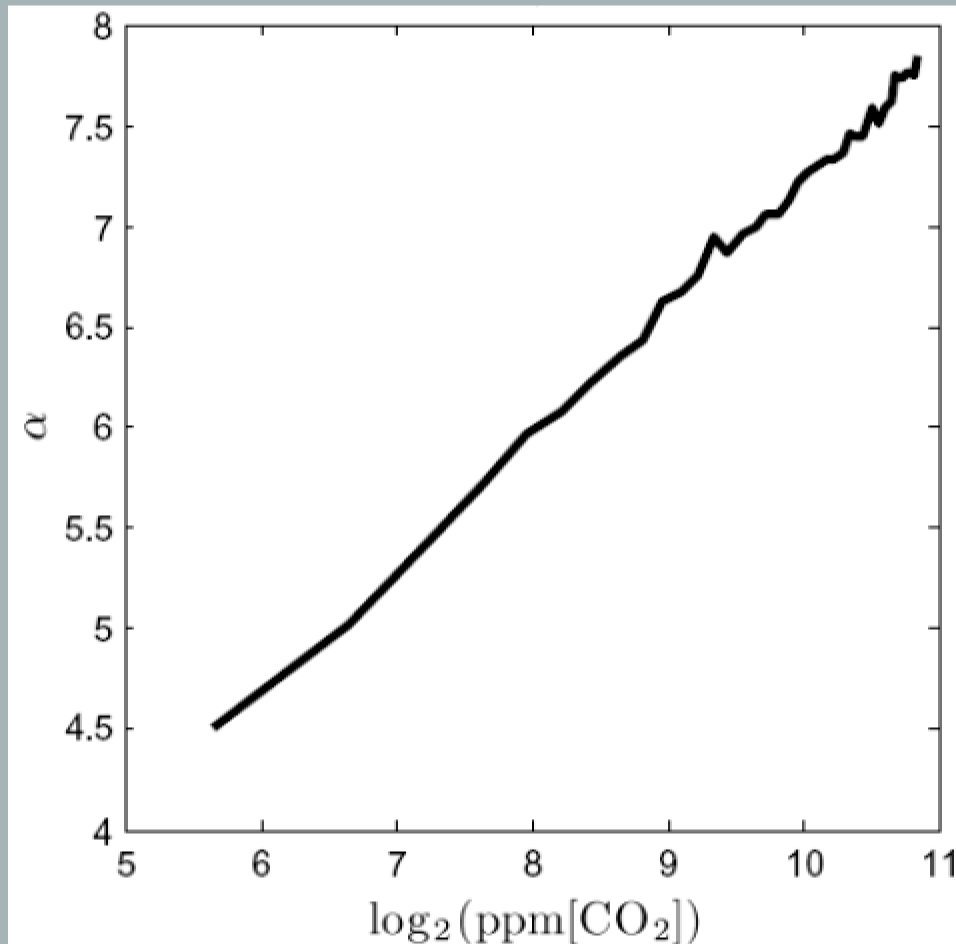
- *Trade-off between higher heat absorption and lower efficiency: Lorenz energy cycle is weaker when [CO_2]↑*
- *Relative decrease is 3% per [CO_2]doubling*
- *Same applies for dissipation: lower surface winds*

Entropy Production



- *Lower dissipation & higher temperature → lower entropy production by viscous processes*
- *Nevertheless, total material entropy production increases*
  - *Greater role by heat transport contributions → LH fluxes*

Irreversibility



- *The irreversibility of the system increases with [CO_2]*
- *The system becomes closer to a "conductive" system producing negligible mechanical work*

Generalized Sensitivities

Definition	Value
$\Lambda_{\overline{T_S}}$	2.55 K
Λ_{Θ^+}	1.65 K
Λ_{Θ^-}	2.35 K
Λ_{η}	−0.002
$\Lambda_{\overline{\dot{W}}}$	$-0.06\ \mathrm{Wm^{-2}}$
$\Lambda_{\overline{\dot{S}_{in}}}$	$0.0004\ \mathrm{Wm^{-2}\,K^{-1}}$
Λ_{α}	0.7

Parameterisations

Definition	Value
$d\Theta^{+} / d\overline{T_s}$	0.65
$d\Theta^{-} / d\overline{T_s}$	0.92
$d\eta / d\overline{T_s}$	$-0.0008\,\mathrm{K}^{-1}$
$d\overline{\dot{W}} / d\overline{T_s}$	$-0.024\,\mathrm{Wm}^{-2}\,\mathrm{K}^{-1}$
$d\overline{\dot{S}_{\mathrm{in}}} / d\overline{T_s}$	$0.00016\,\mathrm{Wm}^{-2}\,\mathrm{K}^{-2}$
$d\alpha / d\overline{T_s}$	$0.275\,\mathrm{K}^{-1}$

Thermodynamic Bounds

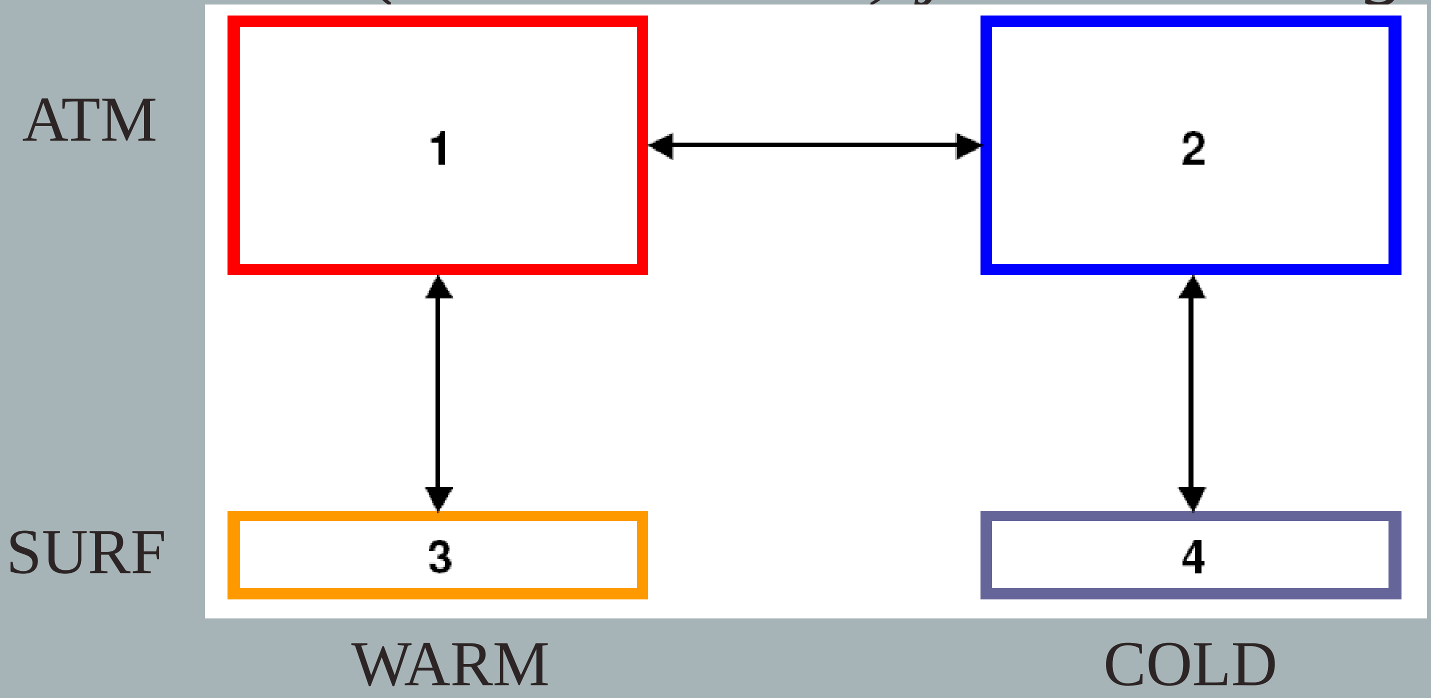
- *All of this looks good, but we need 3D data!*
- *Thermodynamic bounds for entropy production and Lorenz energy cycle based only on average TOA 2D radiative fields*
- *Good for data from planetary objects*

$$\overline{\dot{S}_{mat}} > \overline{\dot{S}_{mat}^{diff}} = \int_A \frac{\overline{\varepsilon^2}}{T_{diss}} d\sigma > \overline{\dot{S}_{mat}^{hor}} = \left(\frac{\left\langle \overline{R_{net}} \right\rangle_{>} \left| A_{>} \right|}{\left\langle T_E \right\rangle_{>}} + \frac{\left\langle \overline{R_{net}} \right\rangle_{<} \left| A_{<} \right|}{\left\langle T_E \right\rangle_{<}} \right)$$

$$\overline{\dot{W}} \geq \overline{\dot{W}_{\min}} = -\left\langle T_E \right\rangle \left(\frac{\left\langle \overline{R_{net}} \right\rangle_{>} \left| A_{>} \right|}{\left\langle T_E \right\rangle_{>}} + \frac{\left\langle \overline{R_{net}} \right\rangle_{<} \left| A_{<} \right|}{\left\langle T_E \right\rangle_{<}} \right)$$

Vertical Structure

- *If no vertical temperature structure, the inequalities become identities!*
- *A minimal model for EP requires 4 boxes (atmo/surf, warm/cold)*
- *2 Boxes (R. Lorenz etc.) just not enough*



Earth, Mars, Venus, Titan

- *Bounds can be easily computed from coarse resolution TOA data*
- *With LW data we obtain effective temperatures*
- *With SW data we obtain the budgets*

Celestial Body	$\|A\|(m^2)$	$\langle T_E \rangle_< (K)$	$\langle T_E \rangle_> (K)$	η_h	$\overline{F_{\max}}/\|A\|$ (Wm^{-2})	$\overline{\dot{S}^{hor}_{mat}}/\|A\|$ $(WK^{-1}m^{-2})$	$\overline{\dot{W}_{\min}}/\|A\|$ (Wm^{-2})
Earth	5.1×10^{14}	248.3	259.8	0.044	10.01	3.8×10^{-3}	9.7×10^{-1}
Mars	1.4×10^{14}	201.9	222.5	0.096	0.32	2.8×10^{-4}	5.9×10^{-2}
Venus	4.1×10^{14}	227.9	231.0	0.014	7.60	8.8×10^{-4}	2.0×10^{-1}
Titan	8.3×10^{13}	82.8	85.0	0.026	0.12	3.7×10^{-5}	3.1×10^{-3}

Energy Scaling

- *We can scale the thermo terms with respect to suitable powers of average SW=S(1-α)*
- *Differences will depend on circulation*
- *All results are within one order of magnitude!*

Celestial Body	$\lvert A\rvert(m^2)$	$S(1-\alpha)$ (Wm^{-2})	r	$\langle T_E\rangle_<$ (K)	$\langle T_E\rangle_>$ (K)	η_h	$\overline{H_{\max}}/\lvert A\rvert$ (Wm^{-2})	$\overline{\dot{S}^{hor}_{mat}}/\lvert A\rvert$ $(WK^{-1}m^{-2})$	$\overline{\dot{W}_{\min}}/\lvert A\rvert$ (Wm^{-2})
Earth	$5.1\cdot10^{14}$	237.0	1.000	248.3	259.8	0.044	10.01	3.8×10^{-3}	9.7×10^{-1}
Mars	$1.4\cdot10^{14}$	116.7	0.492	241.0	265.6	0.096	0.65	4.8×10^{-4}	1.2×10^{-1}
Venus	$4.1\cdot10^{14}$	157.3	0.664	252.5	256.0	0.014	11.41	1.2×10^{-3}	3.0×10^{-1}
Titan	$8.3\cdot10^{13}$	2.81	0.012	250.9	257.6	0.026	10.08	1.0×10^{-3}	2.6×10^{-1}

Conclusions

- *Theoretical framework linking previous finding on the efficiency of the climate system to its entropy production.*
- *Unifying picture connecting the Energy cycle to the MEPP;*
- *Test of these results on Snow Ball hysteresis experiment, and some ideas on mechanisms involved in climate transitions;*
- *Analysis of the impact of [CO_2] increase, with a generalized set of climate sensitivities and a proposal for parameterizations and diagnosic studies*
- *Thermodynamic bounds seem promising! (new results)*
- *Issues:*
  - *Are climate models balanced in terms of energy budgets? No! (Lucarini & Ragone 2010) Can this be a problem? Yes! (Lucarini & Fraedrich 2009)-We have introduced eqs. also for dealing with biases*
  - *What is the role of the ocean?*
  - *Climate tipping points?*
  - *Other celestial bodies?*

References

- *Lucarini V., Thermodynamic Efficiency and Entropy Production in the Climate System, Phys. Rev. E 80, 021118 (2009)*
- *Lucarini V., Fraedrich K., Symmetry breaking, mixing, instability, and low-frequency variability in a minimal Lorenz-like system, Phys. Rev. E 80, 026313 (2009)*
- *Lucarini V., Fraedrich K., Lunkeit F., Thermodynamic Analysis of Snowball Earth Hysteresis Experiment: Efficiency, Entropy Production, and Irreversibility, QJRMS 135, 2-11 (2010)*
- *Lucarini V., Fraedrich K., Lunkeit F., Thermodynamics of Climate Change: Generalized sensitivities, ACPD 10, 3699-3715 (2010)*
- *Lucarini V., Ragone F., Energetics of IPCC4AR Climate Models: Energy Balance and Meridional Enthalpy Transports, submitted to Rev. Geophys. also at arXiv:0911.5689v1 [physics.ao-ph] (2010)*
- *Lucarini V., Fraedrich K, Bounds on the thermodynamical properties of a planet based upon its radiative budget at the top of the atmosphere: theory and results for Earth, Mars, Titan, and Venus, submitted to GRL (2010); also at arXiv:1002.0157v2 [physics.ao-ph] (2010)*