Inertia-gravity waves emitted spontaneously from quasi-balanced flow: properties and consequences

Paul David Williams
(joint work with Thomas W. N. Haine, Peter L. Read [1])

Inertia-gravity waves are observed ubiquitously throughout the stratified parts of the atmosphere and ocean. Orthodox mechanisms for inertia-gravity wave generation include dynamical instability, such as Kelvin-Helmholtz shear instability. Another possible mechanism is the interaction between the flow and a physical obstruction, which is the mechanism by which mountains generate atmospheric gravity waves. Direct forcing of the ocean by the atmosphere is an important source of oceanic gravity waves.

Despite the above insights, our understanding of the sources of inertia-gravity waves remains rudimentary. For instance, a further potential source is spontaneous emission from quasi-balanced flow [2]. This is a generalization of the classical geostrophic adjustment process [3], in which an unbalanced flow tends to establish geostrophic balance by shedding excess energy as transient inertia-gravity waves. It has been shown that even first-order balanced flows undergo a generalized adjustment process that is accompanied by the spontaneous emission of inertia-gravity waves [4]. It is proving extraordinarily difficult to determine whether or not this mechanism is a significant source of inertia-gravity waves in real geophysical flows, however [5]. This hinders the development of parameterisations of the waves in general circulation models.

The spontaneous emission of inertia-gravity waves is intimately related to the concept of the slow manifold [6, 7]. The slow manifold is a putative, invariant sub-manifold of phase space, upon which the fluid remains completely devoid of inertia-gravity waves. The strict existence of the slow manifold, and hence the possibility of a flow evolving without ever spontaneously emitting inertia-gravity waves, has long been debated. The formal non-existence of the slow manifold is now generally accepted, however.

Even accepting the inevitability of spontaneous inertia-gravity wave emission, the possibility remains that the emitted waves will be sufficiently weak that they merely perturb the slow manifold into a quasi-stochastic fuzzy manifold [8] that retains many of its useful properties. The amplitude of spontaneously-emitted inertia-gravity waves, and its dependence on the bulk flow properties (especially the Rossby number), is therefore of great interest.

In this report, I summarize the properties of observed inertia-gravity waves emitted spontaneously from quasi-balanced flow, and the consequences for loss of balance from the atmospheric and oceanic mesoscale. In a laboratory study using a rotating two-layer annulus [9], it is found that all evolving quasi-balanced flows emit inertia-gravity waves spontaneously. It has been shown [10] that the appearance of the waves is well-predicted by the radiation term derived by [4], following [11].

Two important issues arise from this study. First, the quasi-balanced flow leaks roughly 1% of its energy each rotation period into inertia-gravity waves at the peak of their generation. Extrapolation of this result suggests that the spontaneous emission mechanism might make a significant contribution to the energy budgets of the ocean and atmosphere. For example, it is crudely estimated that O(1TW) is being lost from balanced mesoscale ocean eddies into the internal wave field, suggesting that this mechanism might be a significant player in maintaining the deep ocean stratification.

Second, the inertia-gravity wave amplitude shows a broadly linear variation with Rossby number in the range 0.05–0.14, at constant Froude number. This is in disagreement with asymptotic and non-asymptotic theories, which predict algebraic and exponential variation, respectively [12, 13]. This suggests that the fuzzy manifold is not exponentially thin in Rossby number, as previously thought. This has potentially important implications for the fundamental dynamical concepts of balance and potential vorticity inversion.

An asymptotic renormalization theory appears to yield the observed linear Rossby number scaling. The theory is based on that of [14] in the small Rossby number (Ro) limit. The first-order renormalized equation contains only resonant triplet interactions and thus cannot generate inertia-gravity waves from quasi-balanced potential vorticity modes. In fact, if the initial inertia-gravity wave energy is zero, the equation reduces to quasi-geostrophic dynamics. The renormalized solution also contains a first-order slaved term consisting only of inertia-gravity waves, however. This term is zero initially, but increases in a few fast wave periods to be O(Ro). The inertia-gravity waves are slaved to the quasi-balanced flow and are not freely propagating. In this sense a slow manifold still exists because the entire flow can still be deduced from potential vorticity inversion.

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The primary nonlinear dynamics of modal and nonmodal perturbations of monochromatic inertia-gravity waves

Ulrich Achatz

The importance of gravity waves for the dynamics of the middle atmosphere via their momentum and energy deposition has been known for a long time. Since the major part of the corresponding wave spectrum is at scales which cannot be resolved within state-of-the-art general circulation models these can only incorporate gravity-wave effects via parameterizations. With this regard there are still considerable uncertainties since many details of (mostly tropospheric and stratospheric) gravity-wave radiation, propagation through the middle atmosphere, and breaking, predominantly in the mesosphere-lower-thermosphere (MLT), are not sufficiently understood yet [6]. With regard to the breaking process itself, a systematic approach, starting with a linear analysis, and using the thereby identified instability patterns for distinctly perturbing a wave, had not been done yet. This might, however, be useful for the derivation of paradigms of wave breaking which could be used in improved parameterization schemes. As a first step in such a procedure, the linear stability of monochromatic gravity waves has been reinvestigated by [1] and [4, 5]. A fundamental result is that even in the absence of classic normal-mode (NM) instabilities vigorous transient growth of singular vectors (SV) is still possible. Especially for inertia-gravity waves (IGWs) this puts traditionally used instability thresholds, such as those of static instability (negative vertical buoyancy gradient) or dynamic instability (sufficiently strong vertical shear), at question.
Based on these results the breaking of an inertia-gravity wave, initiated by its leading NMs or SVs, and the resulting small-scale eddies have been investigated by means of direct numerical simulations of a Boussinesq fluid characterizing the