Achieving seventh-order amplitude accuracy in leapfrog integrations

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The leapfrog time-stepping scheme makes no amplitude errors when integrating linear oscillations. Unfortunately, the Robert–Asselin filter, which is used to damp the computational mode, introduces first-order amplitude errors. The RAW filter, which was recently proposed as an improvement, eliminates the first-order amplitude errors and yields third-order amplitude accuracy. However, it has not previously been shown how to further improve the accuracy by eliminating the third- and higher-order amplitude errors.

Here, it is shown that leapfrogging over a suitably weighted blend of the filtered and unfiltered tendencies eliminates the third-order amplitude errors and yields fifth-order amplitude accuracy. It is further shown that the use of a more discriminating \((1, -4, 6, -4, 1)\) filter instead of a \((1, -2, 1)\) filter eliminates the fifth-order amplitude errors and yields seventh-order amplitude accuracy. The proposed new schemes are tested successfully in numerical integrations of a simple nonlinear system. They appear to be attractive alternatives to the filtered leapfrog schemes currently used in many atmosphere and ocean models.

Reference