

Var practical

Introduction

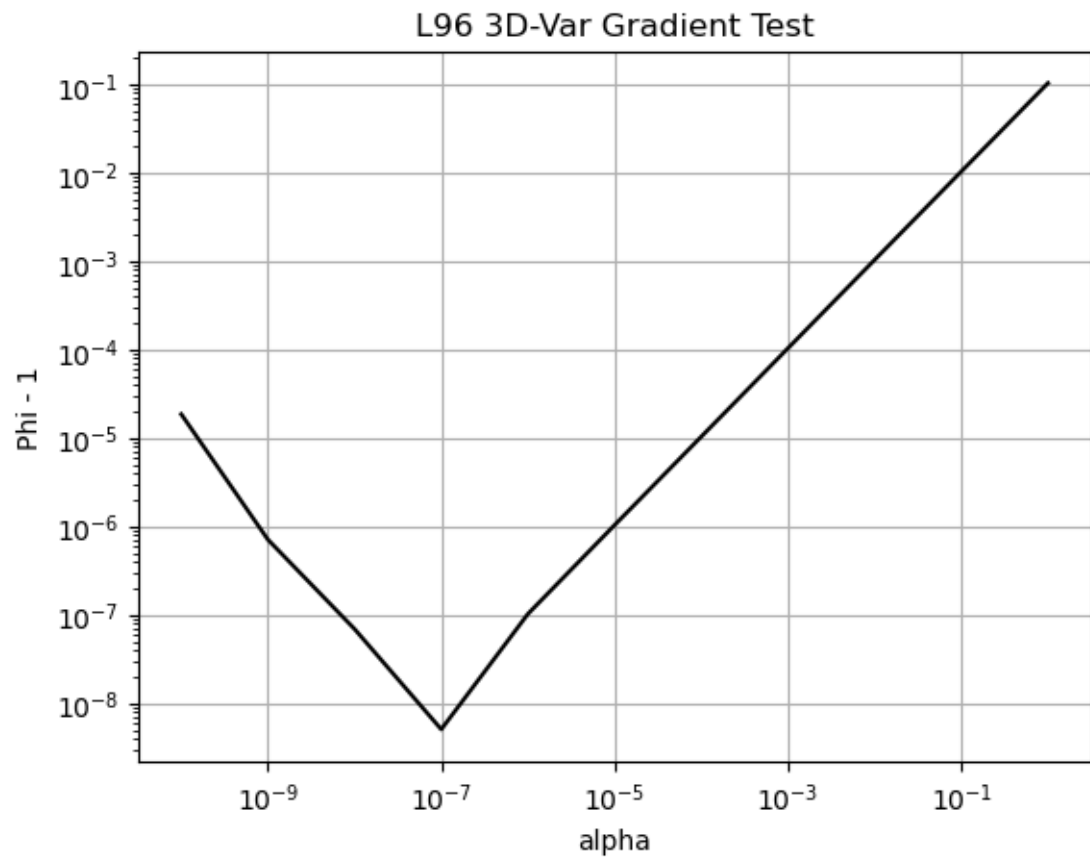
Today we will be applying 3DVar and incremental 4DVar to the Lorenz 96 system.

Some things to think about from Monday's practical:

- The non-linearity of the model: how does this affect the choice of assimilation frequency and assimilation window length?
- The observations assimilated: how does their sampling characteristics and precision affect the quality of the analysis?

3DVar

$$J^{3D}(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}^b)^T \mathbf{B}_0^{-1}(\cdot) + \frac{1}{2} \sum_{i=0}^T (\mathbf{y}_i - \mathcal{H}_i(\mathbf{x}_i^R) - \mathbf{H}_i \mathbf{x})^T \mathbf{R}_i^{-1}(\cdot),$$

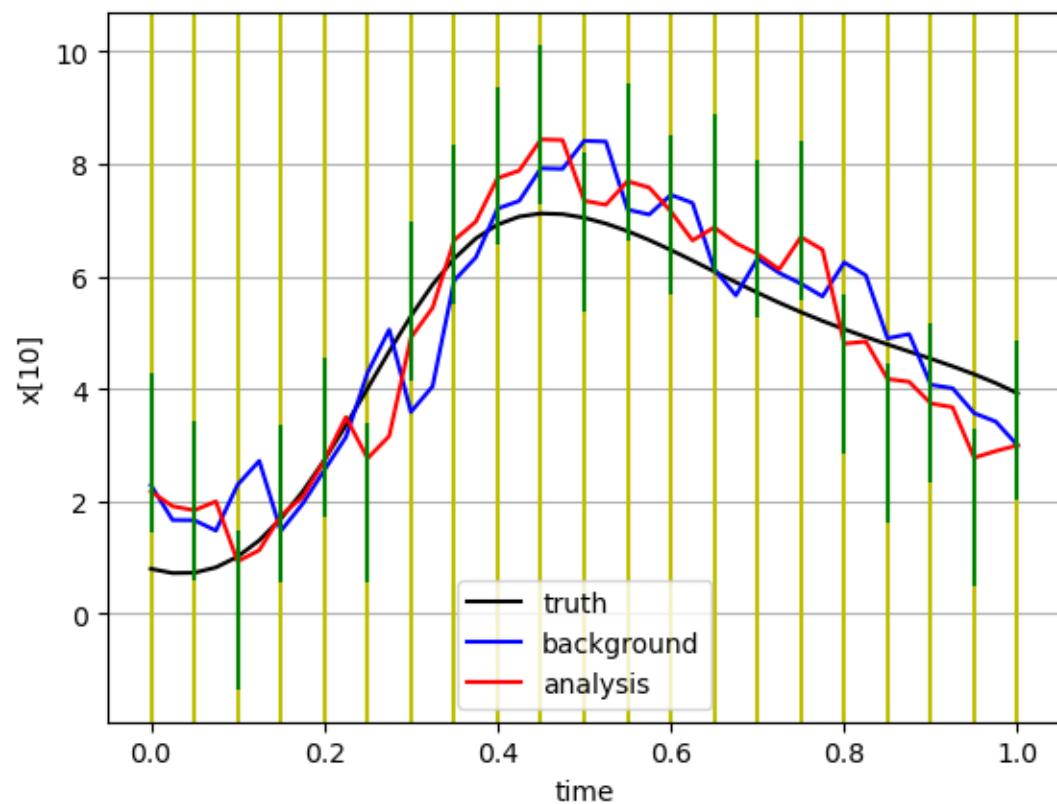


$$\Phi(\alpha) - 1 = \frac{J(\mathbf{x} + \alpha \mathbf{h}) - J(\mathbf{x})}{\alpha \mathbf{h}^T \nabla J(\mathbf{x})} - 1 = O(\alpha),$$

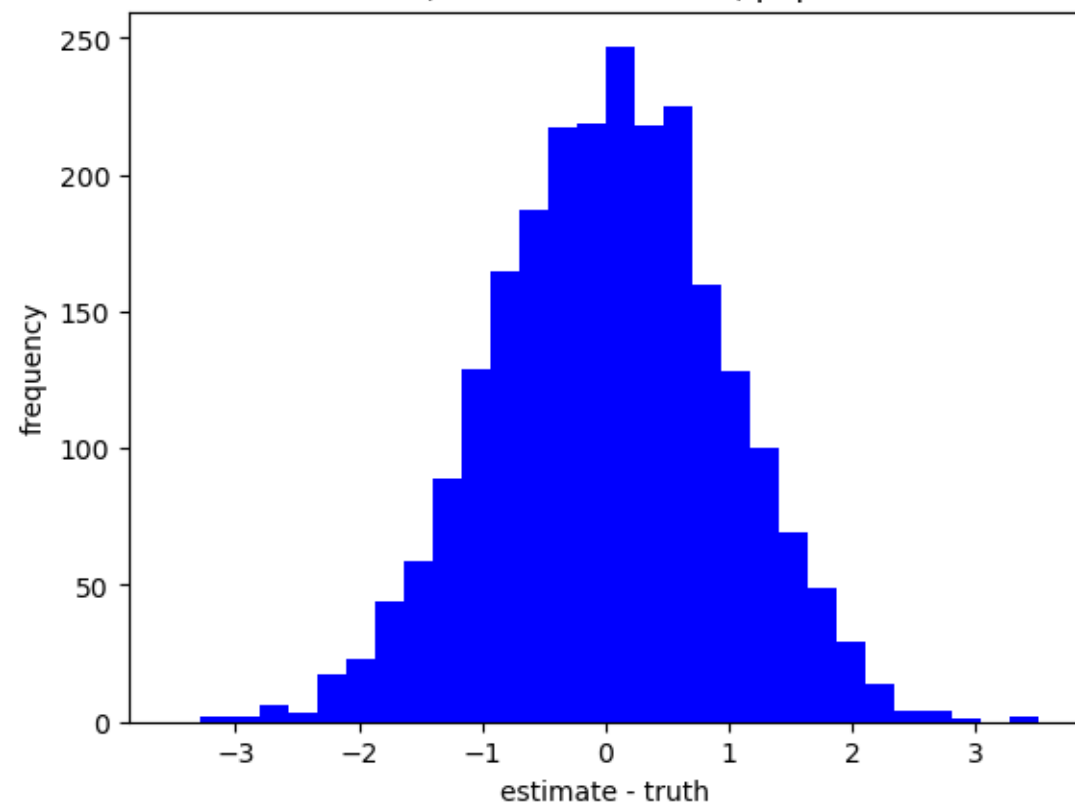
3DVar

Direct observations every other time step. Var=2.

3D-Var results variable 10



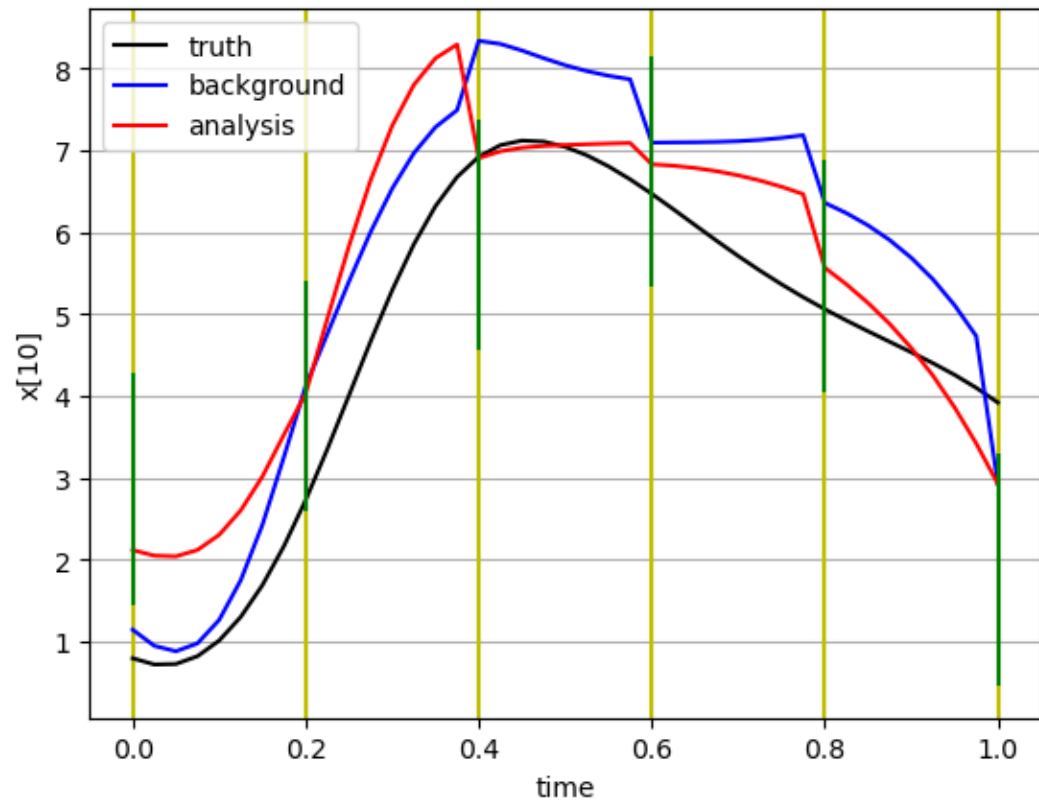
3D-Var analysis error distribution
mean = 0.02456, stddev = 0.94167, population = 2412



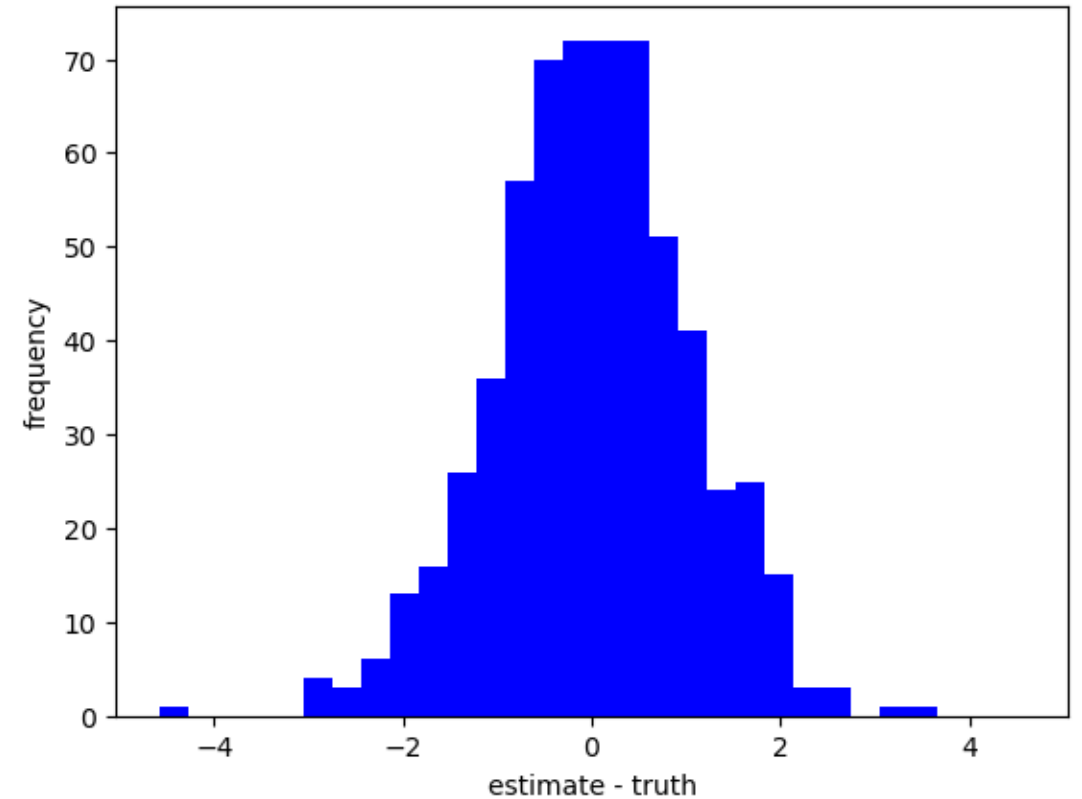
3DVar- reducing frequency of observations

Direct observations every 8th time step. Var=2.

3D-Var results variable 10



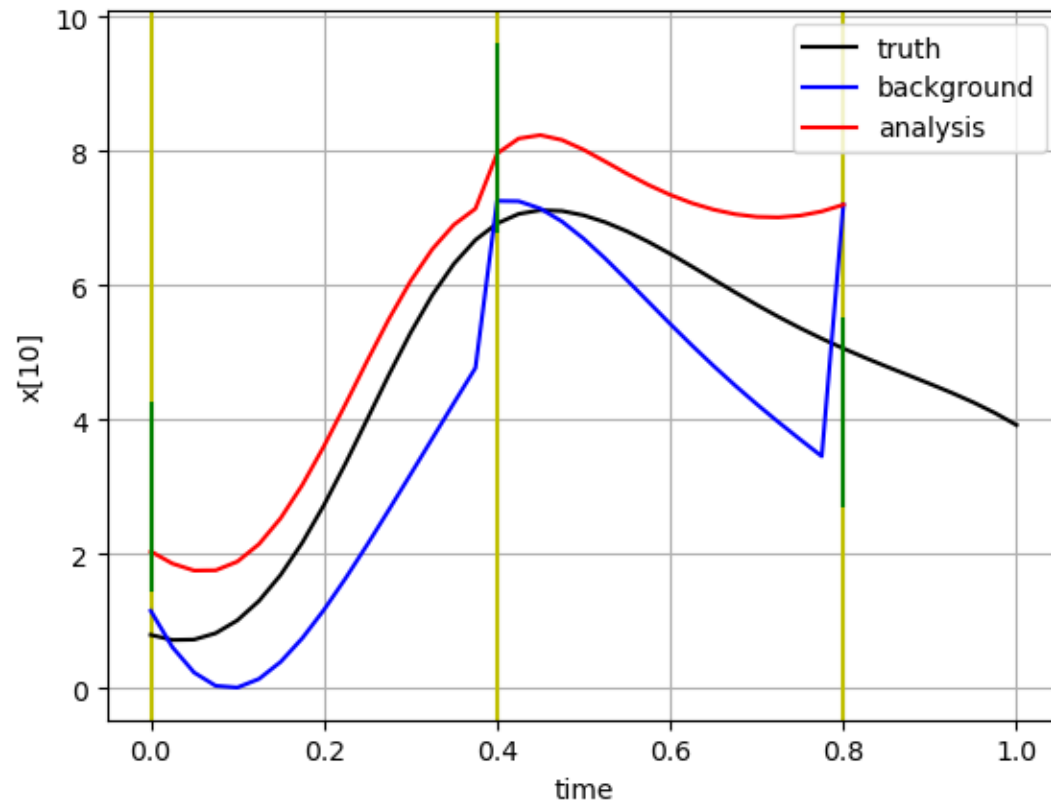
3D-Var analysis error distribution
mean = 0.00583, stddev = 1.04799, population = 612



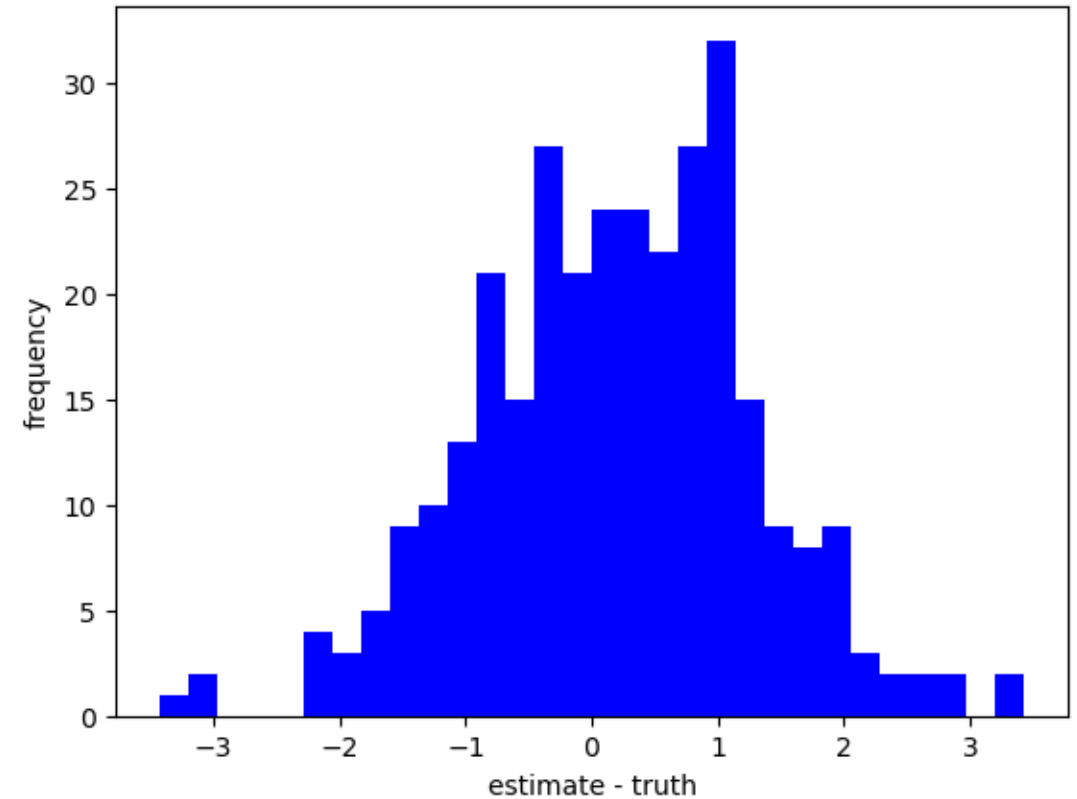
3DVar- reducing frequency of observations

Direct observations every 16th time step. Var=2.

3D-Var results variable 10

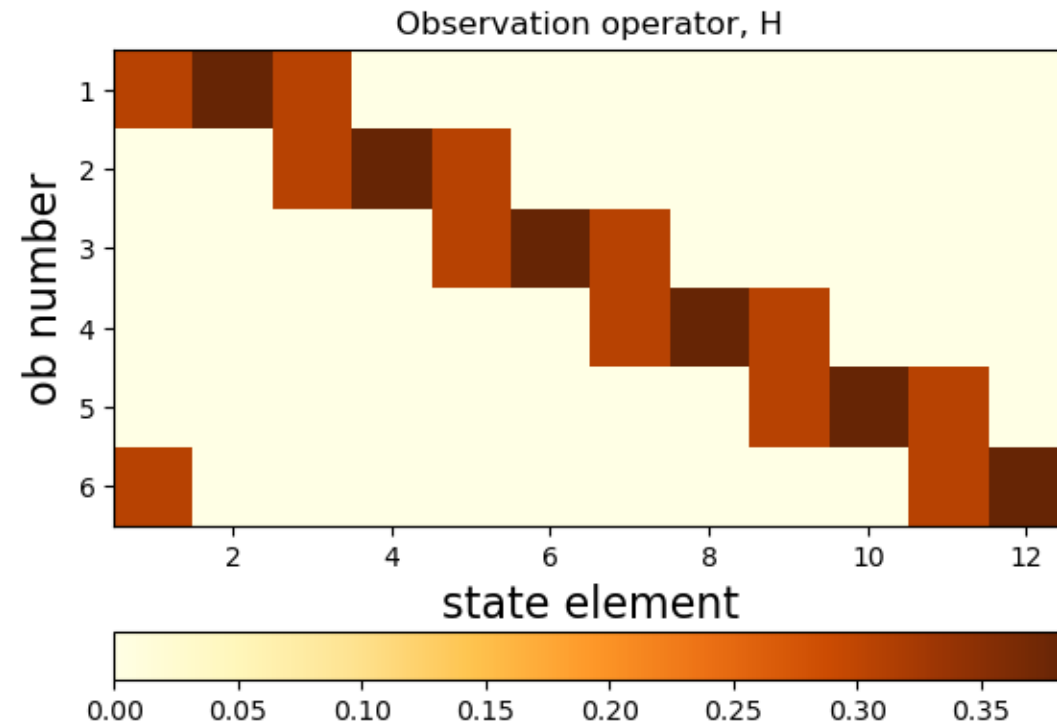


3D-Var analysis error distribution
mean = 0.19751, stddev = 1.09104, population = 312



3Dvar – indirect observations

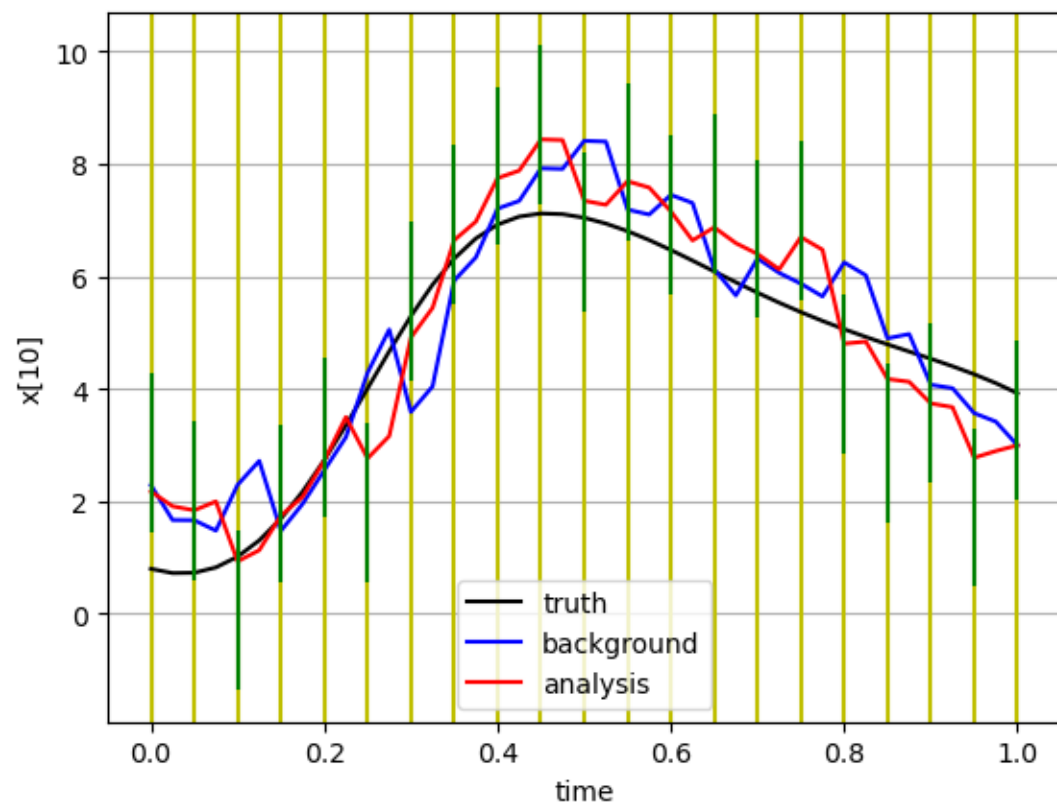
footprint observations every other time step. Var=2.



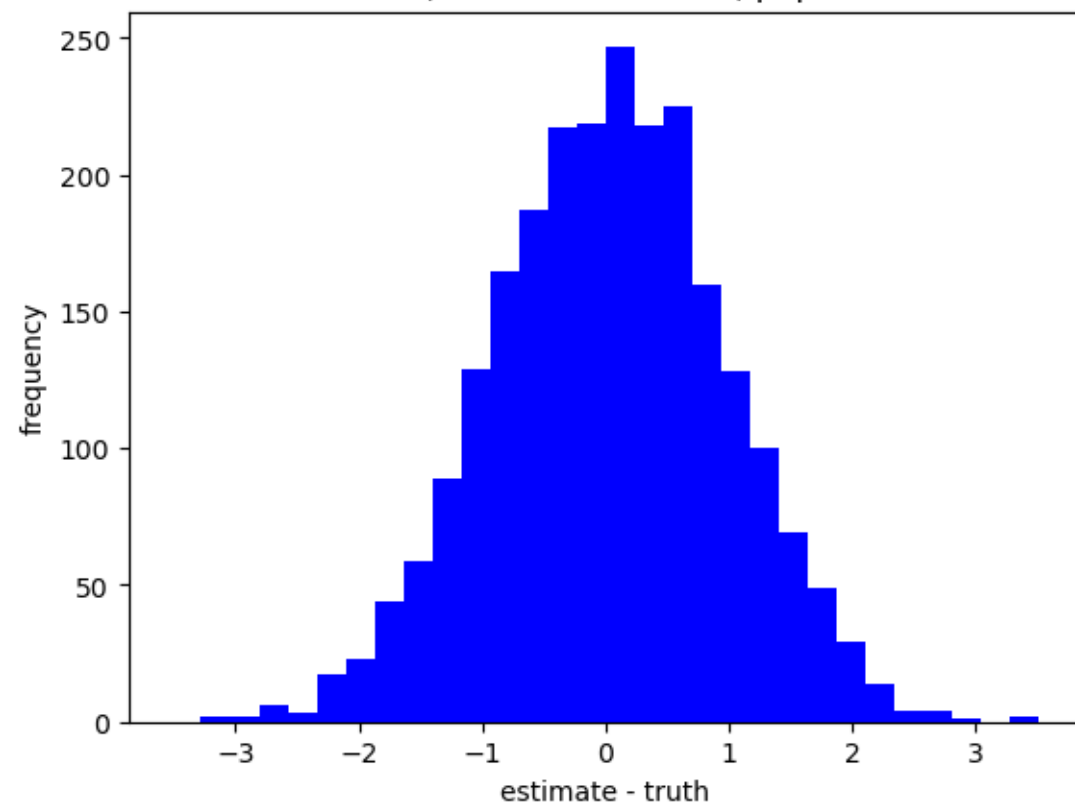
3DVar

Direct observations every other time step. Var=2.

3D-Var results variable 10



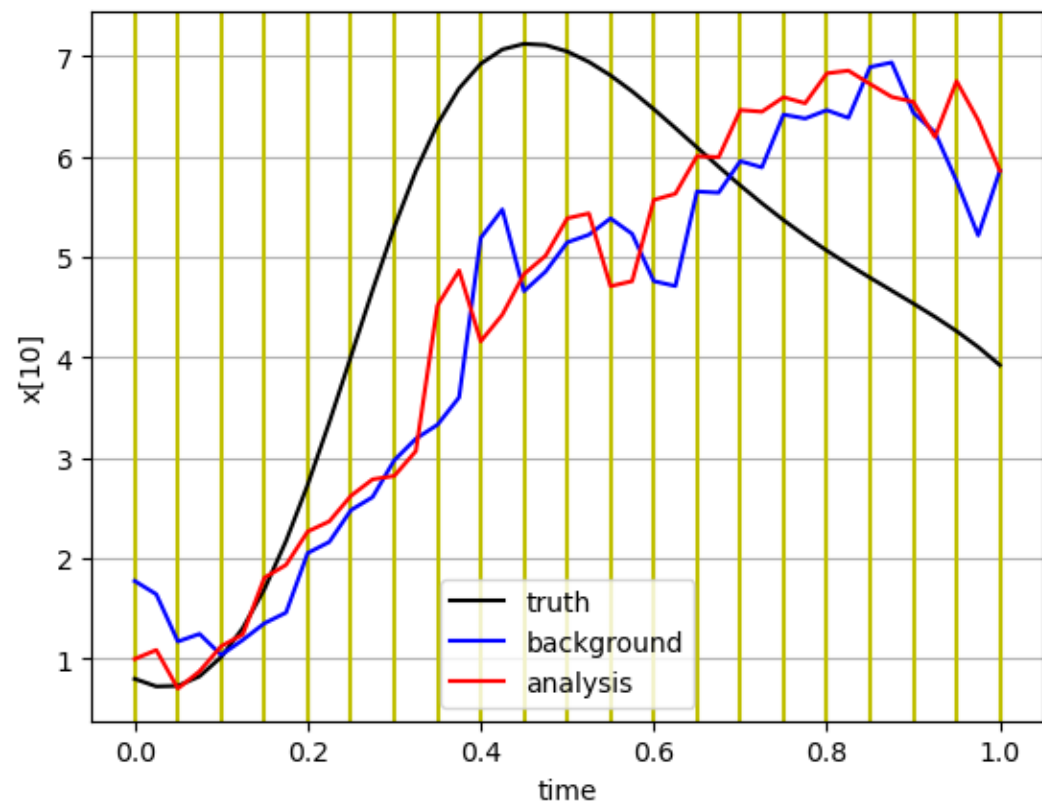
3D-Var analysis error distribution
mean = 0.02456, stddev = 0.94167, population = 2412



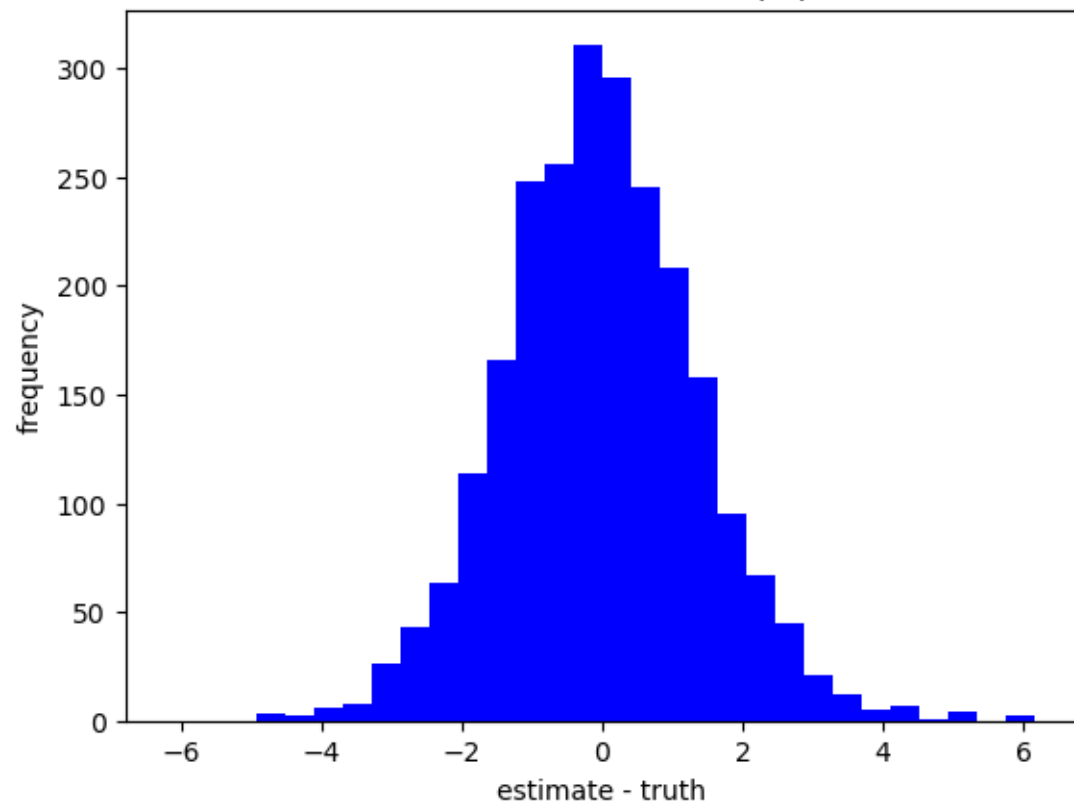
3DVar

footprint observations every other time step. $\text{Var}=2$.

3D-Var results variable 10



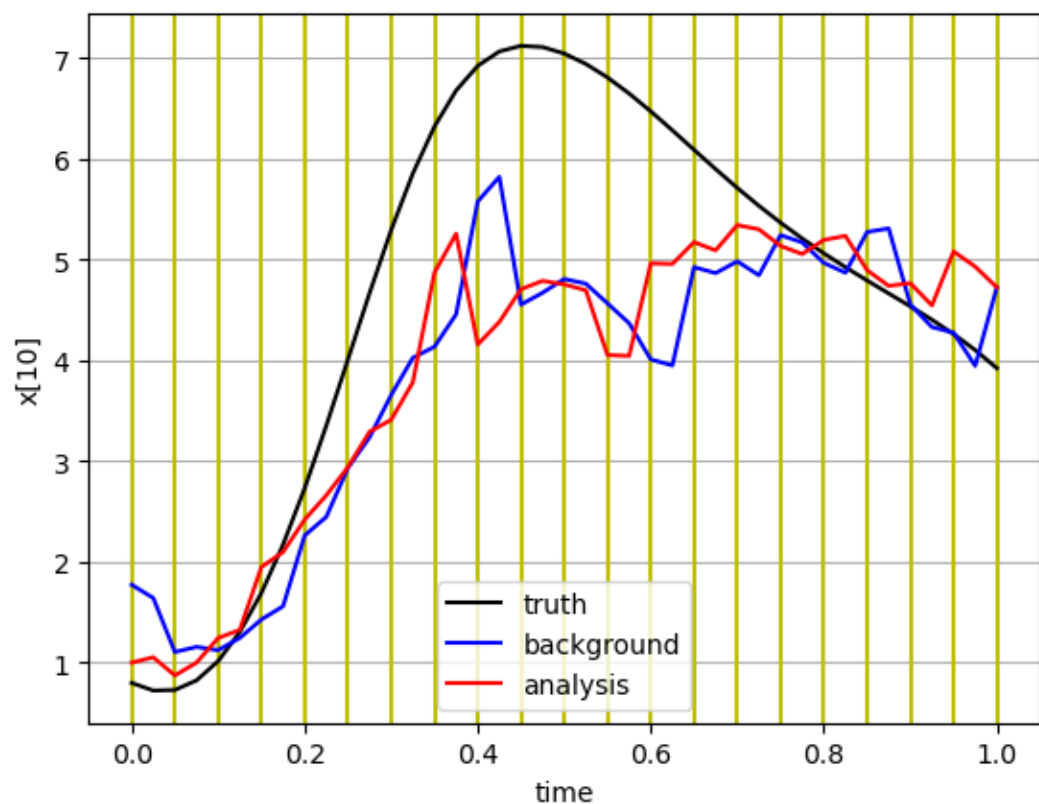
3D-Var analysis error distribution
mean = -0.01848, stddev = 1.38641, population = 2412



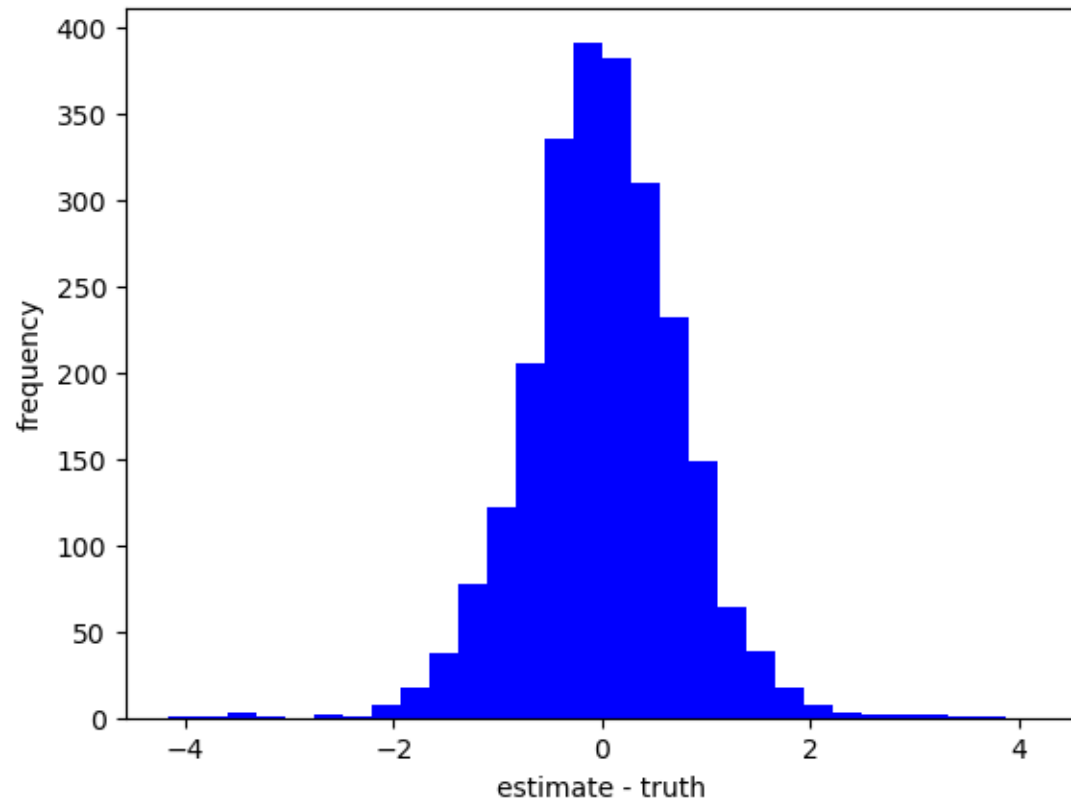
3DVar

footprint observations every other time step. $\text{Var}=0.2$

3D-Var results variable 10



3D-Var analysis error distribution
mean = 0.00742, stddev = 0.74690, population = 2412

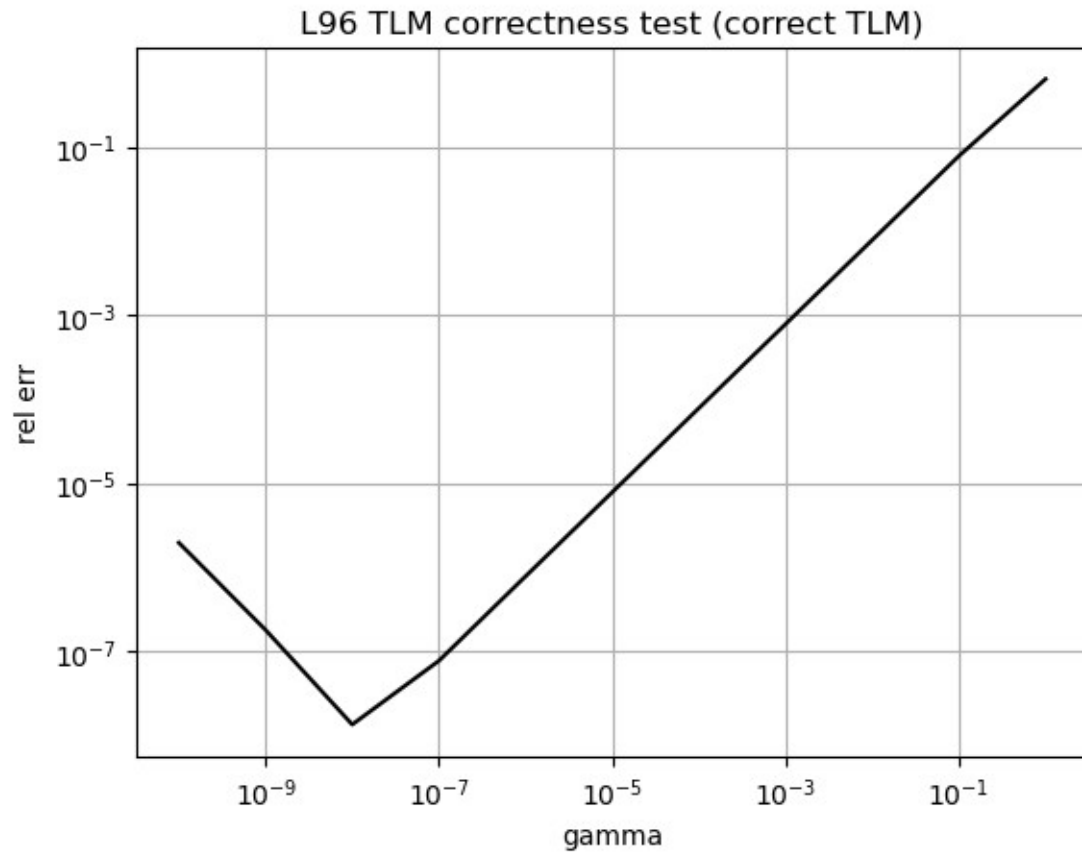


Incremental 4DVar

$$J(\delta \mathbf{x}_0) = \frac{1}{2} \delta \mathbf{x}_0^T \mathbf{B}_0^{-1} \delta \mathbf{x}_0 + \frac{1}{2} \sum_{i=0}^T (\mathbf{y}_i - \mathcal{H}_i(\mathbf{x}_i^b) - \mathbf{H}_i \mathbf{M}_{i-1} \mathbf{M}_{i-2} \dots \mathbf{M}_0 \delta \mathbf{x}_0)^T \mathbf{R}_i^{-1} (\bullet).$$

4DVar- TL and adjoint test

$$\frac{\|\mathcal{M}_{0 \rightarrow \tau}(\mathbf{x}_0' + \gamma \delta \mathbf{x}_0) - \mathcal{M}_{0 \rightarrow \tau}(\mathbf{x}_0) - \mathbf{M}_{0 \rightarrow \tau} \gamma \delta \mathbf{x}_0\|}{\|\mathbf{M}_{0 \rightarrow \tau} \gamma \delta \mathbf{x}_0\|},$$



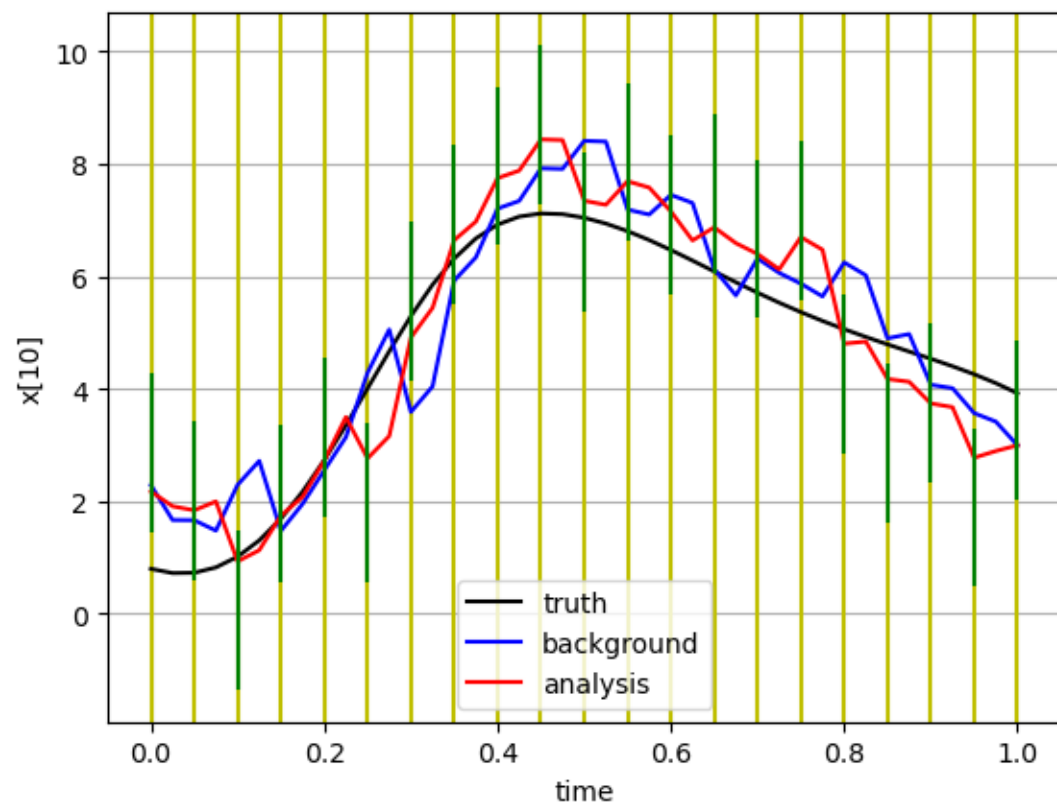
$$(\mathbf{M} \delta \mathbf{x}, \mathbf{M} \delta \mathbf{x}) = (\delta \mathbf{x}, \mathbf{M}^T \mathbf{M} \delta \mathbf{x})$$

lhs = 27.660443950044545
rhs = 27.66044395004454

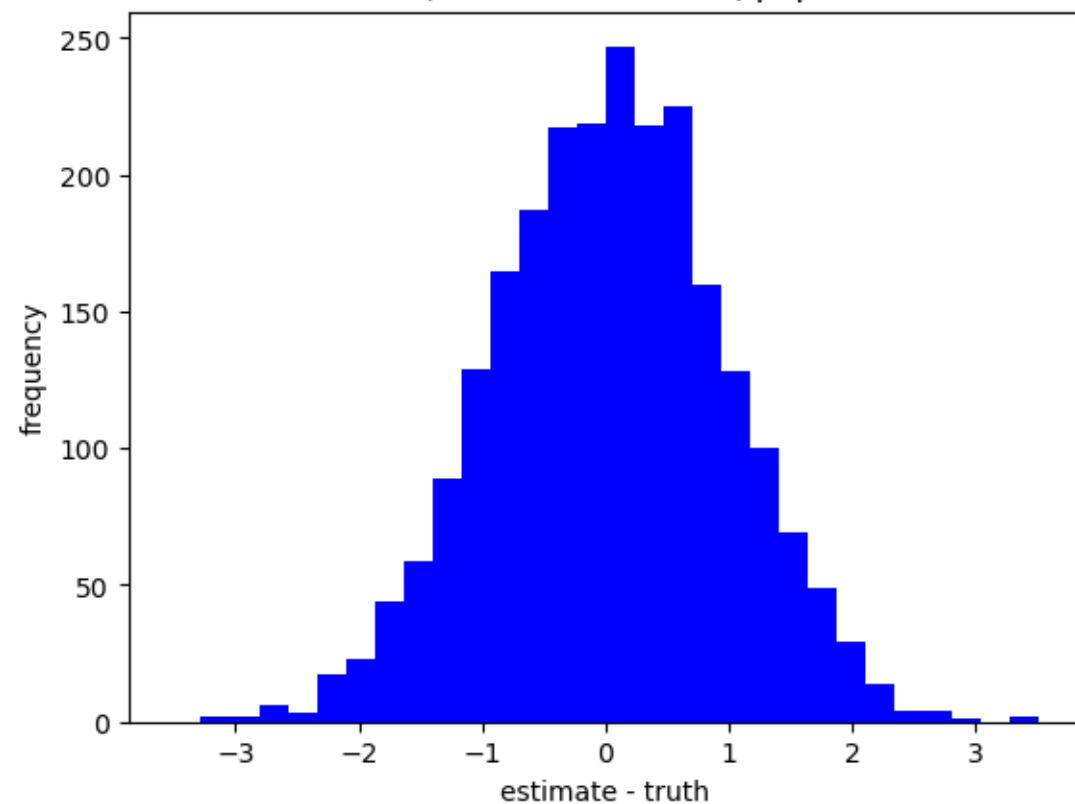
3DVar

Direct observations every other time step. Var=2.

3D-Var results variable 10



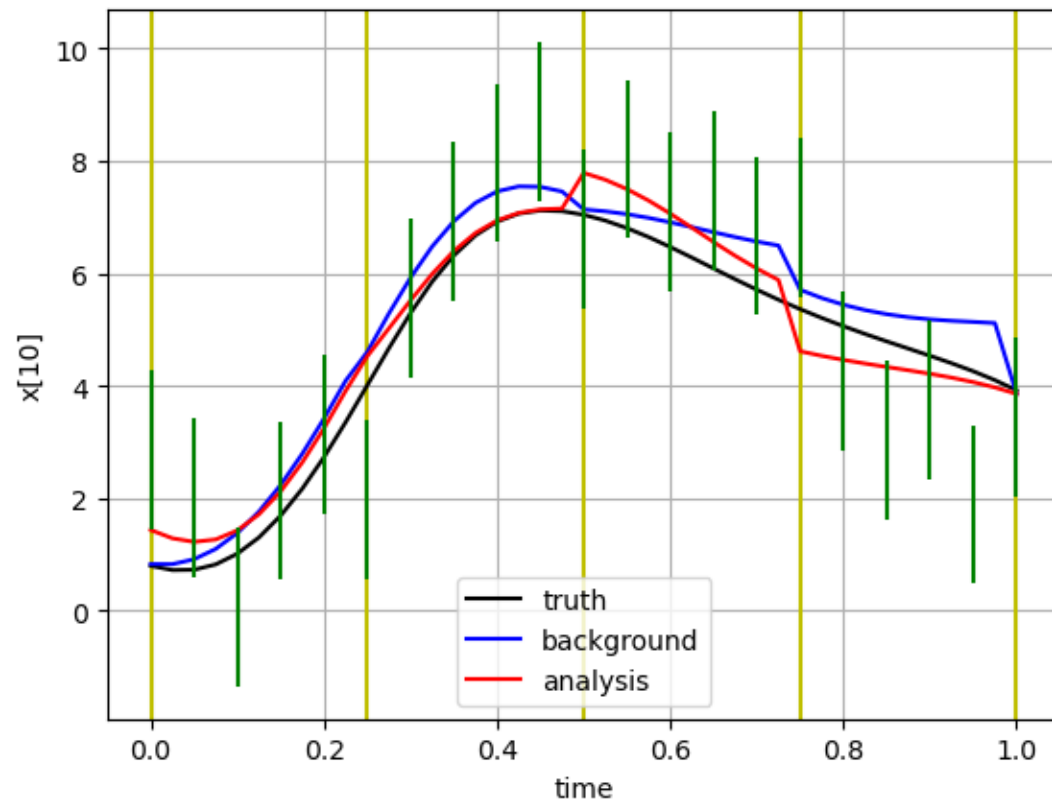
3D-Var analysis error distribution
mean = 0.02456, stddev = 0.94167, population = 2412



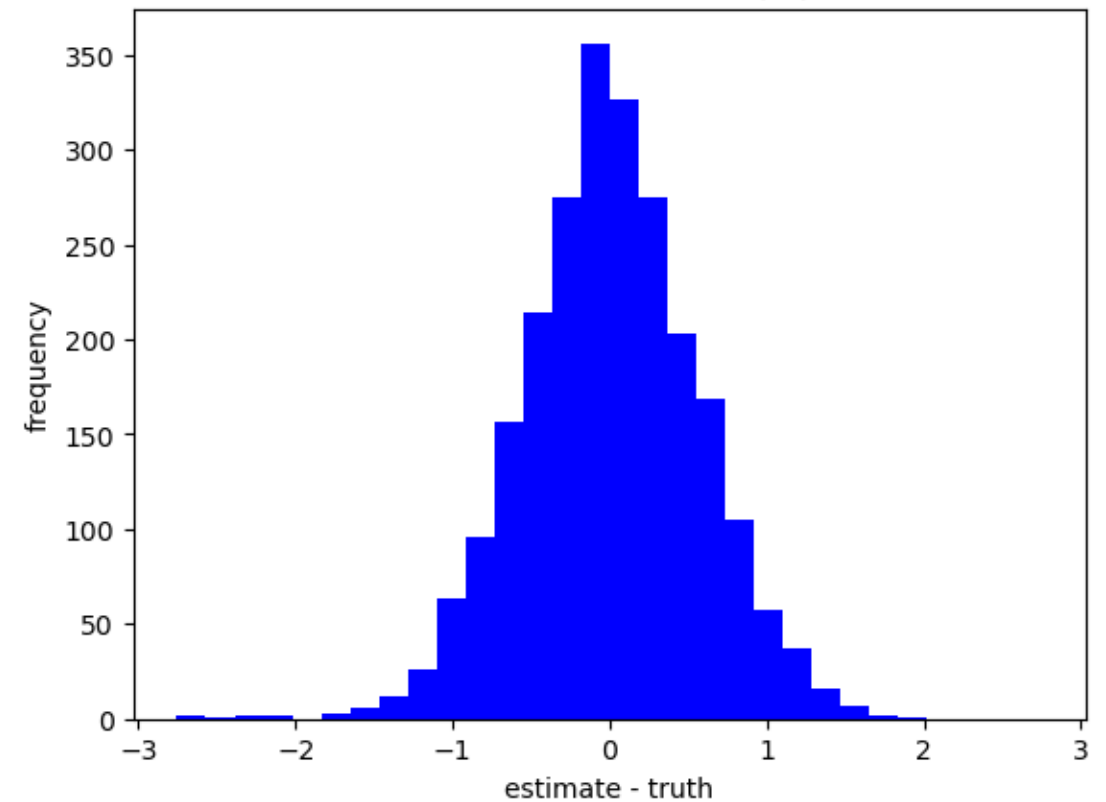
4DVar

Direct observations every other time step. Var=2.
Assim window=5 ob times

4D-Var results variable 10



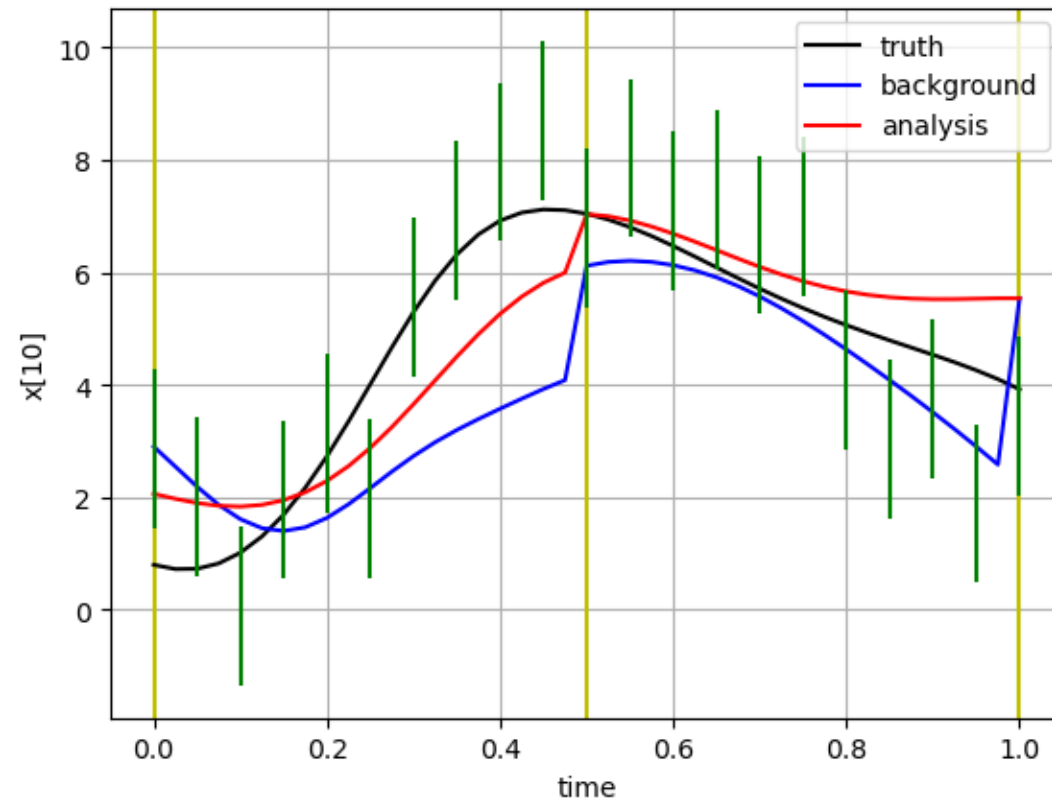
4D-Var analysis error distribution
mean = 0.00223, stddev = 0.55913, population = 2412



4DVar

Direct observations every other time step. Var=2.
Assim window=10 ob times

4D-Var results variable 10



4D-Var analysis error distribution
mean = -0.03619, stddev = 0.99348, population = 2412

