Var practical

Introduction

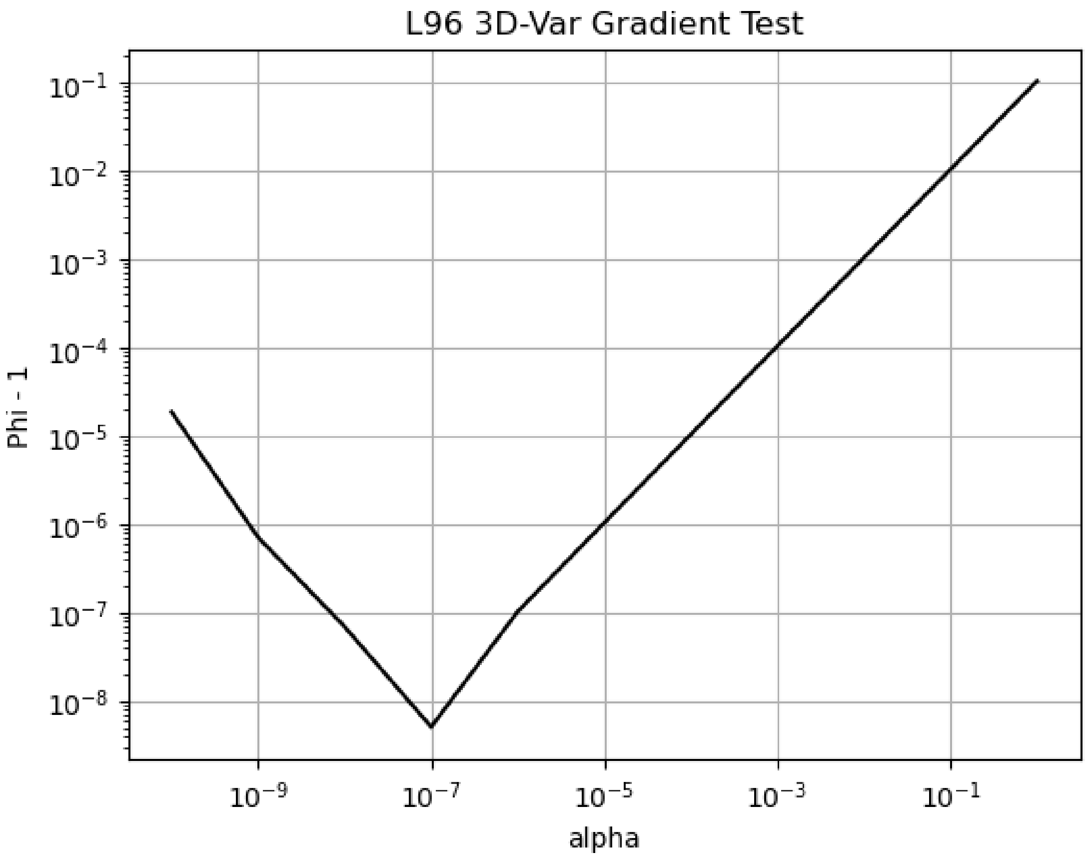
Today we will be applying 3DVar and incremental 4DVar to the Lorenz 96 system.

Some things to think about from Monday’s practical:

- The non-linearity of the model: how does this affect the choice of assimilation frequency and assimilation window length?
- The observations assimilated: how does their sampling characteristics and precision affect the quality of the analysis?

3DVar

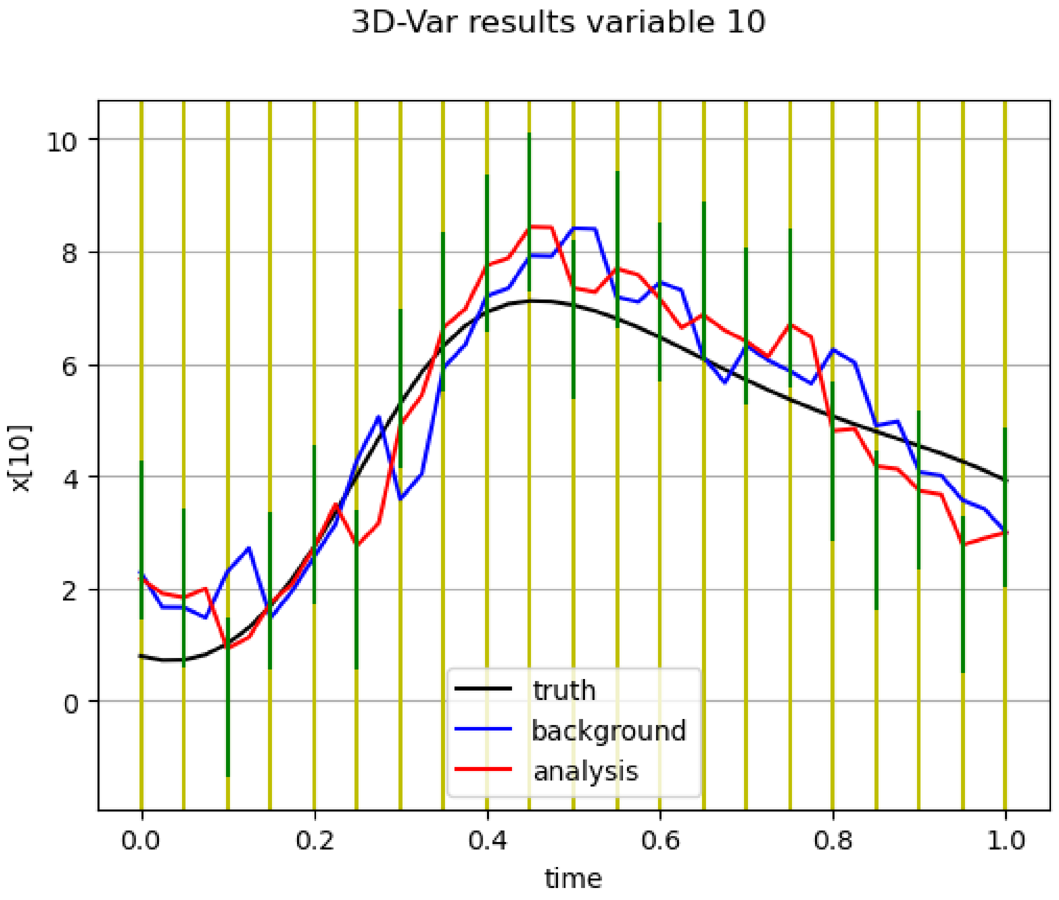
$$J^{3\mathrm{D}}(\mathbf{x}) = \frac{1}{2}\left(\mathbf{x} - \mathbf{x}^{\mathrm{b}}\right)^{\mathrm{T}}\mathbf{B}_0^{-1}\left(\bullet\right) + \frac{1}{2}\sum_{i=0}^{T}\left(\mathbf{y}_i - \mathcal{H}_i(\mathbf{x}_i^{\mathrm{R}}) - \mathbf{H}_i\mathbf{x}\right)^{\mathrm{T}}\mathbf{R}_i^{-1}\left(\bullet\right),$$

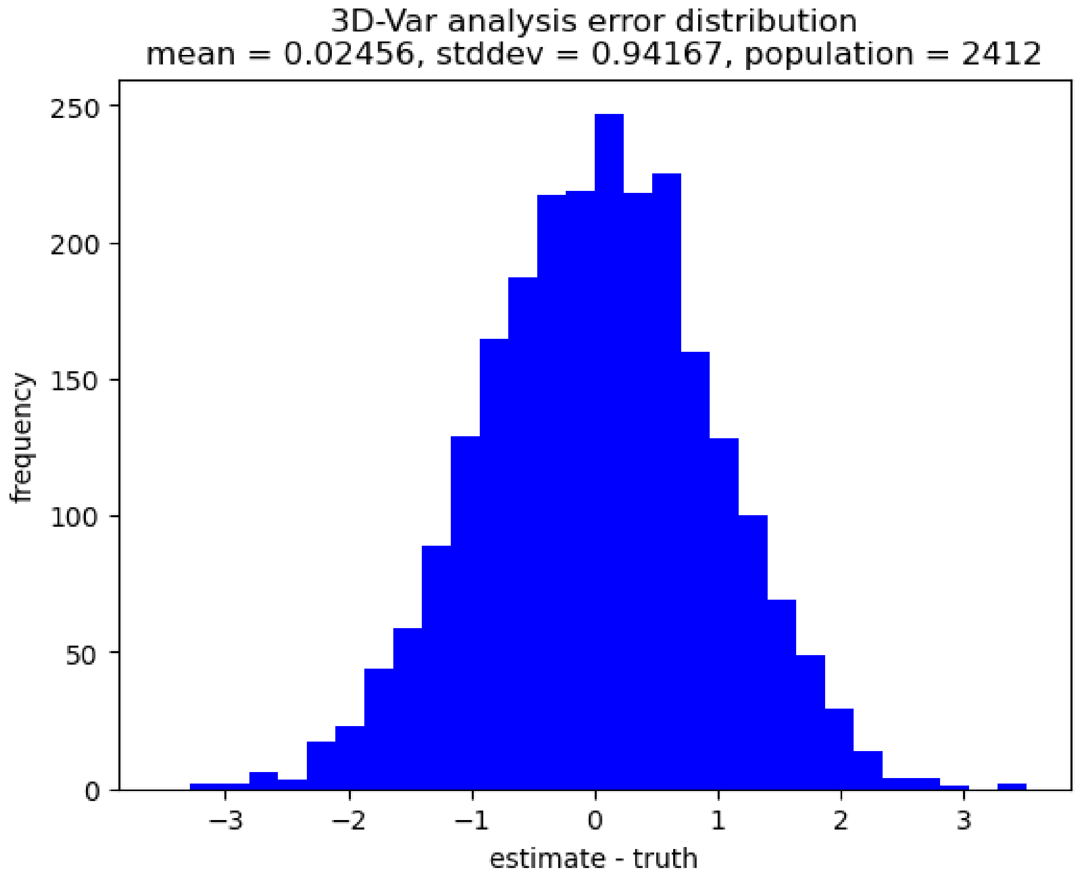


$$\Phi(\alpha) - 1 = \frac{J(\mathbf{x} + \alpha\mathbf{h}) - J(\mathbf{x})}{\alpha\mathbf{h}^{\mathrm{T}}\nabla J(\mathbf{x})} - 1 = O(\alpha),$$

3DVar

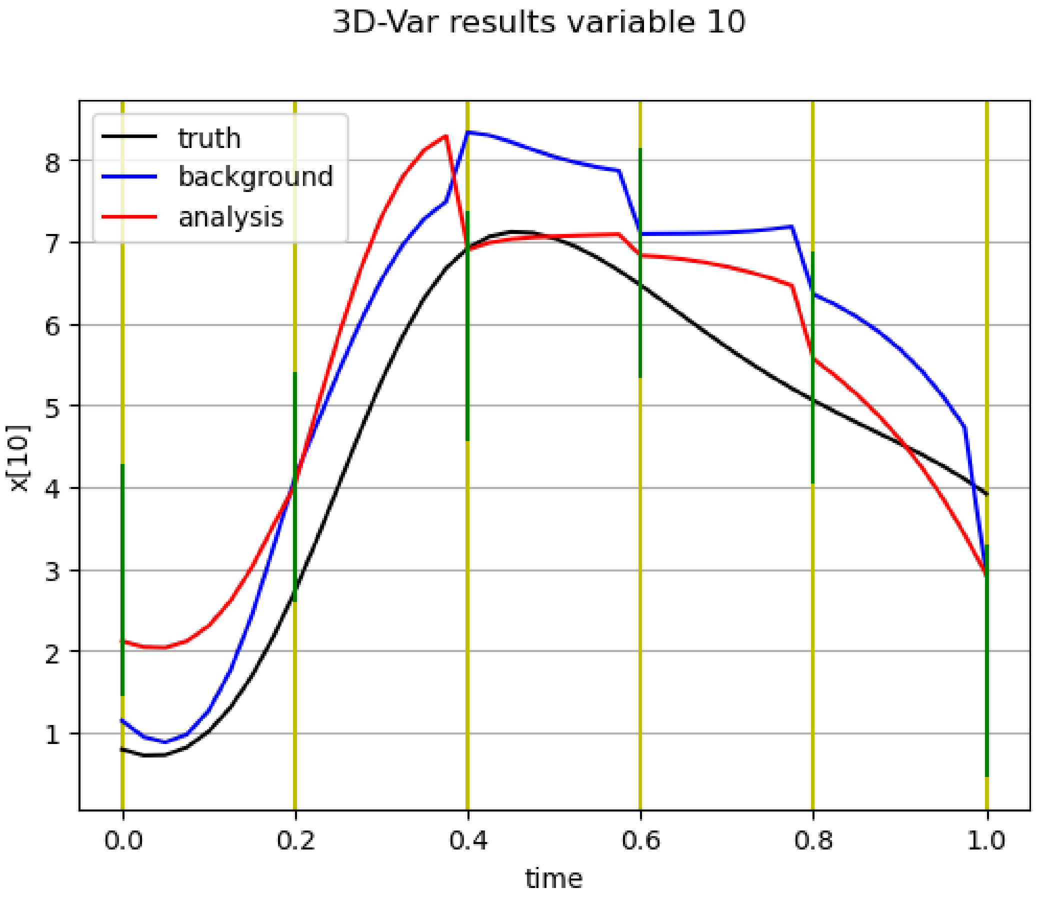
Direct observations every other time step. Var=2.

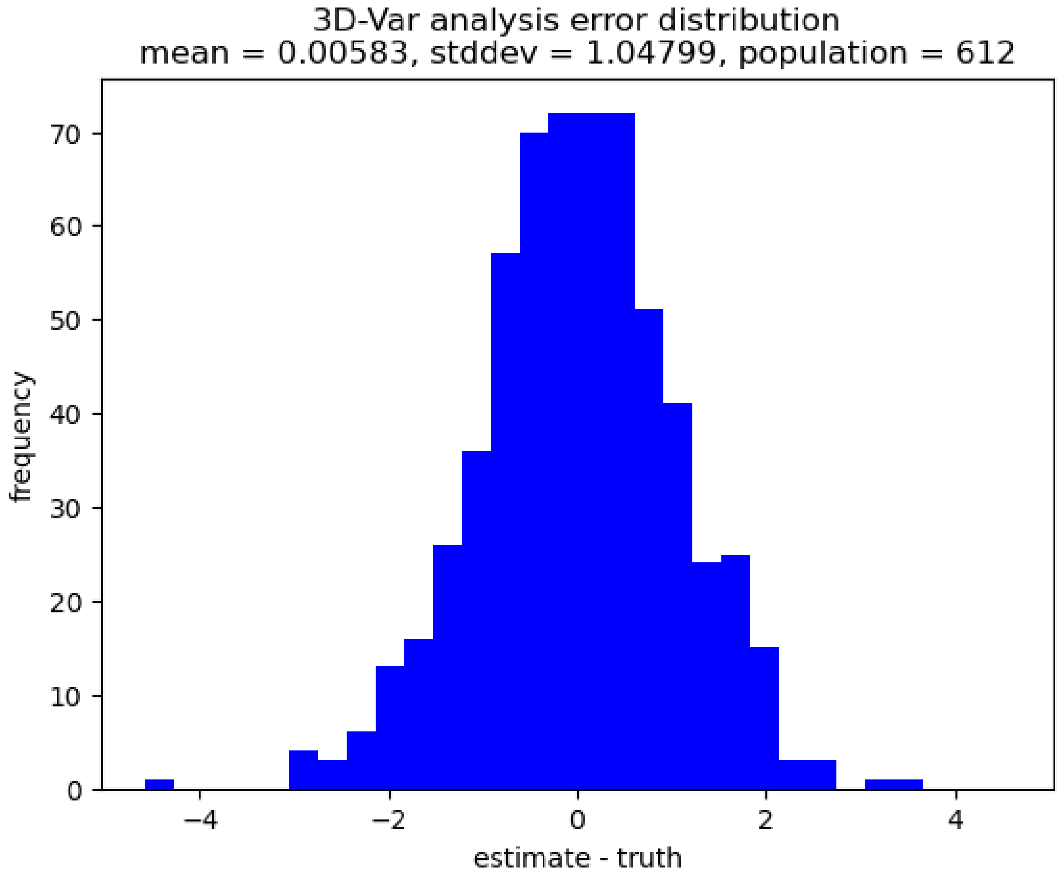




3DVar- reducing frequency of observations

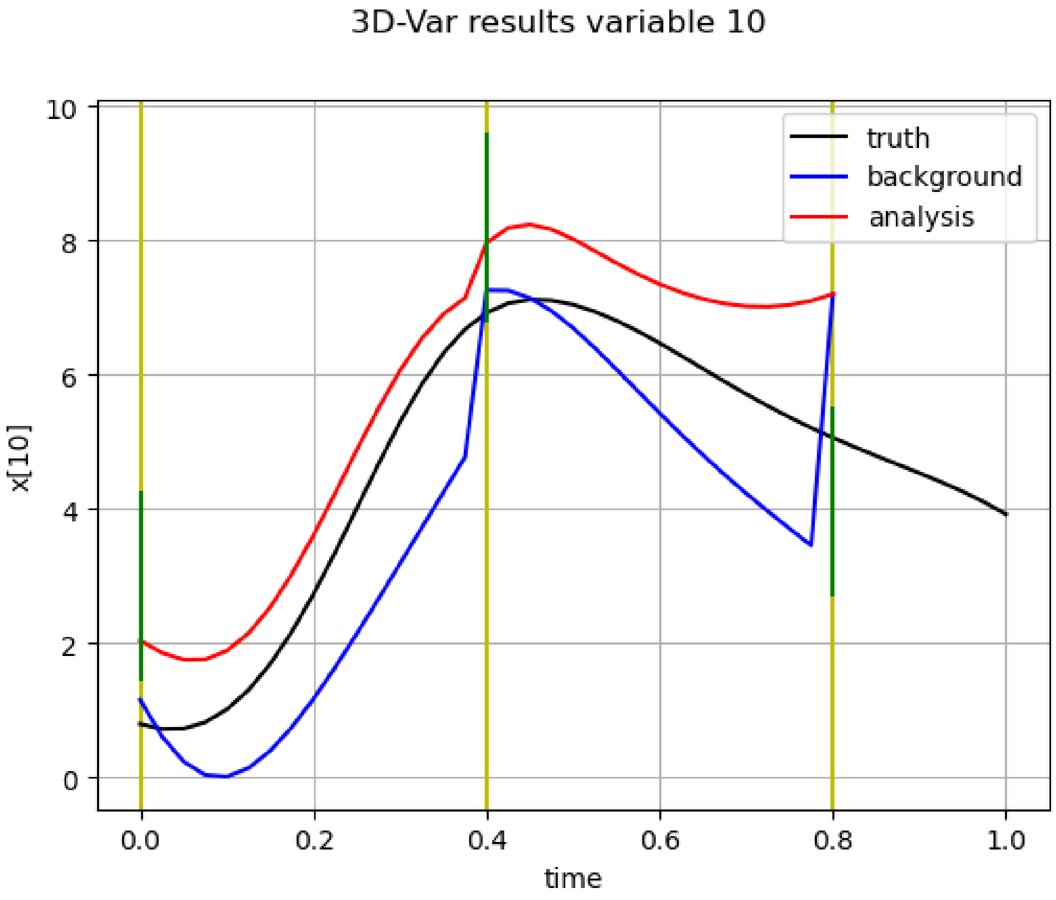
Direct observations every 8th time step. Var=2.

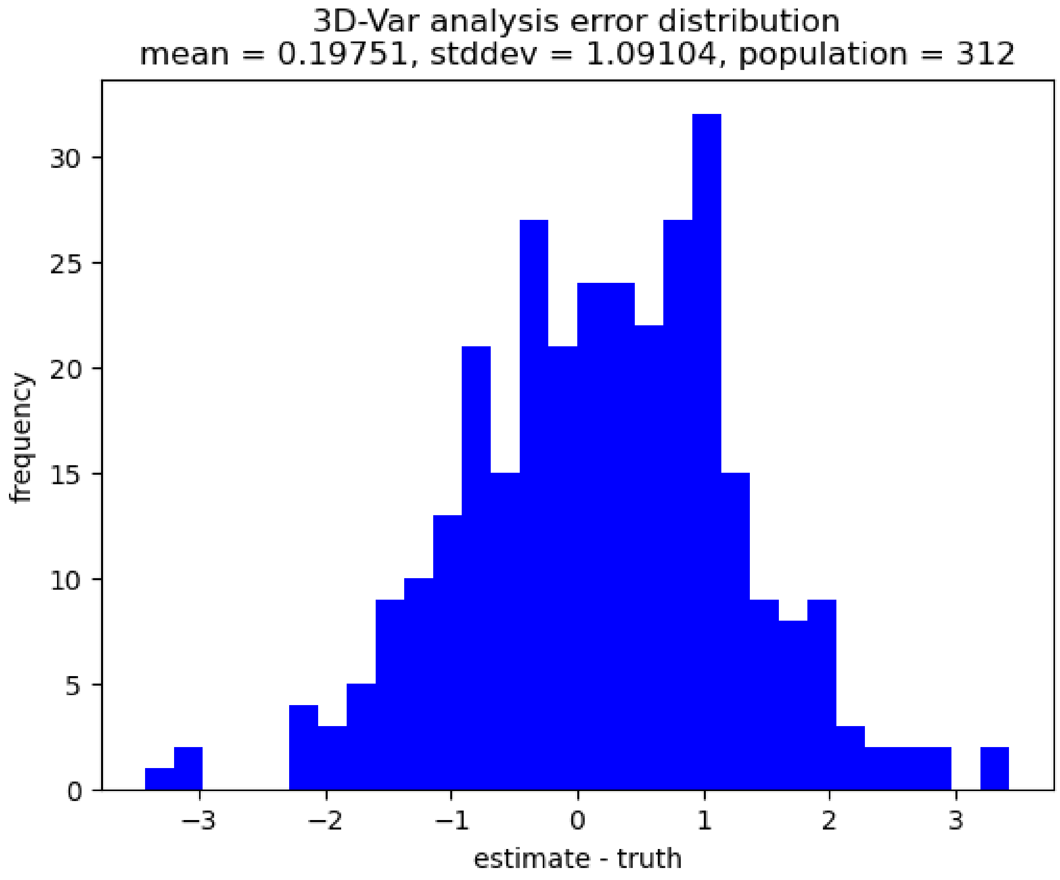




3DVar- reducing frequency of observations

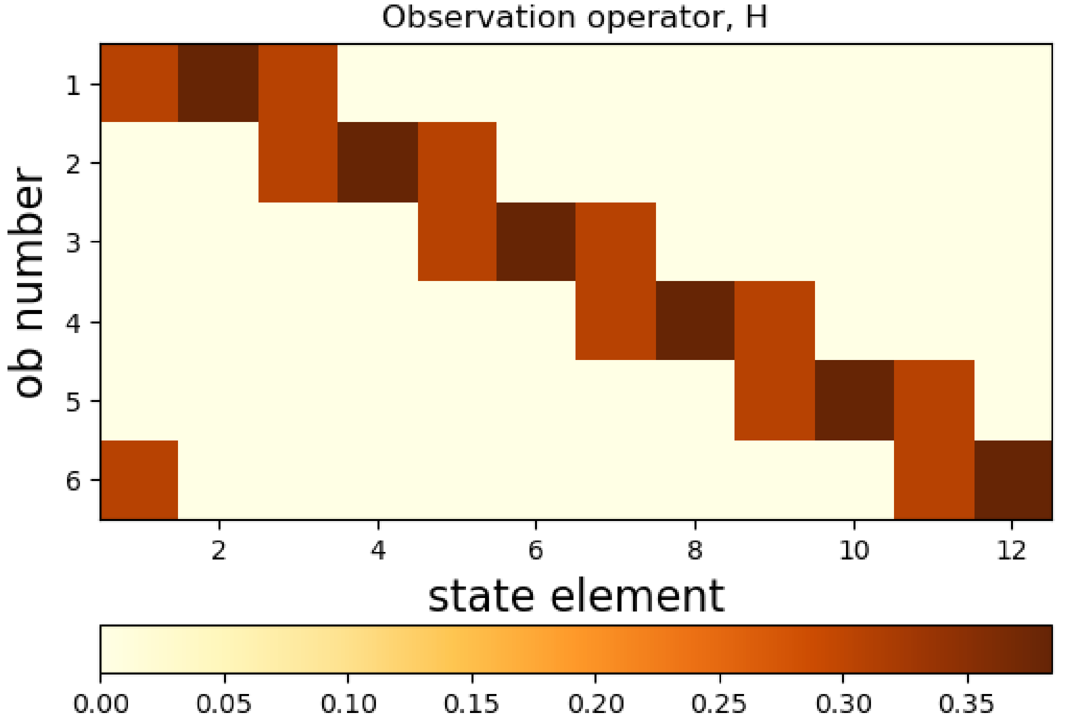
Direct observations every 16th time step. Var=2.





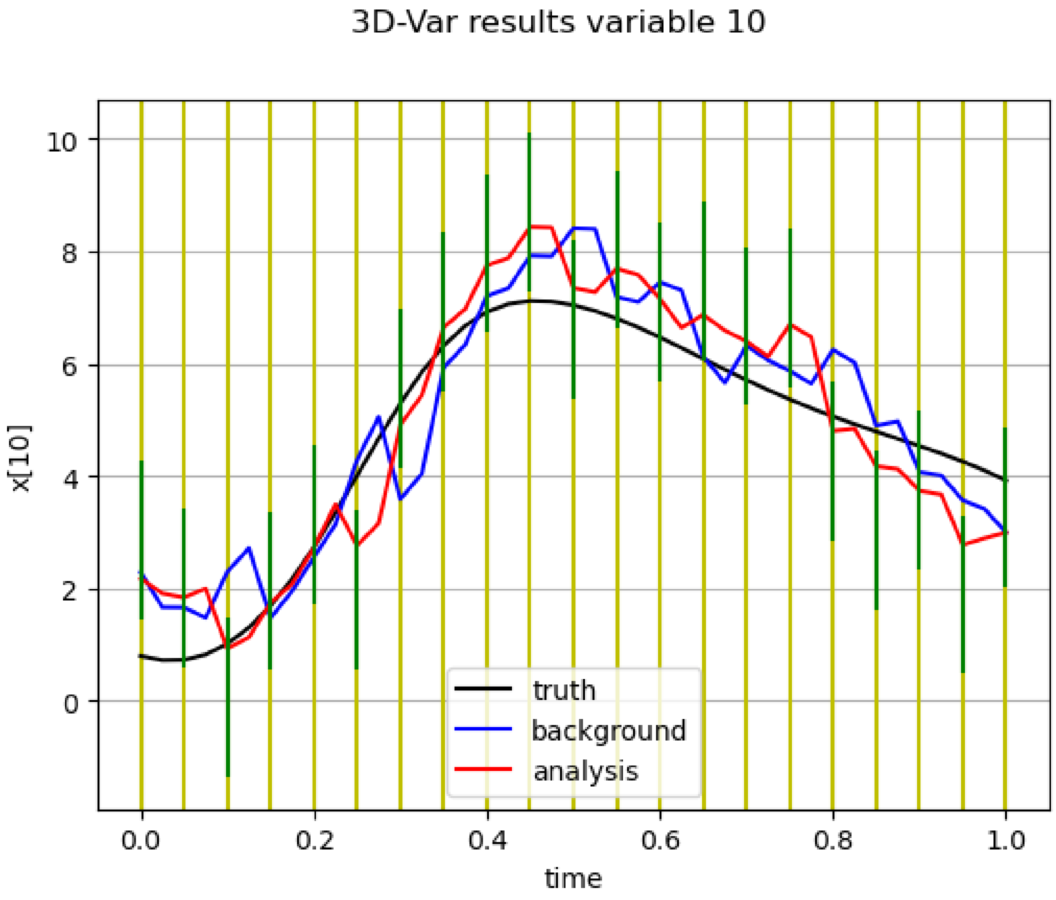
3Dvar – indirect observations

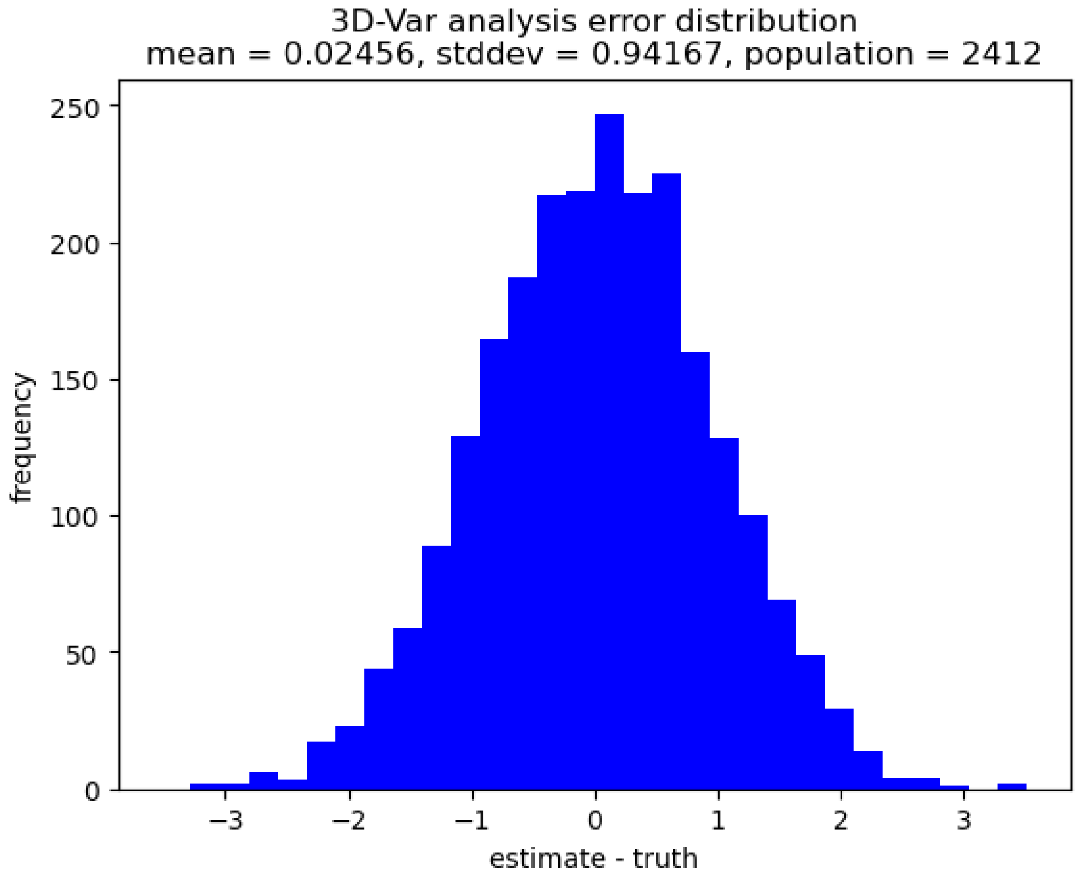
footprint observations every other time step. Var=2.



3DVar

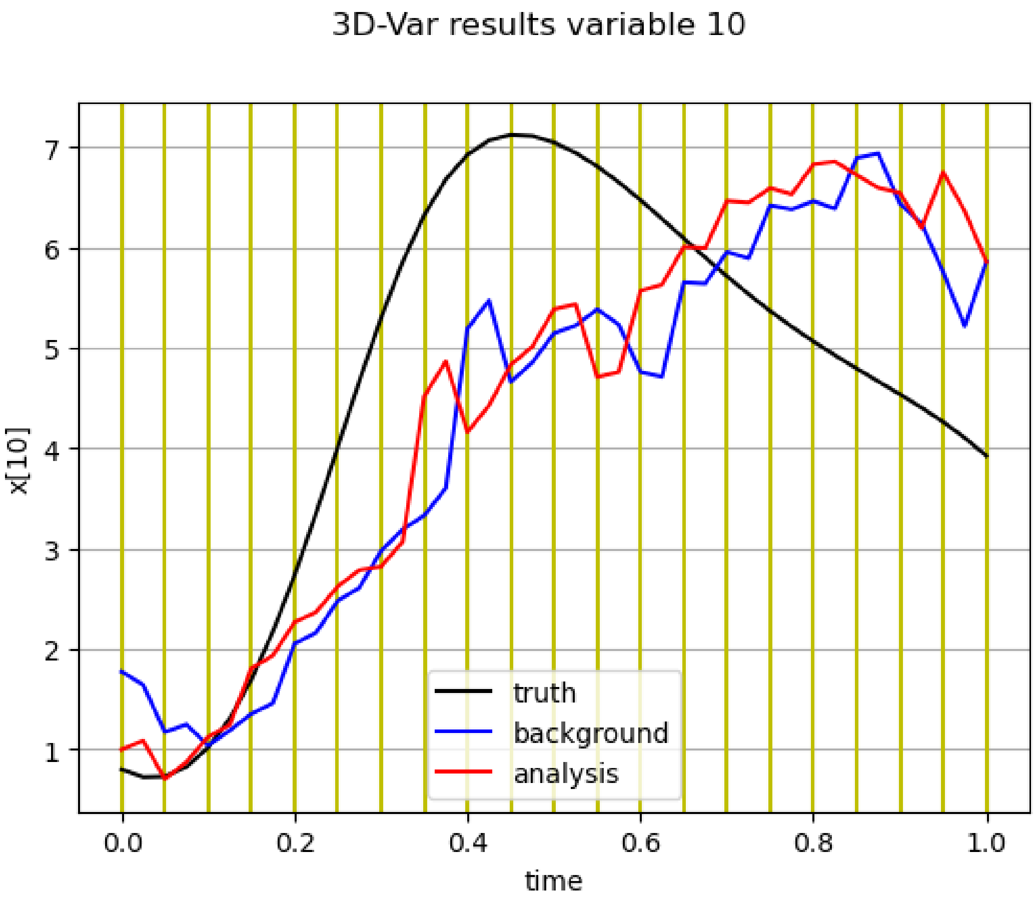
Direct observations every other time step. Var=2.

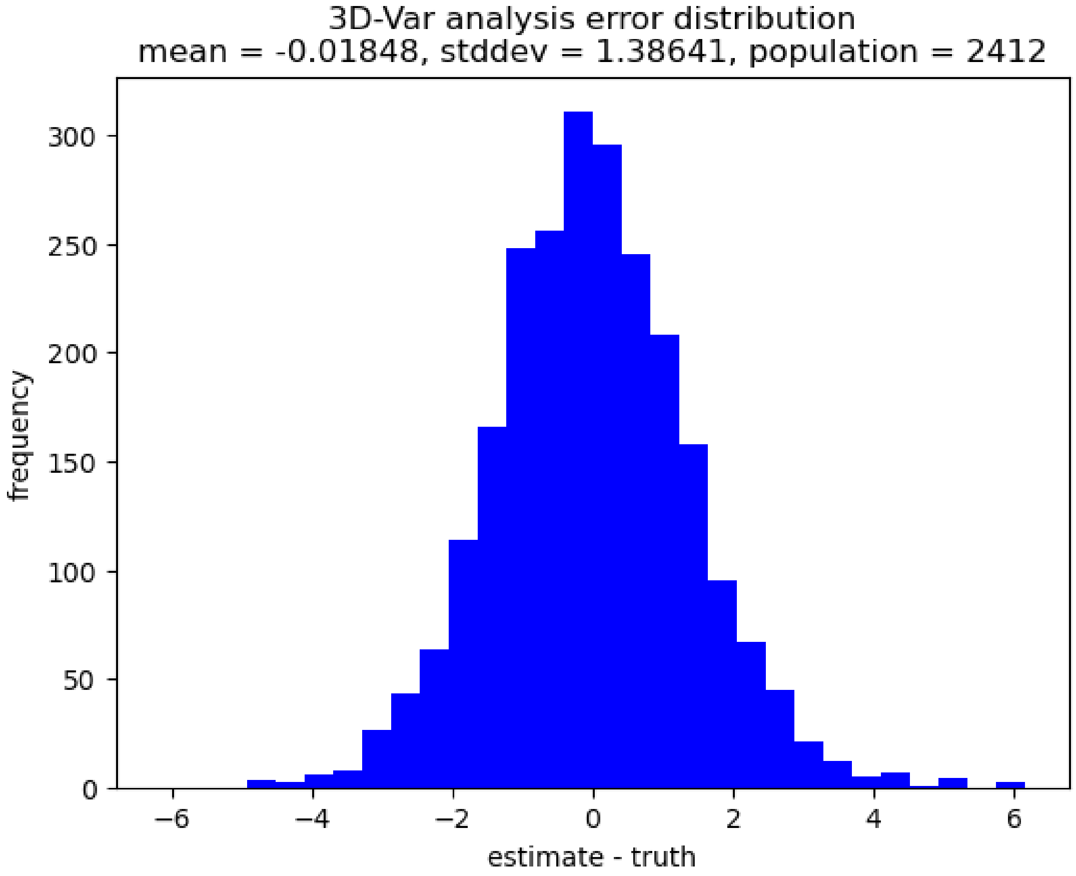




3DVar

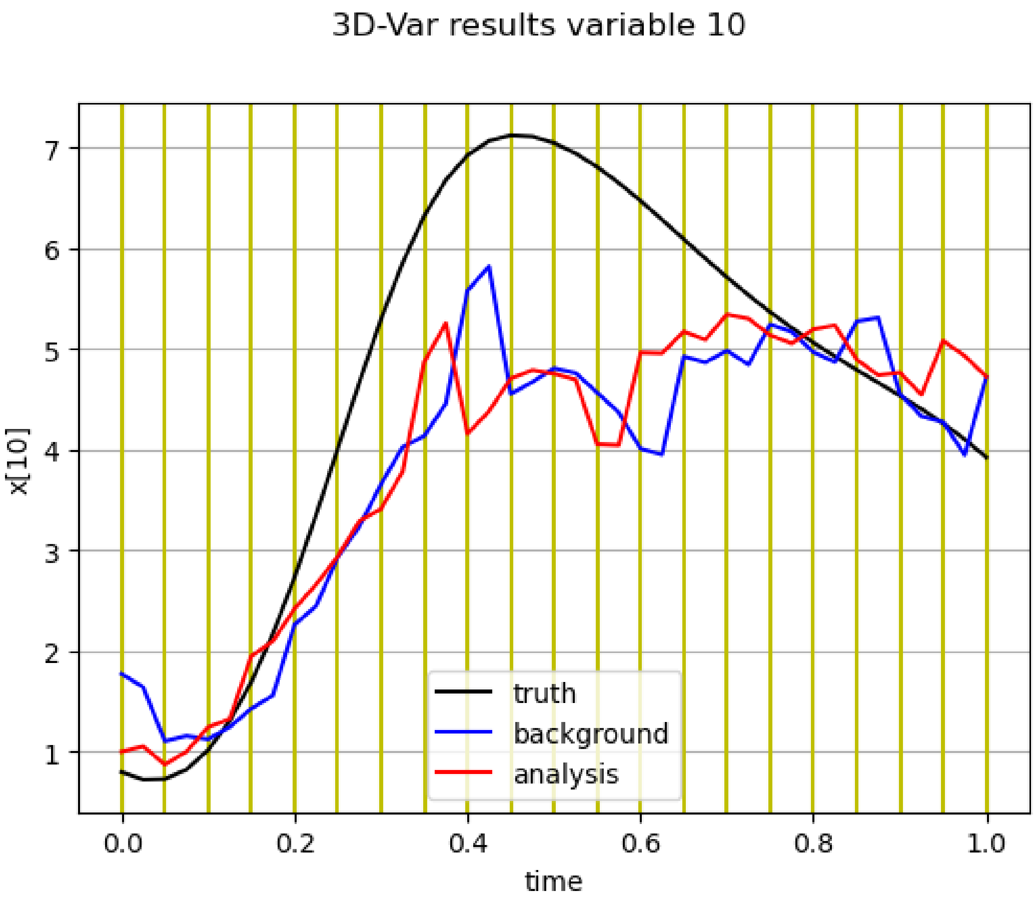
footprint observations every other time step. Var=2.

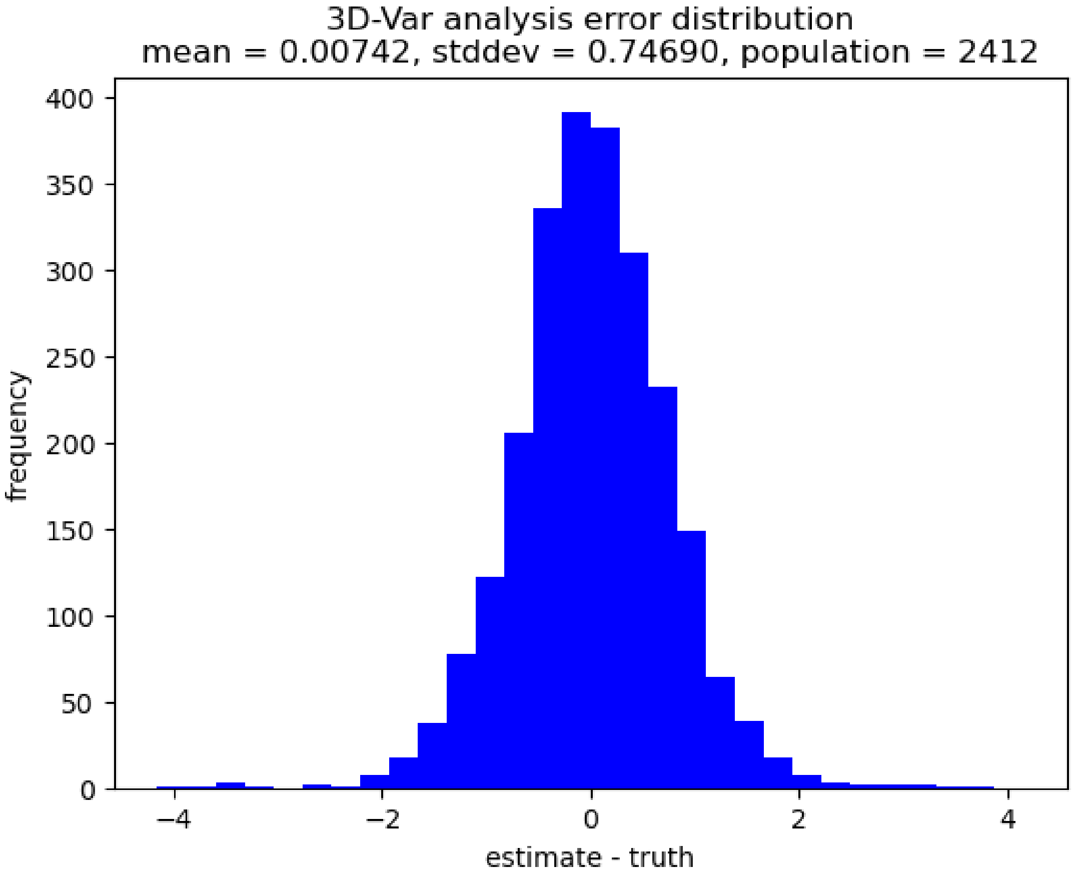




3DVar

footprint observations every other time step. Var=0.2

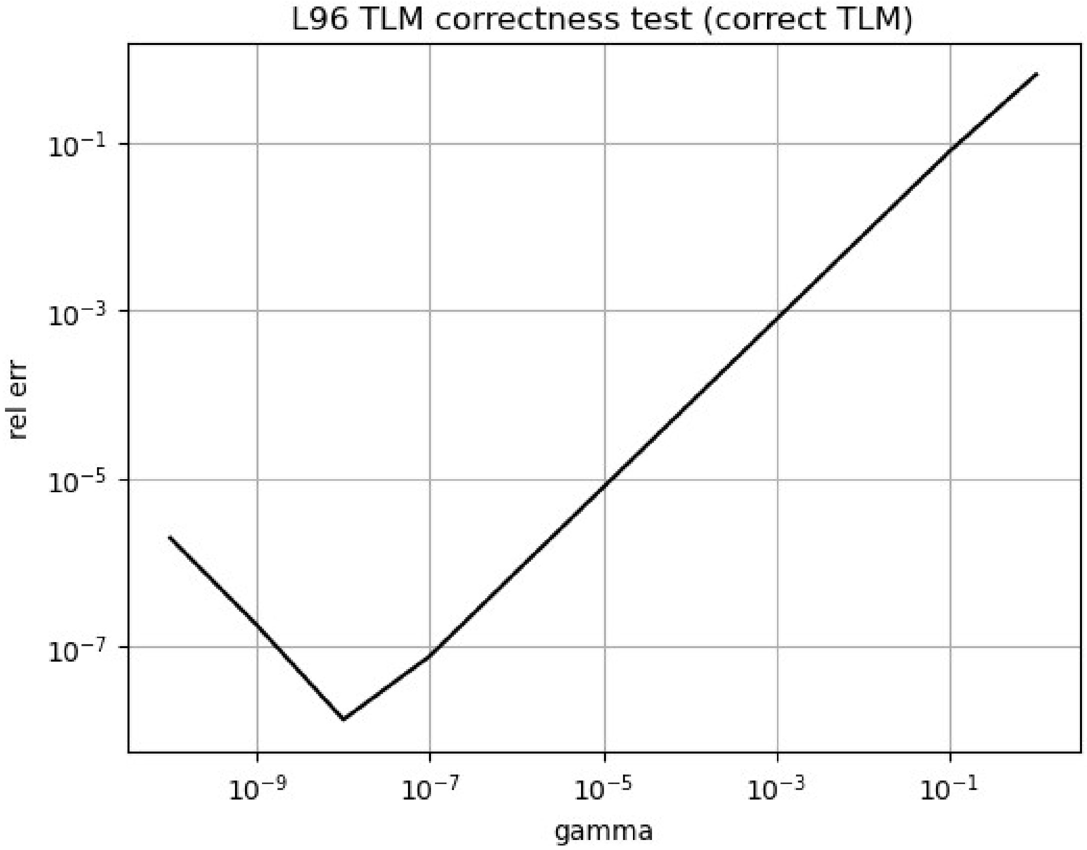




Incremental 4DVar

$$J(\delta\mathbf{x}_0) = \frac{1}{2}\delta\mathbf{x}_0^{\mathrm{T}}\mathbf{B}_0^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{T}\left(\mathbf{y}_i - \mathcal{H}_i(\mathbf{x}_i^{\mathrm{b}}) - \mathbf{H}_i\mathbf{M}_{i-1}\mathbf{M}_{i-2}\ldots\mathbf{M}_0\delta\mathbf{x}_0\right)^{\mathrm{T}}\mathbf{R}_i^{-1}\left(\bullet\right).$$

4DVar- TL and adjoint test



$$\frac{\|\mathcal{M}_{0\to\tau}(\mathbf{x}_0 + \gamma\delta\mathbf{x}_0) - \mathcal{M}_{0\to\tau}(\mathbf{x}_0) - \mathbf{M}_{0\to\tau}\gamma\delta\mathbf{x}_0\|}{\|\mathbf{M}_{0\to\tau}\gamma\delta\mathbf{x}_0\|}.$$

$$(\mathbf{M}\,\delta\mathbf{x}\,,\,\mathbf{M}\,\delta\mathbf{x}) = (\delta\mathbf{x}\,,\,\mathbf{M}^{T}\,\mathbf{M}\,\delta\mathbf{x})$$

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lhs =  27.660443950044545
rhs =  27.66044395004454
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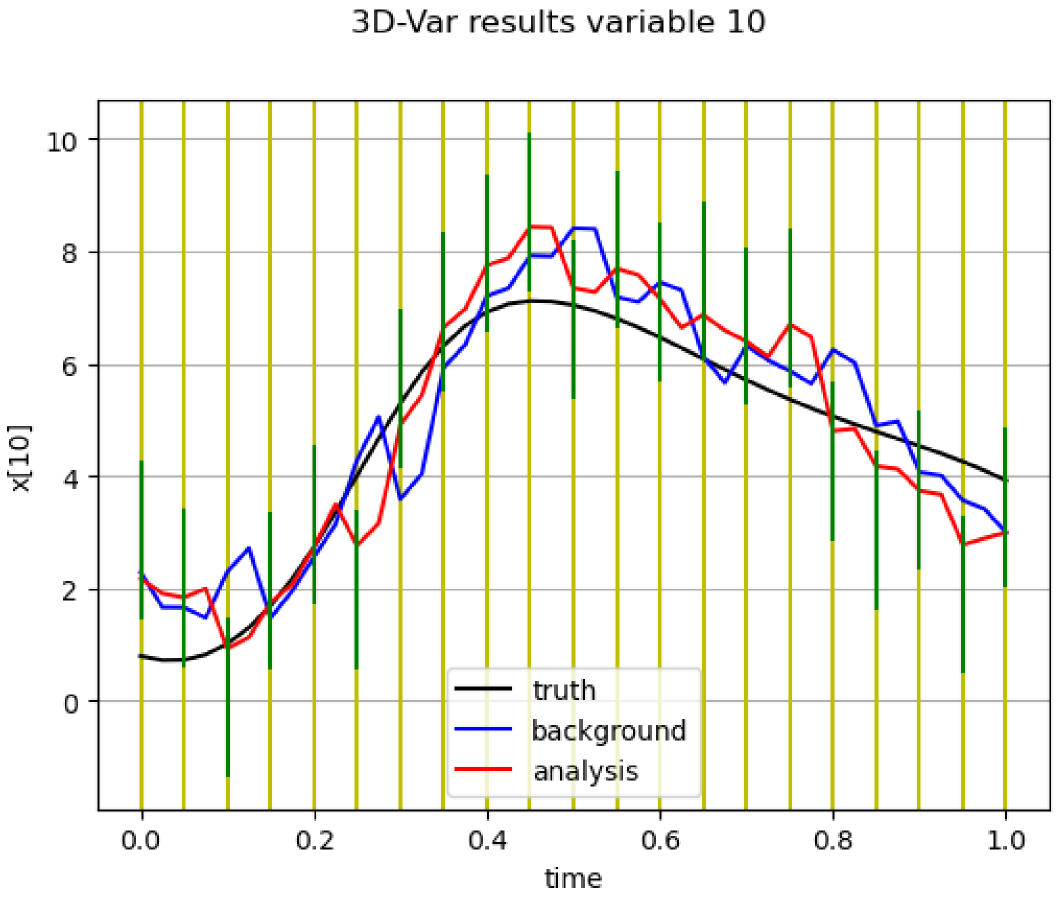
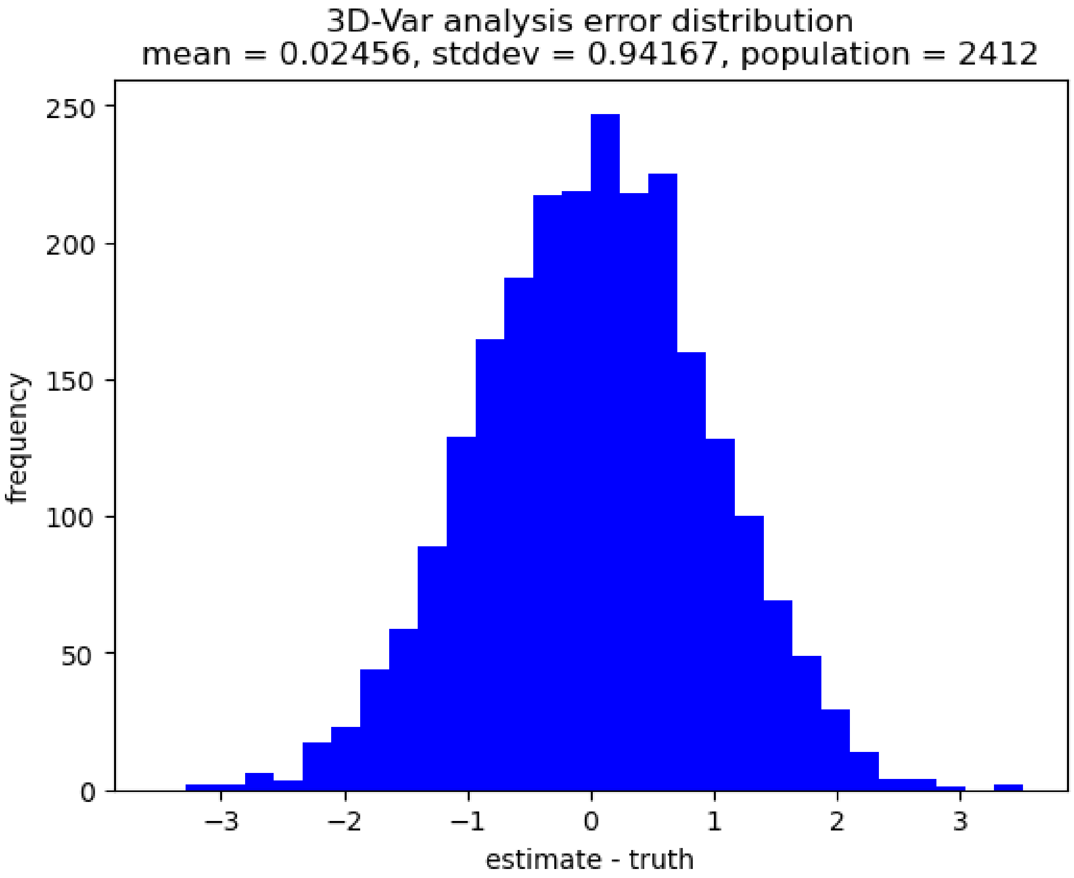
3DVar

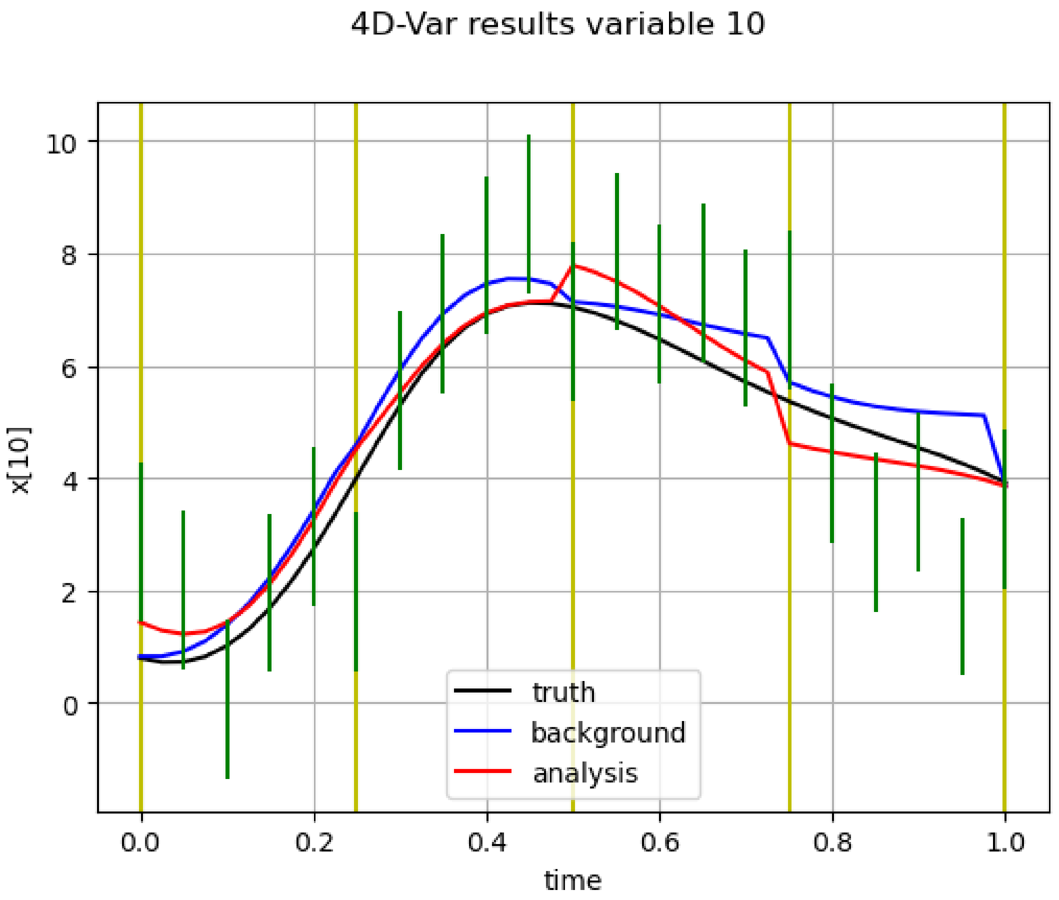
Direct observations every other time step. Var=2.

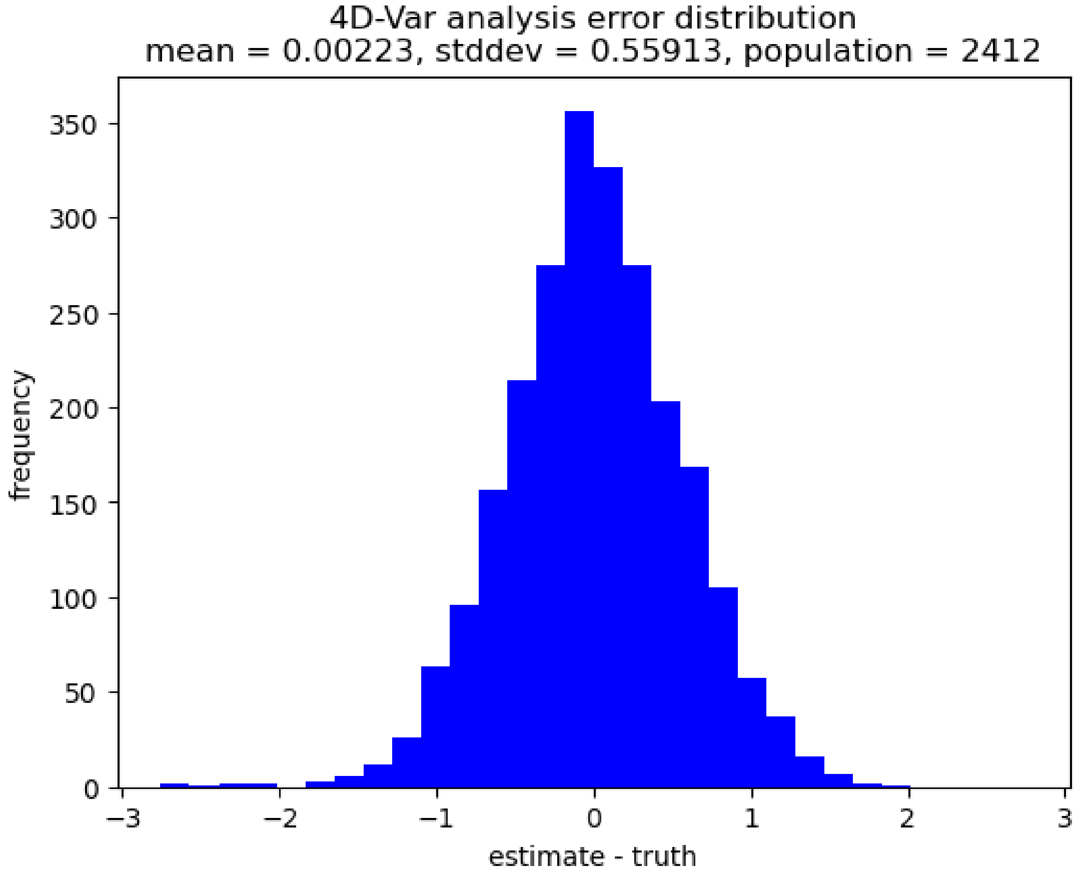




4DVar

Direct observations every other time step. Var=2.
Assim window=5 ob times





4DVar

Direct observations every other time step. Var=2.
Assim window=10 ob times