

Hybrid Data Assimilation II

Hybrid methods

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influunt dependens variationis \approx flow-dependent variational

How do we combine the properties of 'flow-dependence' of ensemble methods with the 'full-rankness' of variational methods?

Quiz: Which of the following is a definition of a hybrid data assimilation method?

- A. An ensemble DA method that uses a variational solution?
- B. A method that combines the \mathbf{B} -matrix of Var with the \mathbf{P}^f -matrix of the EnKF?
- C. A method that takes the arithmetic average of the analysis increments of Var and EnKF?
- D. A method that takes the geometric average of the analysis increments of Var and EnKF?

How do we combine the properties of ‘flow-dependence’ of ensemble methods with the ‘full-rankness’ of variational methods?

Quiz: Which of the following is a definition of a hybrid data assimilation method?

A. An ensemble DA method that uses a variational solution? *This is formally called pure EnVar.*

B. A method that combines the **B**-matrix of Var with the \mathbf{P}^f -matrix of the EnKF? *This is formally called hybrid-EnVar.*

~~**C.** A method that takes the arithmetic average of the analysis increments of Var and EnKF?~~

~~**D.** A method that takes the geometric average of the analysis increments of Var and EnKF?~~

Pure ensemble-variational methods (EnVar), no localisation

The basic idea (initial time, no localisation)

Suppose that we have an ensemble of model states from a separate EnKF. Let an increment at $t = 0$ be a linear combination of scaled forecast ensemble perturbations at $t = 0$ (no time indices means $t = 0$).

$$\begin{aligned}\delta \mathbf{x} &= \sum_{\ell=1}^N [\mathbf{v}_{\text{ens}}]_{\ell} \frac{\left(\mathbf{x}^{\text{f}(\ell)} - \overline{\mathbf{x}}^{\text{f}} \right)}{\sqrt{N-1}} \\ &= \mathbf{X}'^{\text{f}} \mathbf{v}_{\text{ens}}, \quad \mathbf{v}_{\text{ens}} \in \mathbb{R}^N\end{aligned}$$

recall $\mathbf{X}'^{\text{f}} = \frac{1}{\sqrt{N-1}} \begin{pmatrix} \uparrow & & \uparrow & & \uparrow \\ \mathbf{x}^{\text{f}(1)} - \overline{\mathbf{x}}^{\text{f}} & \dots & \mathbf{x}^{\text{f}(\ell)} - \overline{\mathbf{x}}^{\text{f}} & \dots & \mathbf{x}^{\text{f}(N)} - \overline{\mathbf{x}}^{\text{f}} \\ \downarrow & & \downarrow & & \downarrow \end{pmatrix}$

What does this remind us of? A control variable transform: “ $\delta \mathbf{x} = \mathbf{U} \mathbf{v}$ ”, with $\mathbf{U} \rightarrow \mathbf{X}'^{\text{f}}$.

$$\text{Implied covariance } \mathbf{U} \mathbf{U}^{\text{T}} = \mathbf{X}'^{\text{f}} \mathbf{X}'^{\text{f} \text{T}}$$

Pure ensemble-variational methods (EnVar), no localisation

$$\delta \mathbf{x}_0 = \mathbf{X}'_0 \mathbf{v}_{\text{ens}}$$

En3DVar (FGAT)

Start with the incremental formulation of 3DVar-FGAT (reference state is the background)

$$J^{\text{3DVar}}(\delta \mathbf{x}_0) = \frac{1}{2} \delta \mathbf{x}_0^T \mathbf{B}^{-1} \delta \mathbf{x}_0 + \frac{1}{2} \sum_{t=-T/2}^{T/2} (\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^{\text{b}}) - \mathbf{H}_t \delta \mathbf{x}_0)^T \mathbf{R}_t^{-1} (\bullet)$$

In control variable $\mathbf{v}_{\text{ens}} \in \mathbb{R}^N$ space

$$J^{\text{En3DVar}}(\mathbf{v}_{\text{ens}}) = \frac{1}{2} \mathbf{v}_{\text{ens}}^T \mathbf{v}_{\text{ens}} + \frac{1}{2} \sum_{t=0}^T (\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^{\text{b}}) - \mathbf{H}_t \mathbf{X}'_0 \mathbf{v}_{\text{ens}})^T \mathbf{R}_t^{-1} (\bullet)$$

$$\mathbf{x}_0^{\text{a}} = \mathbf{x}_0^{\text{b}} + \mathbf{X}'_0 \operatorname{argmin} (J^{\text{En3DVar}}(\mathbf{v}_{\text{ens}}))$$

Pure ensemble-variational methods (EnVar), no localisation

$$\delta \mathbf{x}_0 = \mathbf{X}'_0 \mathbf{v}_{\text{ens}}$$

En4DVar

Start with the incremental formulation of 4DVar (reference state is the background)

$$J^{4\text{DVar}}(\delta \mathbf{x}_0) = \frac{1}{2} \delta \mathbf{x}_0^T \mathbf{B}_0^{-1} \delta \mathbf{x}_0 + \frac{1}{2} \sum_{t=0}^T \left(\mathbf{y}_t - \mathbf{H}_t(\mathbf{x}_t^b) - \mathbf{H}_t \mathbf{M}_{t-1} \mathbf{M}_{t-2} \dots \mathbf{M}_0 \delta \mathbf{x}_0 \right)^T \mathbf{R}_t^{-1} (\bullet)$$

In control variable $\mathbf{v}_{\text{ens}} \in \mathbb{R}^N$ space

$$J^{\text{En4DVar}}(\mathbf{v}_{\text{ens}}) = \frac{1}{2} \mathbf{v}_{\text{ens}}^T \mathbf{v}_{\text{ens}} + \frac{1}{2} \sum_{t=0}^T \left(\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^b) - \mathbf{H}_t \mathbf{M}_{t-1} \mathbf{M}_{t-2} \dots \mathbf{M}_0 \mathbf{X}'_0 \mathbf{v}_{\text{ens}} \right)^T \mathbf{R}_t^{-1} (\bullet)$$

$$\mathbf{x}_0^a = \mathbf{x}_0^b + \mathbf{X}'_0 \operatorname{argmin} \left(J^{\text{En4DVar}}(\mathbf{v}_{\text{ens}}) \right)$$

Still need the tangent linear model (and adjoint for the gradient w.r.t. \mathbf{v}_{ens}).

Pure ensemble-variational methods (EnVar), no localisation

4DEnVar

Start with the En4DVar cost function:

$$J^{\text{En4DVar}}(\mathbf{v}_{\text{ens}}) = \frac{1}{2} \mathbf{v}_{\text{ens}}^T \mathbf{v}_{\text{ens}} + \frac{1}{2} \sum_{t=0}^T \left(\mathbf{y}_t - \mathbf{H}_t(\mathbf{x}_t^{\text{b}}) - \mathbf{H}_t \underbrace{\overbrace{\mathbf{M}_{t-1} \mathbf{M}_{t-2} \dots \mathbf{M}_0}^{\mathbf{M}_{0 \rightarrow t}} \mathbf{X}'_0}_{\mathbf{X}'_t} \mathbf{v}_{\text{ens}} \right)^T \mathbf{R}_t^{-1}(\bullet).$$

Consider the ℓ th column of \mathbf{X}_0^{f} (call $\mathbf{x}_0^{\text{f}(\ell)}$) and do a Taylor expansion of $\mathcal{M}_{0 \rightarrow t}$:

$$\begin{aligned} \mathcal{M}_{0 \rightarrow t} \underbrace{\left(\mathbf{x}_0^{\text{f}(\ell)} \right)}_{\substack{\ell\text{th column} \\ \text{of } \mathbf{X}_0^{\text{f}}}} &\approx \mathcal{M}_{0 \rightarrow t} \left(\overline{\mathbf{x}_t^{\text{f}}} \right) + \underbrace{\mathbf{M}_{0 \rightarrow t} \left(\mathbf{x}_t^{\text{f}(\ell)} - \overline{\mathbf{x}_t^{\text{f}}} \right)}_{\substack{\ell\text{th column of } \mathbf{X}'_0^{\text{f}} \\ \ell\text{th column of } \mathbf{X}'_t^{\text{f}}}}. \\ &\approx \overline{\mathcal{M}_{0 \rightarrow t} \left(\mathbf{x}_t^{\text{f}} \right)} + \mathbf{M}_{0 \rightarrow t} \left(\mathbf{x}_t^{\text{f}(\ell)} - \overline{\mathbf{x}_t^{\text{f}}} \right) \end{aligned}$$

Pure ensemble-variational methods (EnVar), no localisation

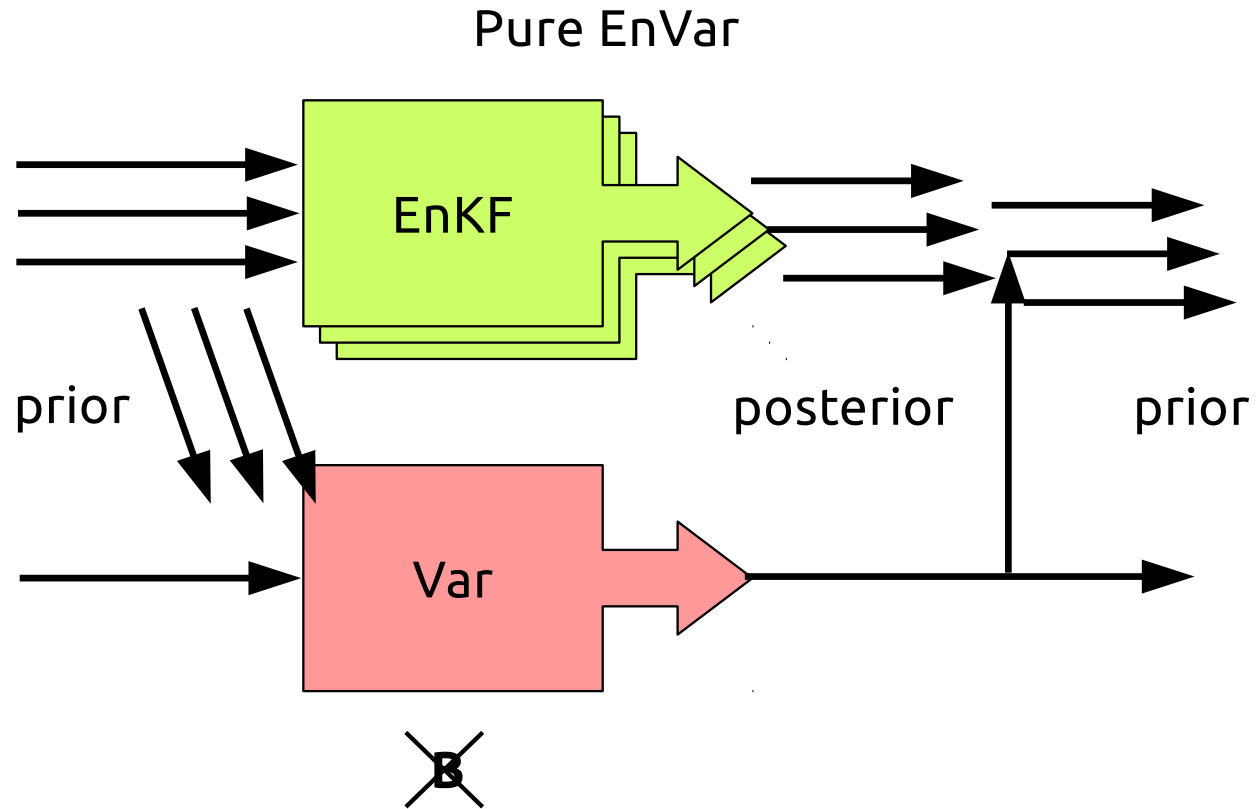
4DEnVar

$$\underbrace{\mathbf{M}_{0 \rightarrow t} \left(\mathbf{x}_t^{\text{f}(\ell)} - \overline{\mathbf{x}_t^{\text{f}}} \right)}_{\ell\text{th column of } \mathbf{X}'_t} \approx \mathcal{M}_{0 \rightarrow t} \left(\mathbf{x}_0^{\text{f}(\ell)} \right) - \overline{\mathcal{M}_{0 \rightarrow t} \left(\mathbf{x}_t^{\text{f}} \right)}$$

- Use the above to build columns of $\mathbf{M}_{0 \rightarrow t} \mathbf{X}'_0^{\text{f}} \equiv \mathbf{X}'_t^{\text{f}}$ (N runs of the model).
- This eliminates the need for a TLM (and its adjoint for the gradient).
- The method is called 4DEnVar.

$$J^{\text{4DEnVar}}(\mathbf{v}_{\text{ens}}) = \frac{1}{2} \mathbf{v}_{\text{ens}}^{\text{T}} \mathbf{v}_{\text{ens}} + \frac{1}{2} \sum_{t=0}^T \left(\mathbf{y}_t - \mathbf{H}_t(\mathbf{x}_t^{\text{b}}) - \mathbf{H}_t \mathbf{X}'_t^{\text{f}} \mathbf{v}_{\text{ens}} \right)^{\text{T}} \mathbf{R}_t^{-1} (\bullet)$$

Combining En and Var (“pure EnVar”)



Pure ensemble-variational methods (EnVar), localisation (loc)

- Remember localisation: $\mathbf{X}'^f \mathbf{X}'^{fT} \rightarrow \Omega \circ (\mathbf{X}'^f \mathbf{X}'^{fT})$.
- In the unlocalised system, \mathbf{v}_{ens} is an N -element set of scalars.
- Introduce loc: change each element of \mathbf{v}_{ens} from scalar $[\mathbf{v}_{\text{ens}}]_\ell \rightarrow n$ -element vector $\mathbf{v}_{\text{ens}}^{(\ell)}$.

- Introduce control matrix $\mathbf{V}_{\text{ens}} \in \mathbb{R}^{n \times N}$ and new CVT

$$\mathbf{V}_{\text{ens}} = \begin{pmatrix} \uparrow & & \uparrow \\ \mathbf{v}_{\text{ens}}^{(1)} & \cdots & \mathbf{v}_{\text{ens}}^{(N)} \\ \downarrow & & \downarrow \end{pmatrix}, \quad \delta \mathbf{x} = \frac{1}{\sqrt{N-1}} \sum_{\ell=1}^N \left(\Omega^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right) \circ \left(\mathbf{x}^{f(\ell)} - \overline{\mathbf{x}^f} \right)$$

- Let the background error covariance of each $\mathbf{v}_{\text{ens}}^{(\ell)}$ be \mathbf{I} , and let $\mathbf{v}_{\text{ens}}^{(\ell)}$ be uncorrelated with $\mathbf{v}_{\text{ens}}^{(\ell')}$ ($\ell \neq \ell'$).

- Cost function with localisation (back to En3DVar e.g.)

$$J^{\text{En3DVar}}(\mathbf{V}_{\text{ens}}) = \frac{1}{2} \sum_{\ell=1}^N \mathbf{v}_{\text{ens}}^{(\ell)T} \mathbf{v}_{\text{ens}}^{(\ell)} + \frac{1}{2} \sum_{t=-T/2}^{T/2} \left(\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^b) - \mathbf{H}_t \frac{1}{\sqrt{N-1}} \sum_{\ell=1}^N \left(\Omega^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right) \circ \left(\mathbf{x}^{f(\ell)} - \overline{\mathbf{x}^f} \right) \right)^T \mathbf{R}_t^{-1} \left(\bullet \right)$$

Pure ensemble-variational methods (EnVar), localisation

Why does this work?

$$\mathbf{V}_{\text{ens}} = \begin{pmatrix} \uparrow & & \uparrow \\ \mathbf{v}_{\text{ens}}^{(1)} & \cdots & \mathbf{v}_{\text{ens}}^{(N)} \\ \downarrow & & \downarrow \end{pmatrix} \quad \delta \mathbf{x} = \frac{1}{\sqrt{N-1}} \sum_{\ell=1}^N \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right) \circ \left(\mathbf{x}^{\text{f}(\ell)} - \bar{\mathbf{x}}^{\text{f}} \right) \quad \left\langle \mathbf{v}_{\text{ens}}^{(\ell)} \mathbf{v}_{\text{ens}}^{(\ell')\text{T}} \right\rangle_{\text{f}} = \mathbf{I} \delta_{\ell\ell'}$$

$$[\delta \mathbf{x}]_i = \frac{1}{\sqrt{N-1}} \sum_{\ell=1}^N \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right)_i \left([\mathbf{x}^{\text{f}(\ell)}]_i - [\bar{\mathbf{x}}^{\text{f}}]_i \right)$$

$$\langle (\delta \mathbf{x})_i \times (\delta \mathbf{x})_j \rangle_{\text{f}} = \frac{1}{N-1} \sum_{\ell=1}^N \left\langle \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right)_i \left([\mathbf{x}^{\text{f}(\ell)}]_i - [\bar{\mathbf{x}}^{\text{f}}]_i \right) \times \sum_{\ell'=1}^N \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell')} \right)_j \left([\mathbf{x}^{\text{f}(\ell')}]_j - [\bar{\mathbf{x}}^{\text{f}}]_j \right) \right\rangle$$

$$= \frac{1}{N-1} \sum_{\ell=1}^N \sum_{\ell'=1}^N \underbrace{\left\langle \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right)_i \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell')} \right)_j \right\rangle_{\text{f}}}_{\boldsymbol{\Omega}_{ij} \delta_{\ell\ell'}} \left([\mathbf{x}^{\text{f}(\ell)}]_i - [\bar{\mathbf{x}}^{\text{f}}]_i \right) \left([\mathbf{x}^{\text{f}(\ell')}]_j - [\bar{\mathbf{x}}^{\text{f}}]_j \right)$$

$$= \boldsymbol{\Omega}_{ij} \frac{1}{N-1} \sum_{\ell=1}^N \left([\mathbf{x}^{\text{f}(\ell)}]_i - [\bar{\mathbf{x}}^{\text{f}}]_i \right) \left([\mathbf{x}^{\text{f}(\ell)}]_j - [\bar{\mathbf{x}}^{\text{f}}]_j \right)$$

$$= \boldsymbol{\Omega}_{ij} \left(\mathbf{X}'^{\text{f}} \mathbf{X}'^{\text{f}\text{T}} \right)_{ij} \quad \therefore \langle \delta \mathbf{x} \delta \mathbf{x}^{\text{T}} \rangle_{\text{f}} = \boldsymbol{\Omega} \circ \left(\mathbf{X}'^{\text{f}} \mathbf{X}'^{\text{f}\text{T}} \right)$$

Pure ensemble-variational methods (EnVar), localisation

Notes

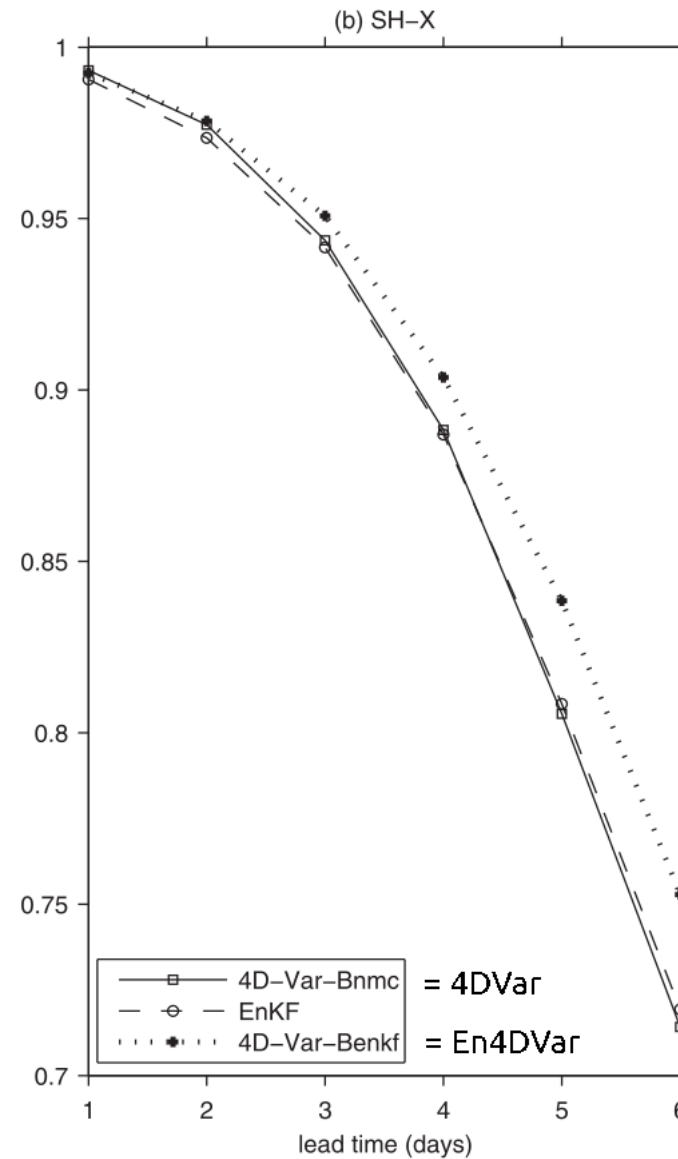
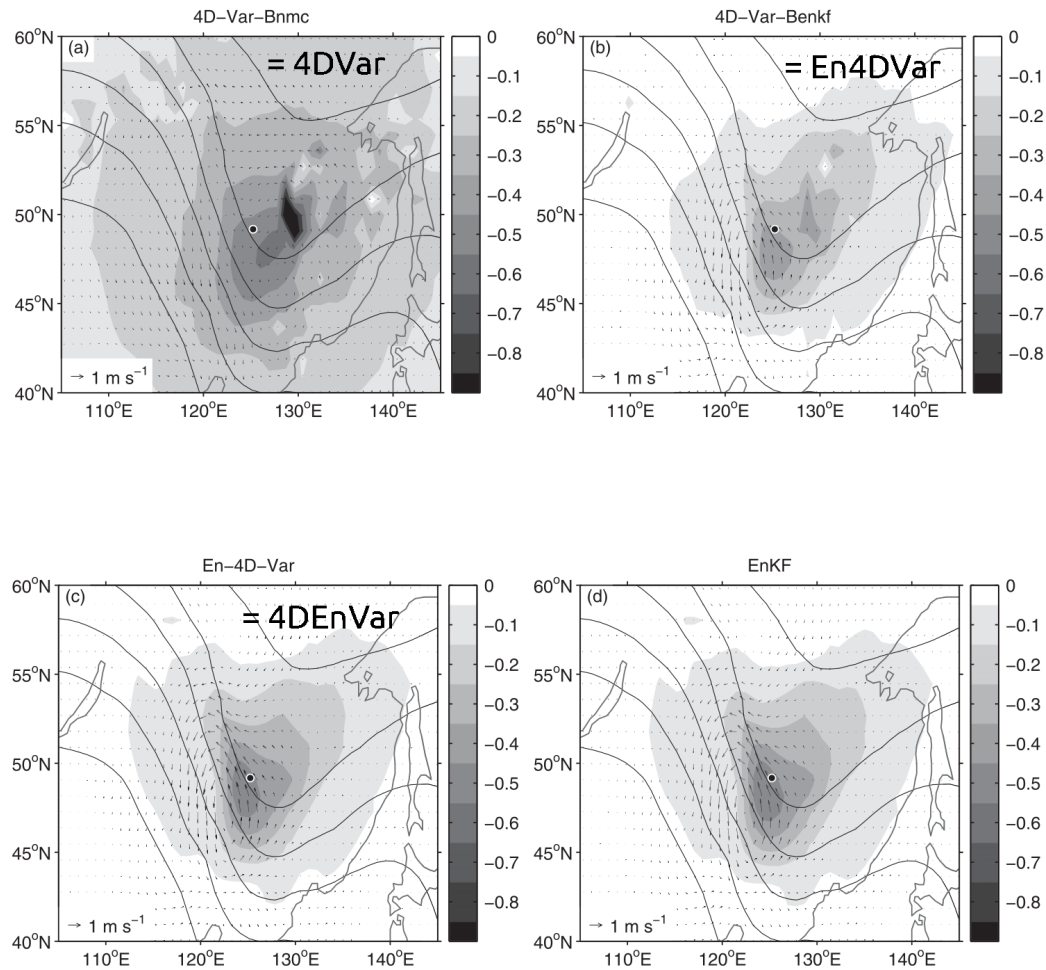
- Compact form of this CVT

$$\mathbf{V} = \begin{pmatrix} \uparrow & & \uparrow \\ \mathbf{v}_{\text{ens}}^{(1)} & \cdots & \mathbf{v}_{\text{ens}}^{(N)} \\ \downarrow & & \downarrow \end{pmatrix} \quad \delta \mathbf{x} = \frac{1}{\sqrt{N-1}} \sum_{\ell=1}^N \left(\boldsymbol{\Omega}^{1/2} \mathbf{v}_{\text{ens}}^{(\ell)} \right) \circ \left(\mathbf{x}^{\text{f}(\ell)} - \overline{\mathbf{x}^{\text{f}}} \right)$$

$$\delta \mathbf{x} = \left(\left(\boldsymbol{\Omega}^{1/2} \mathbf{V}_{\text{ens}} \right) \circ \mathbf{X}^{\text{f}} \right) \mathbf{1} \quad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

- This is the method of localisation from Lorenz 2003 [6]. Another method is from Buehner 2005 [2].
- The Schur product (\circ) messes-up the usual matrix manipulation.
- Don't need to represent background error covariances explicitly.

Single ob experiments and performance [3, 4]



500hPa analysis increments of T (shading) and (u, v) due to T ob $y - H(\mathbf{x}^b) = -1\text{K}$; contours are bg geopotential height.

Southern hemisphere anomaly correlations for 500 hPa geopotential height.

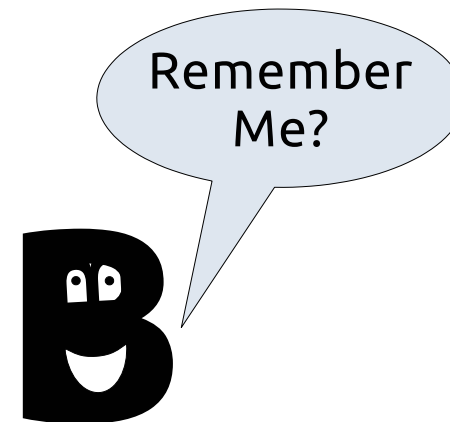
Hybrid ensemble-variational methods (Hybrid-EnVar)

- Up to now we have only considered (localised) *ensemble-derived* covariances, \mathbf{P}^f :
 - \mathbf{P}^f is flow-dependent (good), but rank deficient, etc. (bad) and not completely mitigated for with localisation.
- Remember we also have the \mathbf{B} -matrix from traditional variational assimilation:
 - \mathbf{B} is not fully flow-dependent (bad), but can be full-rank (good), and can have some useful properties (e.g. produces nearly balanced increments).

- Propose to combine them [5]:

$$\mathbf{P}_h = (1 - \beta)\mathbf{B} + \beta\mathbf{P}^f$$

- Trick is to represent this as a CVT.



Hybrid ensemble-variational methods (Hybrid-EnVar)

No localisation – the hybrid control variable transform

$$\delta \mathbf{x} = \mathbf{U}_h \mathbf{v}_h$$

$$\underbrace{\begin{pmatrix} \sqrt{1-\beta} \mathbf{U} & \sqrt{\beta} \mathbf{X}'^f \end{pmatrix}}_{\text{hybrid CVT}} \underbrace{\begin{pmatrix} \mathbf{v}_B \\ \mathbf{v}_{\text{ens}} \end{pmatrix}}_{\text{hybrid control variable}}$$

Implied background error covariance

$$\begin{pmatrix} \sqrt{1-\beta} \mathbf{U} & \sqrt{\beta} \mathbf{X}'^f \end{pmatrix} \begin{pmatrix} \sqrt{1-\beta} \mathbf{U}^T \\ \sqrt{\beta} \mathbf{X}'^{fT} \end{pmatrix} = (1-\beta) \mathbf{U} \mathbf{U}^T + \beta \mathbf{X}'^f \mathbf{X}'^{fT}$$

$$J^{\text{hEn4DVar}}(\mathbf{v}_h) = \frac{1}{2} \begin{pmatrix} \mathbf{v}_B^T & \mathbf{v}_{\text{ens}}^T \end{pmatrix} \begin{pmatrix} \mathbf{v}_B \\ \mathbf{v}_{\text{ens}} \end{pmatrix} + \frac{1}{2} \sum_{t=0}^T (\mathbf{y}_t - \mathbf{H}_t(\mathbf{x}_t^b) - \mathbf{H}_t \mathbf{M}_{0 \rightarrow t} \mathbf{U}_h \mathbf{v}_h)^T \mathbf{R}_t^{-1}(\bullet)$$

Hybrid ensemble-variational methods (Hybrid-EnVar)

No localisation – the control variable transform for Hybrid En3/4DVar

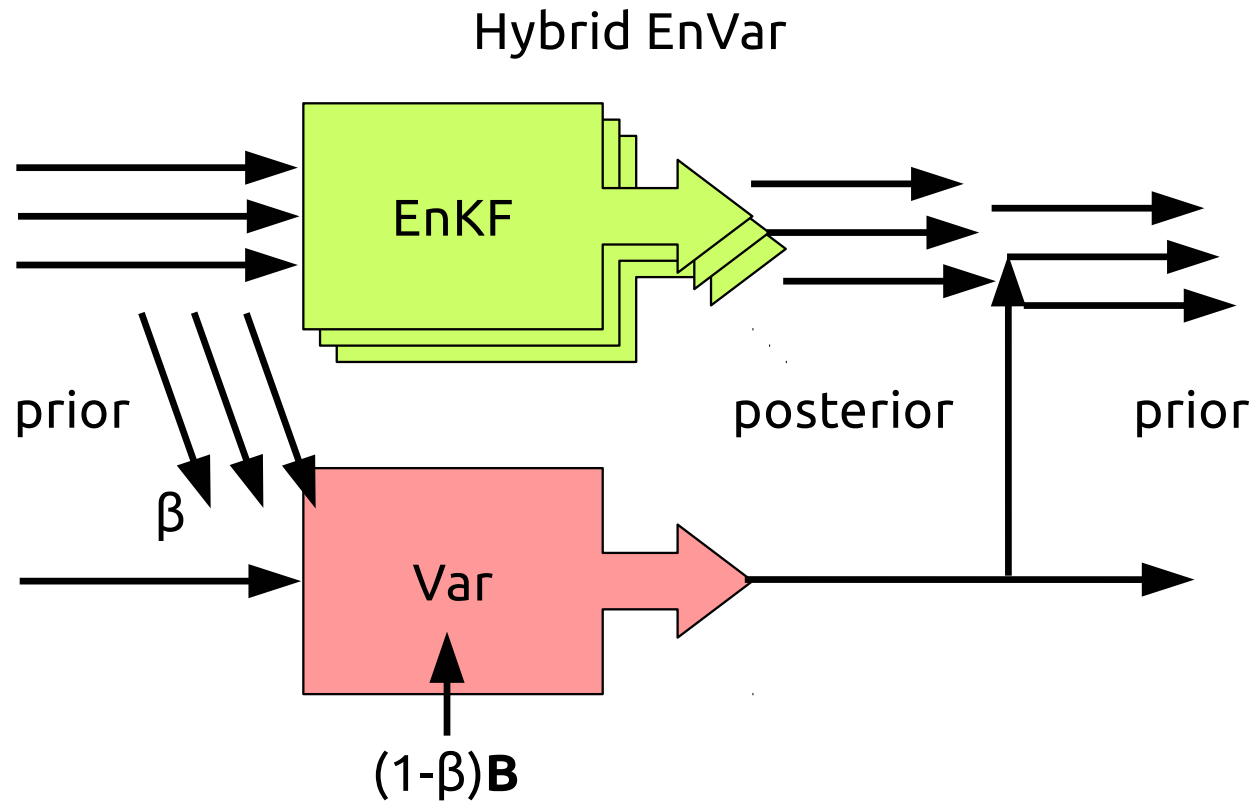
$$\delta \mathbf{x} = \sqrt{1 - \beta} \mathbf{U} \mathbf{v}_B + \sqrt{\beta} \mathbf{X}'^f \mathbf{v}_{\text{ens}} \quad \mathbf{v}_h = \begin{pmatrix} \mathbf{v}_B \\ \mathbf{v}_{\text{ens}} \end{pmatrix}$$

With localisation

$$\delta \mathbf{x} = \sqrt{1 - \beta} \mathbf{U} \mathbf{v}_B + \sqrt{\beta} \left(\mathbf{v}_{\text{ens}} \circ \mathbf{X}'^f \right) \mathbf{1} \quad \mathbf{v}_h = \begin{pmatrix} \mathbf{v}_B \\ \mathbf{v}_{\text{ens}} \end{pmatrix}$$

Things get complicated!

Combining En and Var with B (“hybrid EnVar”)



Summary of different schemes, see e.g. [1], nomenclature [7] (* defining “Hybrid” as a mixture of static and ens-based bg err cov matrices)

All designed to improve the bg err covs (i.e. flow-dependency); all assume cvs are uncorrelated/unit variance.

Scheme	CV	CVT	Length	TLM/adj?	Comments
En3DVar	\mathbf{v}_{ens}	$\delta \mathbf{x}_0 = \mathbf{X}'_0^f \mathbf{v}_{\text{ens}}$	N	n/a	3DVar with $\mathbf{X}'^f \mathbf{X}'^{fT}$
En3DVar (loc)	\mathbf{V}_{ens}	$\delta \mathbf{x} = \left(\left(\Omega^{1/2} \mathbf{V}_{\text{ens}} \right) \circ \mathbf{X}'^f \right) \mathbf{1}$	nN	n/a	3DVar with $\Omega \circ \left(\mathbf{X}'^f \mathbf{X}'^{fT} \right)$
En4DVar	\mathbf{v}_{ens}	As En3DVar	N	Y	4DVar with $\mathbf{X}'^f \mathbf{X}'^{fT}$
En4DVar (loc)	\mathbf{V}_{ens}	As En3DVar (loc)	nN	Y	4DVar with $\Omega \circ \left(\mathbf{X}'^f \mathbf{X}'^{fT} \right)$
4DEnVar	\mathbf{v}_{ens}	$\delta \mathbf{x}_t = \mathbf{X}'_t^f \mathbf{v}_{\text{ens}}$	N	N	$\mathbf{X}'_t^f \mathbf{X}'_t^{fT}$
4DEnVar (loc)	\mathbf{V}_{ens}	4D-version of En3DVar (loc)	nN	N	4D version of En3DVar (loc)
Hybrid*-En3DVar (loc)	$\mathbf{v}_B, \mathbf{V}_{\text{ens}}$	$\delta \mathbf{x}_0 = \mathbf{U} \mathbf{v}_B + \left(\left(\Omega^{1/2} \mathbf{V}_{\text{ens}} \right) \circ \mathbf{X}'^f \right) \mathbf{1}$	$n + nN$	n/a	3DVar with $(1 - \beta) \mathbf{B} + \beta \Omega \circ \mathbf{P}^f$
Hybrid*-En4DVar (loc)	$\mathbf{v}_B, \mathbf{V}_{\text{ens}}$	As Hybrid-En3DVar (loc)	$n + nN$	Y	4DVar with $(1 - \beta) \mathbf{B} + \beta \Omega \circ \mathbf{P}^f$
Hybrid*-4DEnVar (loc)	4D-version of Hybrid-En3DVar (loc)	4D-version of Hybrid-En3DVar (loc)	Many options	N	4D-version of Hybrid-En3DVar (loc)
Weak constraint ...					

Bibliography

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