Climate Thermodynamics 2010

Wed 21 April
12:15 Arrival
12:30 Lunch
13:30 Workshop starts in **1L61**
13:30 Maarten Ambaum: Welcome, introduction; global heat flows in the atmosphere.
14:00 Tim Palmer: Climate Model Bias and the Fluctuation-Dissipation Theorem.
14:30 TBA.
15:00 Break.
15:30 Jonathan Gregory: Energetic analysis of changes in the AMOC under increasing CO2.
16:00 Kevin Oliver: An approximation for the structure of global meridional overturning in the ocean, as a function of the gravitational potential energy generation and surface density fields.
16:30 Valerio Lucarini: Efficiency and Entropy Production in the Climate System.
Drinks and dinner.

Thu 22 April
9:00 Workshop reconvenes in **1L43**.
9:00 Peter Jan van Leeuwen: Information transfer and entropy in large-dimensional systems.
9:30 Salvatore Pascale: Entropy production in HadCM3 model and MEP conjecture for objective tuning.
10:00 Richard Allan: Thermodynamic and Energy Constraints on Precipitation.
10:30 Break.
11:00 Bob Plant: Self-organized criticality in tropical convection.
11:30 Christopher Dancel: The sensitivity of an Ocean Model's Architecture to the latent heat transport in the Atmosphere.
12:00: Remi Tailleux: Dynamics/Thermodynamics coupling in the incompressible Boussinesq model.
Close: short discussion on the way forward (future workshops, publications, consortium bids, etc.)

Global Heat Flows in the Atmosphere

Maarten Ambaum
Department of Meteorology
University of Reading

1 The global energy budget
2 The global entropy budget
3 Closing thoughts

The global energy budget

Earth system at equilibrium:

energy in = energy out ≈ 120 PW (120×10^{15} W)

The global energy budget

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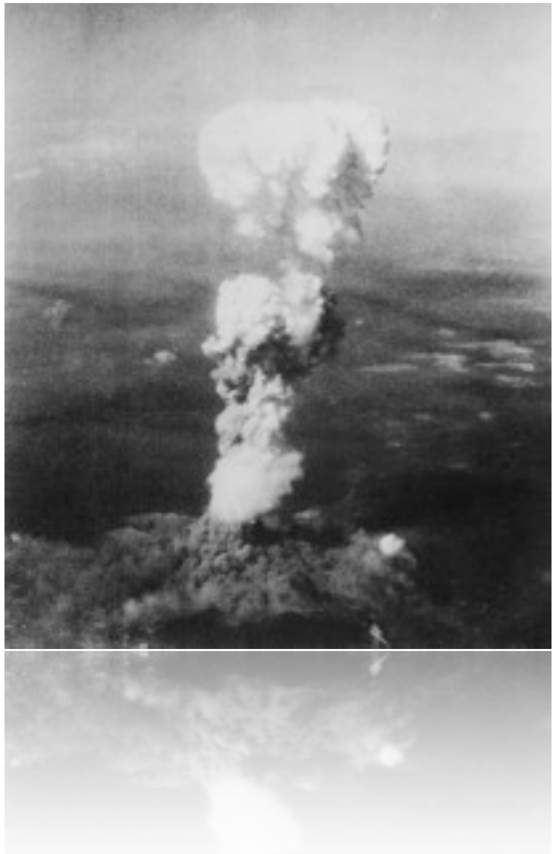


= 1500×Hiroshima *each second*

The energy in = 120 PW = 240 W/m^2 on average

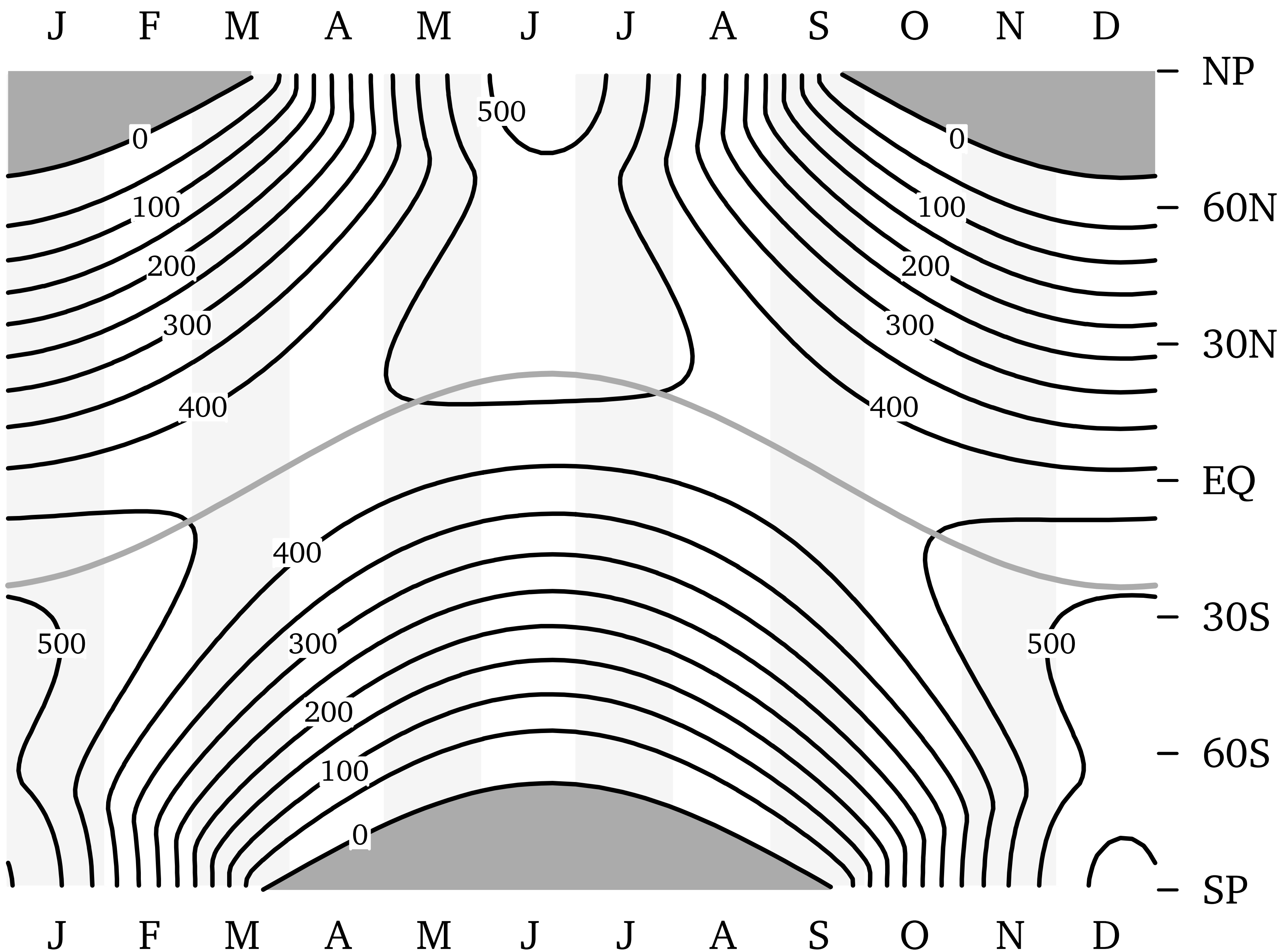
For Reading area (55 km^2):

Energy in = 10 GW = 3 × Didcot power station

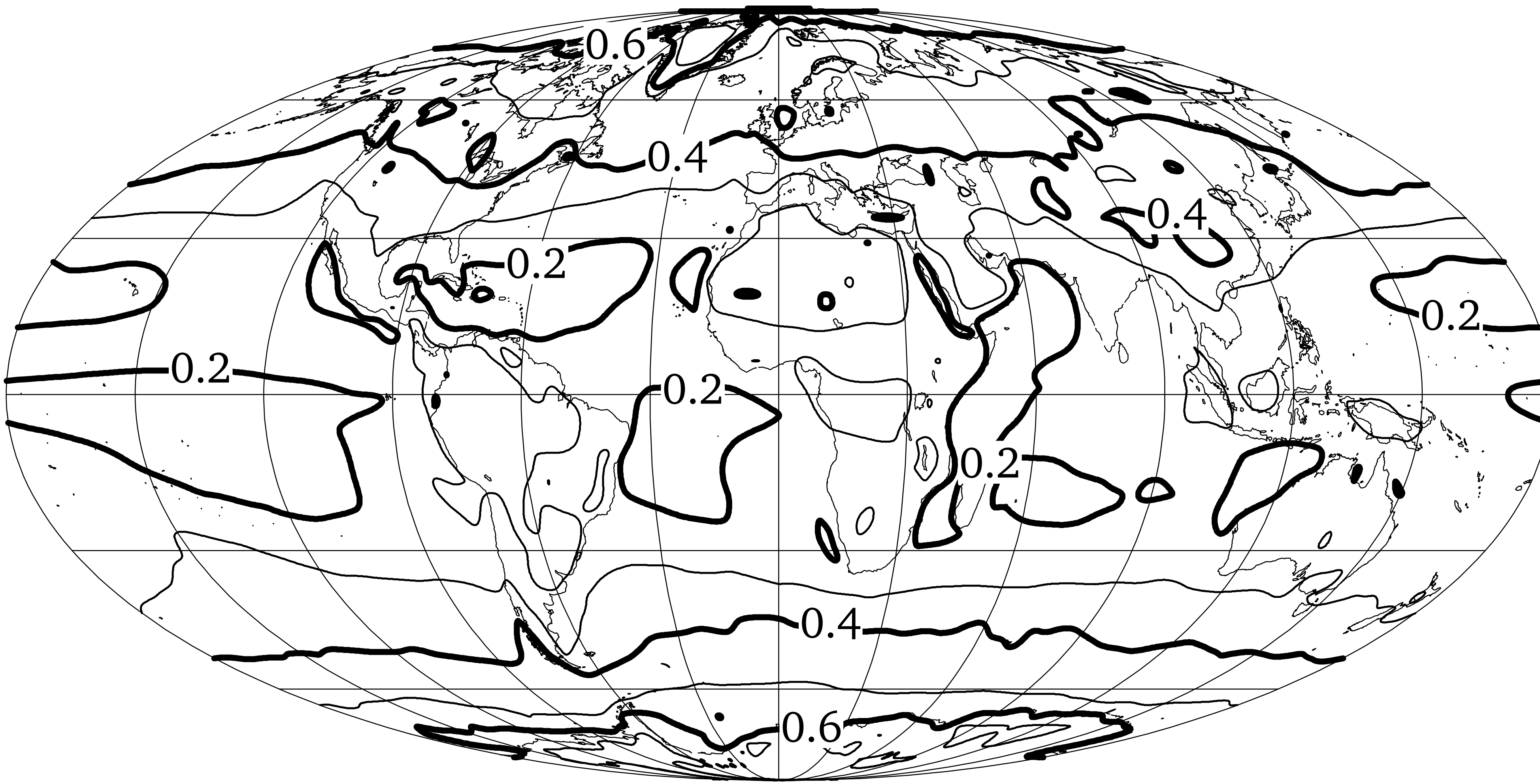
The energy in = 120 PW

= insolated – reflected
= 170 PW – 50 PW

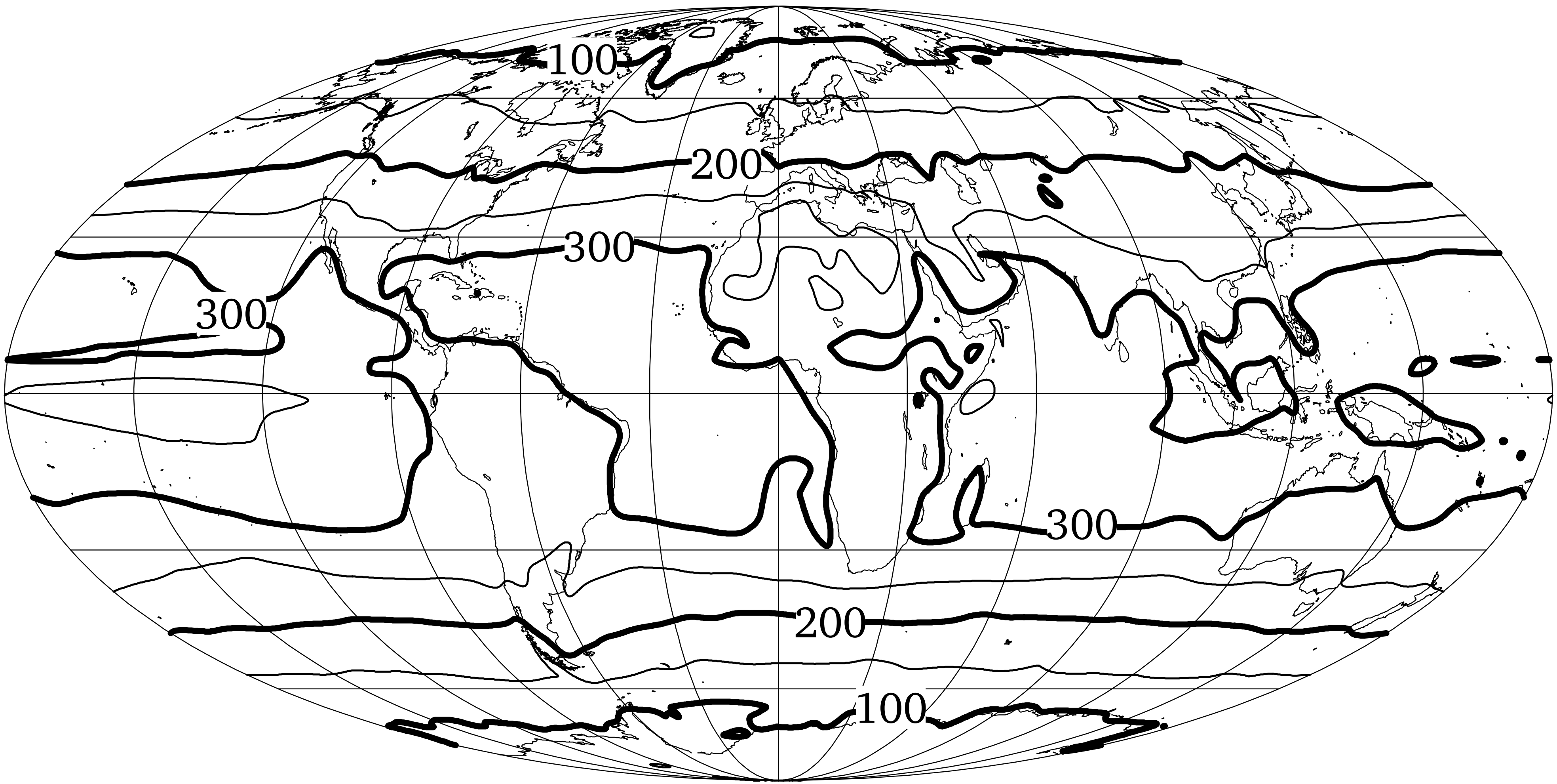
Insolation



Reflected fraction (albedo)



Absorbed short-wave



(units W/m^2)

Of the energy in = 120 PW,

80 PW go to the tropics,
40 PW go to the extratropics.

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80 PW go to the tropics,
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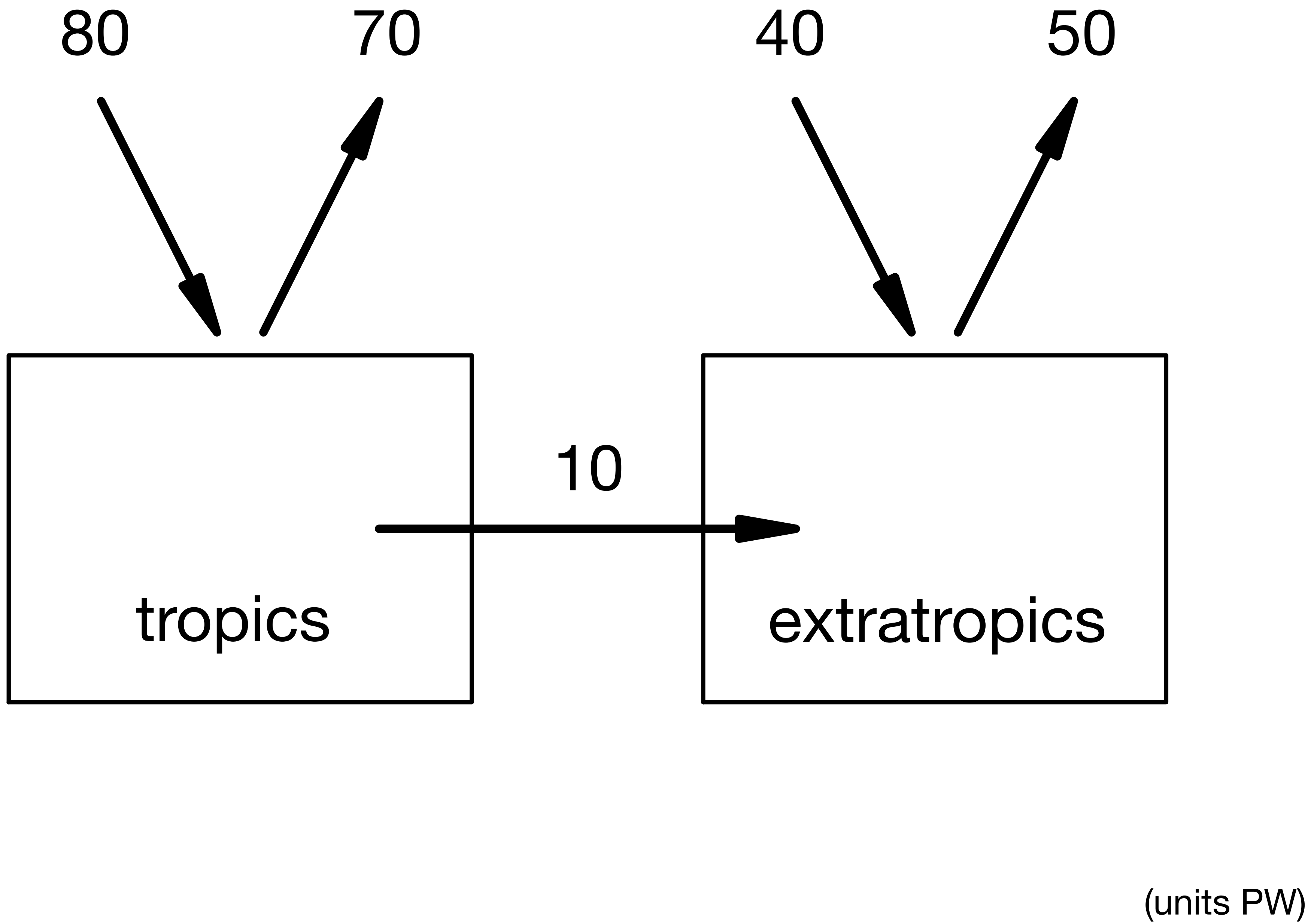
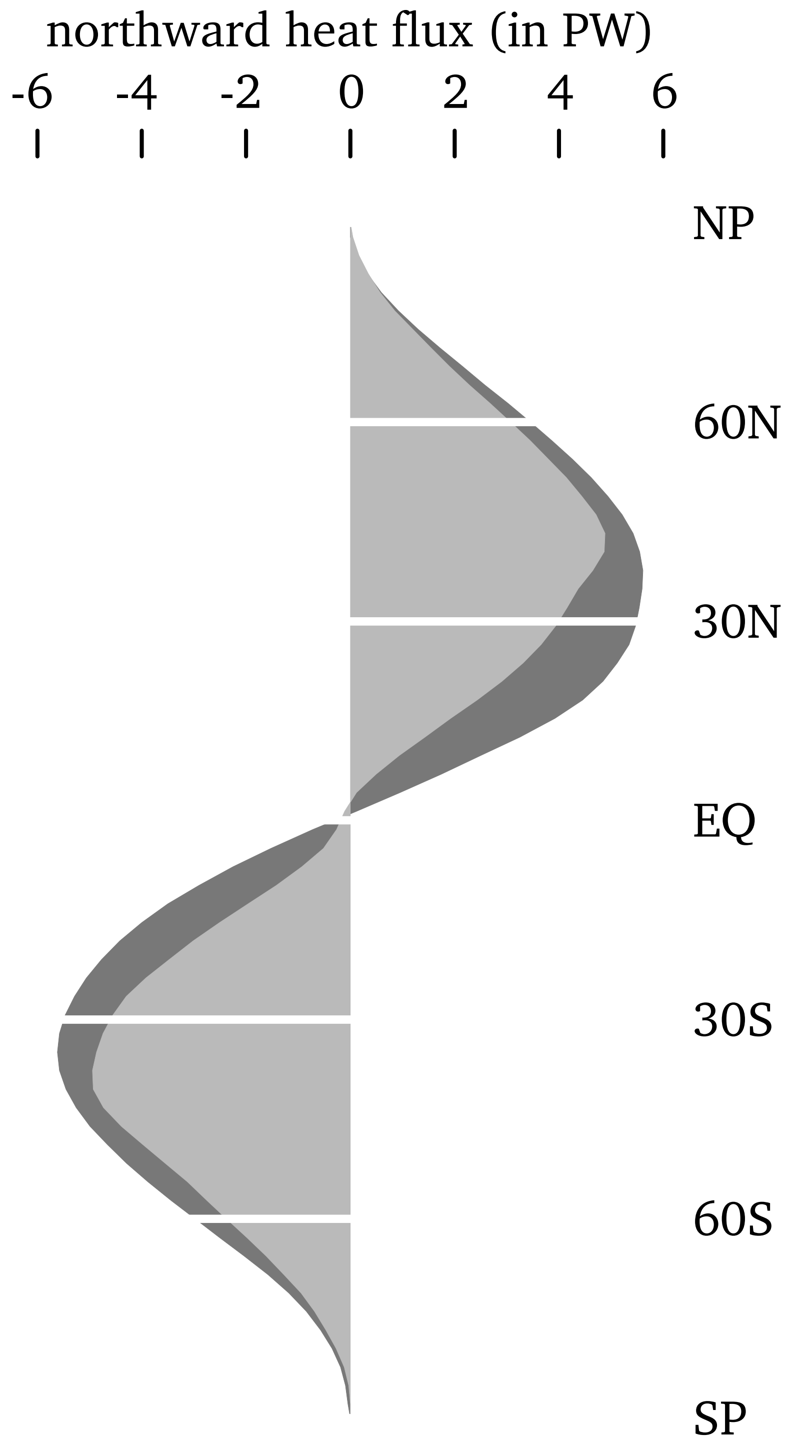
Of the energy out = 120 PW,

70 PW come from the tropics,
50 PW come from the extratropics,
10 PW is transported from tropics to extratropics.


80
70
40
50
10
tropics
extratropics
(units PW)



after Trenberth & Caron, *J. Clim.*, 2001

The global entropy budget

The entropy budget for a body of fluid

The first law: $\mathrm{d}U/\mathrm{d}t = \dot{Q} + \dot{W}$

$$\dot{Q} = -\int_A \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} \, \mathrm{d}A \qquad \dot{W} = -\int_A p\boldsymbol{U} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}A.$$

Now define:

$$F_{\mathrm{in}} = \begin{cases} 0 & \text{if } \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} > 0 \\ -\boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} & \text{if } \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} < 0 \end{cases} \qquad F_{\mathrm{out}} = \begin{cases} \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} & \text{if } \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} > 0 \\ 0 & \text{if } \boldsymbol{F}_q \cdot \hat{\boldsymbol{n}} < 0 \end{cases}$$

$$\Rightarrow \qquad \dot{Q} = \int_A F_{\mathrm{in}} \, \mathrm{d}A - \int_A F_{\mathrm{out}} \, \mathrm{d}A.$$

The entropy budget: $\mathrm{d}S/\mathrm{d}t = \mathrm{d}_e S/\mathrm{d}t + \mathrm{d}_i S/\mathrm{d}t$

$$\frac{\mathrm{d}_e S}{\mathrm{d}t} = -\int_A \frac{\boldsymbol{F}_q \cdot \hat{\boldsymbol{n}}}{T}\,\mathrm{d}A \quad \text{and} \quad \frac{\mathrm{d}_i S}{\mathrm{d}t} \geq 0.$$

then:

$$\frac{\mathrm{d}_e S}{\mathrm{d}t} = \frac{1}{T_{\text{in}}}\int_A F_{\text{in}}\,\mathrm{d}A - \frac{1}{T_{\text{out}}}\int_A F_{\text{out}}\,\mathrm{d}A,$$

where

$$\frac{1}{T_{\text{in}}} = \left(\int_A F_{\text{in}}\,\mathrm{d}A\right)^{-1}\int_A \frac{F_{\text{in}}}{T}\,\mathrm{d}A,$$

(analogous for T_{out})

We can now derive for the work output $\dot{L} = -\dot{W}$:

$$\dot{L} = \left(1 - \frac{T_{\mathrm{out}}}{T_{\mathrm{in}}}\right) \int_A F_{\mathrm{in}} \, \mathrm{d}A - T_{\mathrm{out}} \frac{\mathrm{d}_i S}{\mathrm{d}t}$$

(This general expression for the second law includes the Carnot theorem and the Guoy-Stodola theorem)

Back to the Earth system ....

First write:

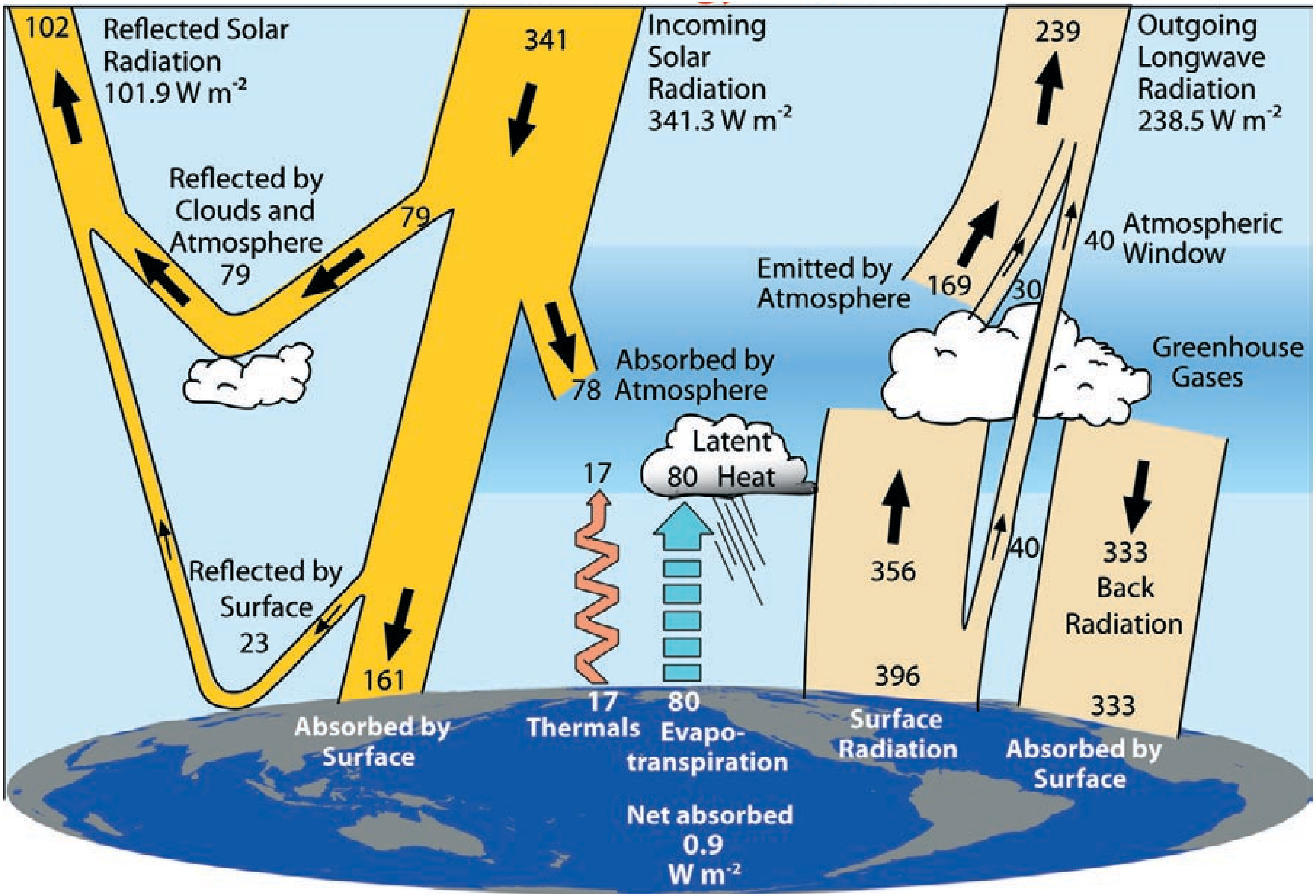
$$\dot{R}_{\text{in}} = \int_A F_{\text{in}}\,\mathrm{d}A \quad \text{and} \quad \dot{R}_{\text{out}} = \int_A F_{\text{out}}\,\mathrm{d}A$$

Both are equal to 120 PW.

At equilibrium:

$$\frac{\mathrm{d}_i S}{\mathrm{d}t} = -\frac{\mathrm{d}_e S}{\mathrm{d}t} = \dot{R}_{\text{in}} \left(\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}} \right)$$

Make T_{in} and T_{out} a weighted average of surface and mean bolometric temperature



from Trenberth, Fasullo & Kiehl, *Bull. Am. Met. Soc.*, 2009

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Make T_{in} and T_{out} a weighted average of surface and mean bolometric temperature to find:

$$T_{\text{in}} \approx 276\,\mathrm{K} \quad \text{and} \quad T_{\text{out}} \approx 260\,\mathrm{K}.$$

... substitute to find the *material entropy production:*

$$\mathrm{d}_i S/\mathrm{d}t \approx 53\,\mathrm{mW\,m^{-2}\,K^{-1}}$$

(Beware confused discussions about entropy in the radiation field!)

$$T_{\text{in}} \approx 276\,\text{K} \quad \text{and} \quad T_{\text{out}} \approx 260\,\text{K}.$$

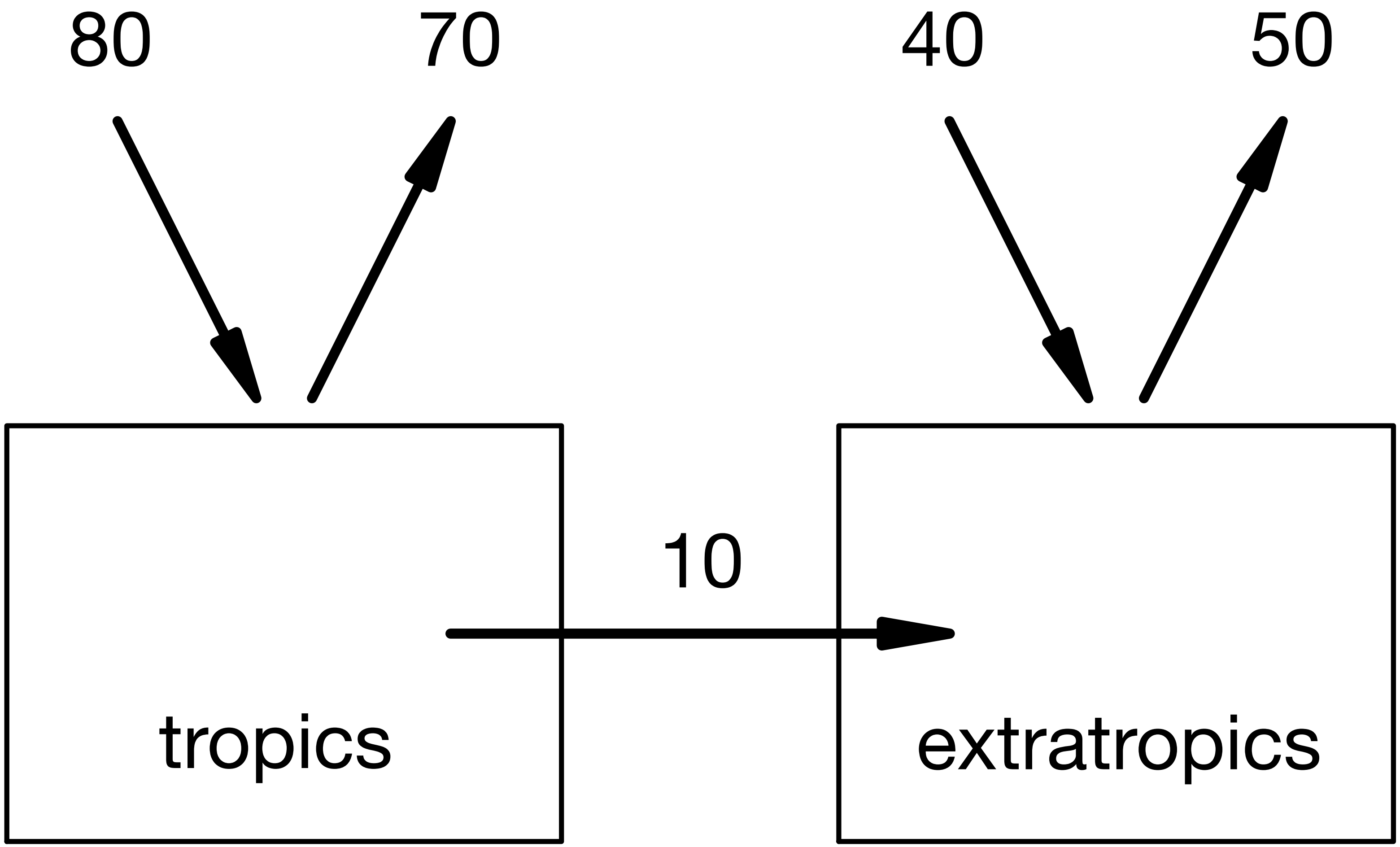
Let’s assume these temperatures are the effective mean temperatures between which the atmosphere operates

max efficiency: $$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}} \approx 0.06$$

max power: $$\eta_{\text{endoreversible}} = 1 - \sqrt{\frac{T_{\text{out}}}{T_{\text{in}}}} \approx 0.03$$

This corresponds to 14 Wm^{-2} and 7 Wm^{-2}; compare to “observed” dissipation rate of 3.5 Wm^{-2}.

Closing thoughts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70
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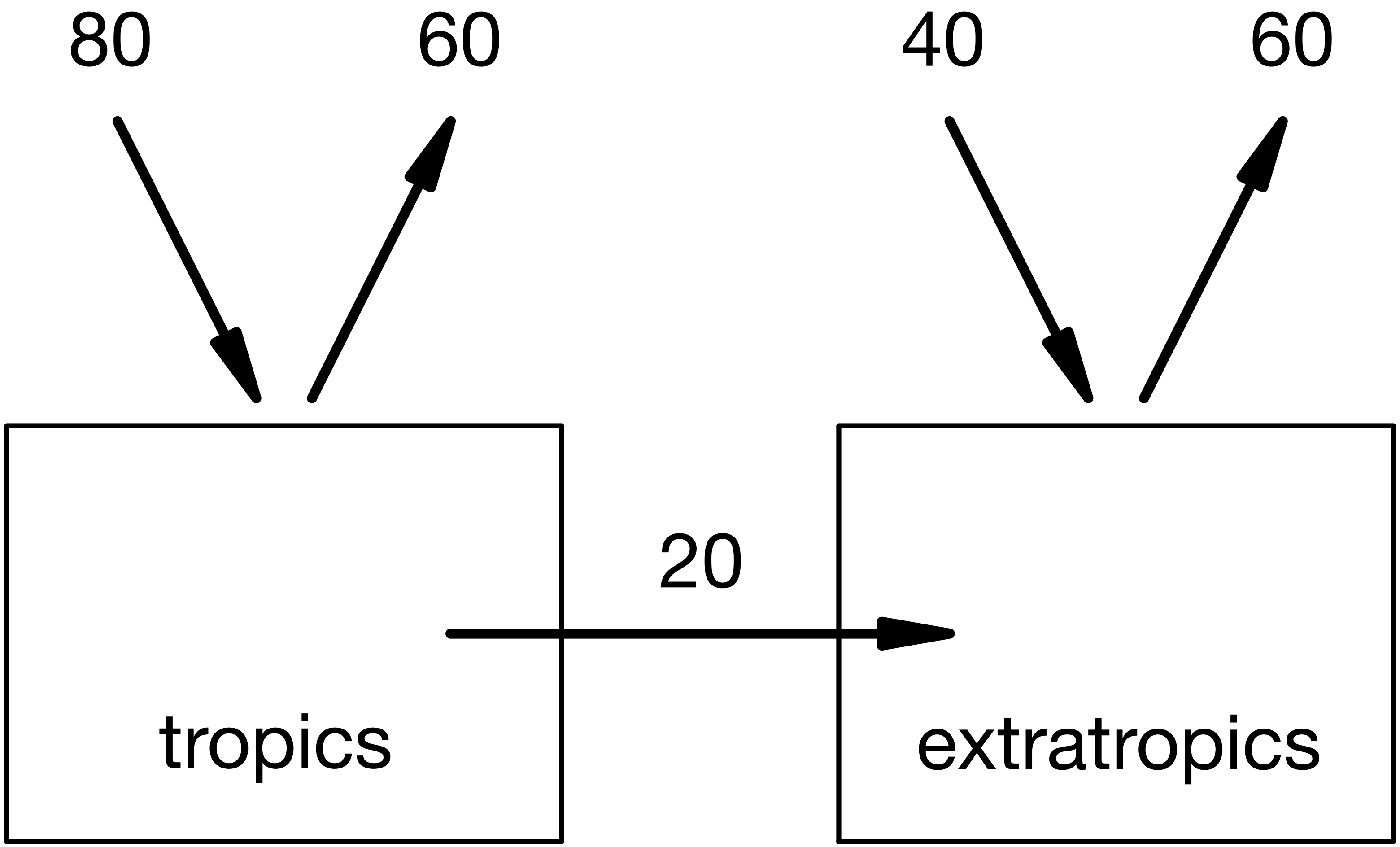
10
tropics
extratropics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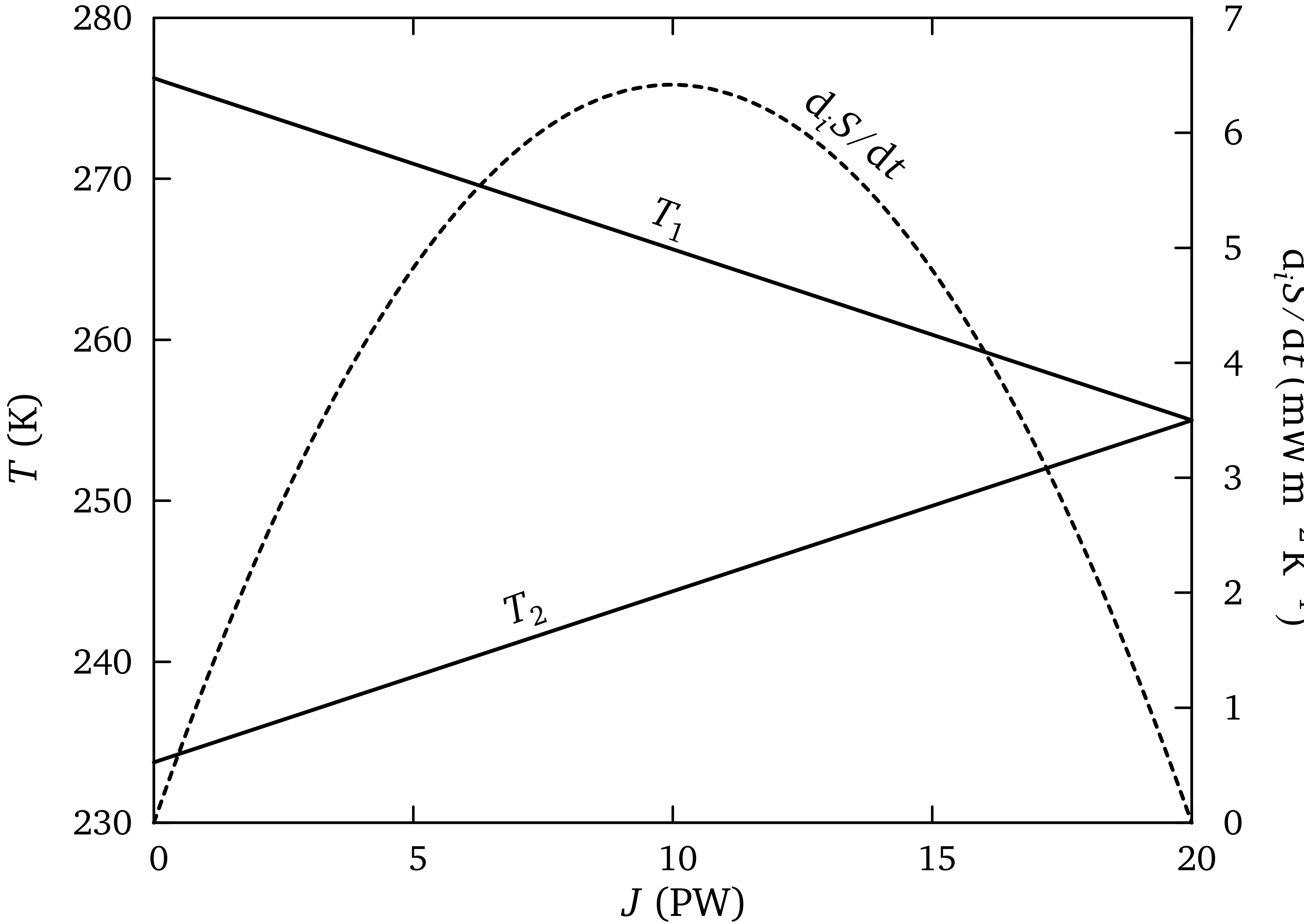
280
270
260
250
240
230
T (K)
0
5
10
15
20
J (PW)
7
6
5
4
3
2
1
0
d_iS/dt (mW m^{-2} K^{-1})
d_iS/dt
T_1
T_2

Back to our general formula:

$$\dot{L} = \left(1 - \frac{T_{\mathrm{out}}}{T_{\mathrm{in}}}\right) \int_A F_{\mathrm{in}}\, \mathrm{d}A - T_{\mathrm{out}} \frac{\mathrm{d}_i S}{\mathrm{d}t}$$

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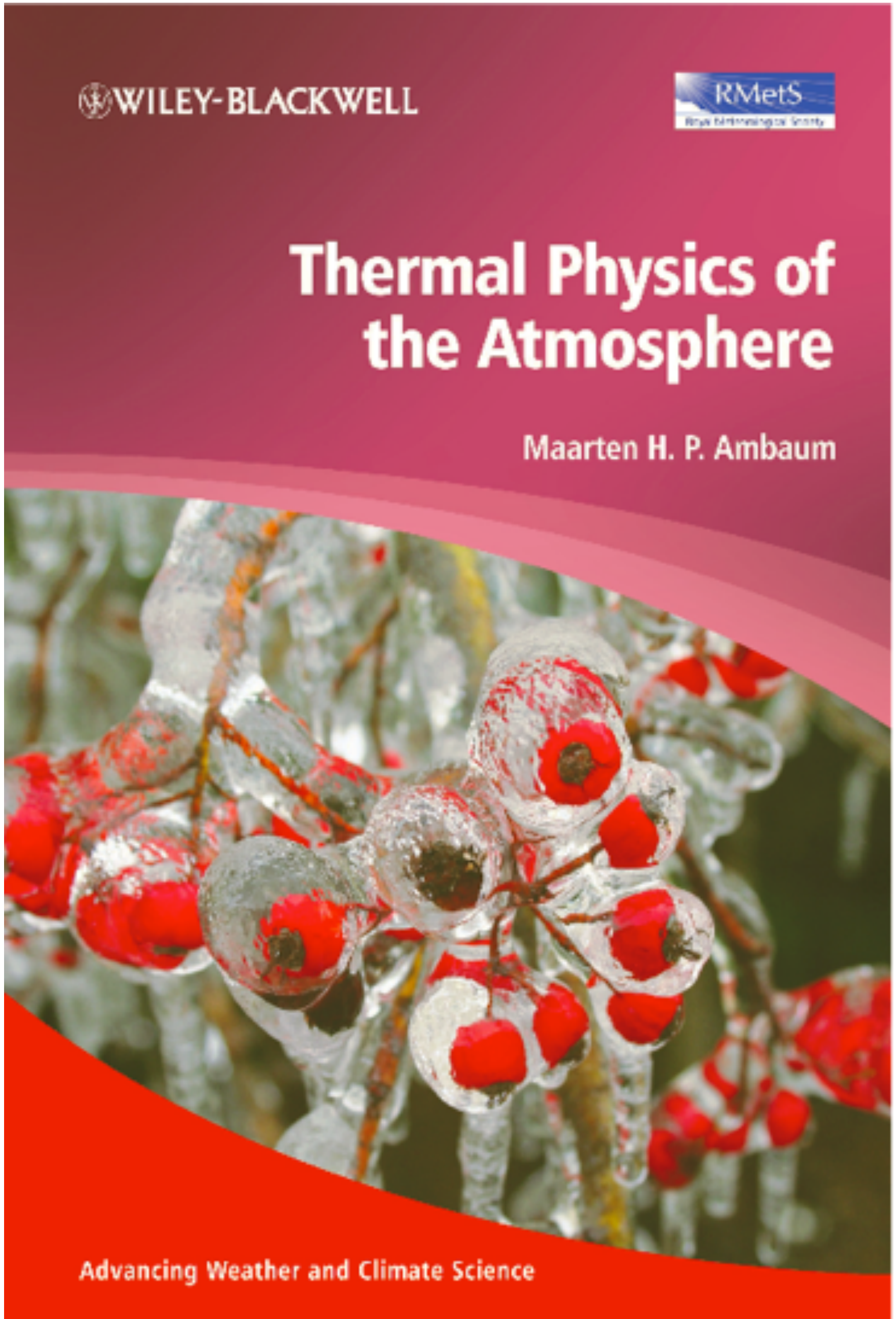
This equation defines a relevant efficiency.

Back to our general formula:

$$\dot{L} = \left(1 - \frac{T_{\rm out}}{T_{\rm in}}\right) \int_A F_{\rm in}\, {\rm d}A - T_{\rm out} \frac{{\rm d}_i S}{{\rm d}t}\cdot$$

This equation defines a relevant efficiency.

This equation implies the Sandström “theorem”.



Now for sale. See:
http://www.met.rdg.ac.uk/~sws97mha/thermal