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A NEW DIRECTION IN CLEAR-AIR TURBULENCE FORECASTING BASED ON SPONTANEOUS IMBALANCE
PART 1: APPLICATION OF THEORY

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1. INTRODUCTION

Clear-air turbulence (CAT) is in-flight bumpiness detected by aircraft at high altitudes in regions devoid of significant cloudiness or nearby thunderstorms. Commercial aircraft encounter severe-or-greater turbulence about 5,000 times each year, the majority of which occur above FL100 (flight level 10,000 feet above mean sea level); these incidents lead to tens of millions of dollars in injury claims per year (Sharman et al. 2006).

However, a significant limitation for the forecasting of all types of aviation turbulence is identifying the source of gravity waves (McCann 2001). Partly as a result, current Federal goals for aviation turbulence forecasting are "currently not achievable by either automated or experienced human forecasters" (Sharman et al. 2006).

Williams et al. (2005) employed laboratory experiments and quasi-geostrophic model results to explore generation mechanisms of gravity waves in a rotating, two-layer vertically sheared flow. To diagnose wave activity, the authors calculated five dynamical indicators, several of which were originally devised as CAT forecasting indices (Roach 1970; Brown 1973). The most accurate indicator tested by Williams et al., however, derived from the Lighthill-Ford theory of spontaneous balance. Williams et al. concluded, "Further work is required to determine in more detail how to properly interpret the Lighthill/Ford indicator," its "geophysical relevance and applicability," and its relationship to other indicators of imbalance. This is the motivation for the present work.

2. THEORY

Lighthill (1952) derived the theory for the generation of sound waves by large-scale motions in a three-dimensional compressible adiabatic gas. Ford (1994) extended Lighthill's theory to rotating stratified flow and inertia-gravity wave generation, as did Medvedev and Gavrilov (1995). Ford's derivation is based on the flux forms of the momentum and conservation of mass equations in shallow-water flow on the *f*-plane. By forming the divergence and vorticity equations, and then combining them with conservation of mass and its second derivative, Ford obtained the following wave equation:

$$\left(\frac{\partial^2}{\partial t^2} + f^2 - gh_0\nabla^2\right)\frac{\partial h}{\partial t} = \frac{\partial^2}{\partial x_i \partial x_j}T_{ij} \qquad (1)$$

in which g is the acceleration due to gravity, h is the layer depth, and h_0 is the layer depth far from the region of vortical motion; and in standard tensor notation T_{ij} is:

$$T_{ij} = \frac{\partial}{\partial t}\left(hu_iu_j\right) + \frac{f}{2}\left(\varepsilon_{ik}hu_ju_k + \varepsilon_{jk}hu_iu_k\right) + \frac{1}{2}g\frac{\partial}{\partial t}\left(h-h_0\right)^2\delta_{ij} \qquad (2)$$

Ford (1994) and Williams et al. (2005) indicated that nonzero values of right-hand side of (1) should be regarded as a source of gravity waves. Strictly speaking, this interpretation is not exact because the variable h is not isolated on the left-hand side of (1). However, the separation of timescales and the weakness of the gravity waves compared to the large-scale flow allow for this interpretation, and this interpretation has also been confirmed by Williams et al.'s laboratory experiments.

Williams et al. (2005) referred to the right-hand side of (1) as the "Lighthill/Ford radiation term" and re-expressed it as

$$R = \underbrace{\frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{G}\right)}_{\text{Term 1}} + \underbrace{f\mathbf{k}\cdot\nabla\times\mathbf{G}}_{\text{Term 2}} + \underbrace{\frac{g}{2}\frac{\partial}{\partial t}\nabla^2\left(h-h_0\right)^2}_{\text{Term 3}} \qquad (3)$$

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in which

$$\mathbf{G} = \mathbf{u}\nabla\cdot(h\mathbf{u}) + (h\mathbf{u}\cdot\nabla)\mathbf{u} \qquad (4)$$

The three right-hand side terms in (3) are the Lighthill-Ford gravity wave radiation terms.

As an extension of previous work, therefore, we expand each term in (3) and discuss each term separately regarding its contribution to spontaneous imbalance. [Derivatives of *h* arising from (4) may be shown to sum to zero via conservation of mass; we omit them below.]

Term 1 can be expressed as a function of a common diagnostic of imbalance, the horizontal divergence *D*:

$$\frac{\partial}{\partial t}(\nabla\cdot\mathbf{G}) = 2h\left[\underbrace{\frac{\partial}{\partial t}D^2}_{\text{Term 1A}} + \underbrace{\frac{\partial}{\partial t}\mathbf{u}\cdot\nabla D}_{\text{Term 1B}} - \underbrace{\frac{\partial}{\partial t}J(u,v)}_{\text{Term 1C}}\right] \qquad (5)$$

These subterms are obtained from the time derivative of the divergence equation in the derivation of (1). Term 1A is a source term due to the local change of divergence; Term 1B is a source of gravity waves via the local change of the horizontal advection of horizontal divergence. Term 1C is the time derivative of the familiar Jacobian term found in both the divergence equation and in its approximated form, the nonlinear balance equation (NBE; Zhang et al. 2000).

Term 2 is expressible as a combination of the horizontal divergence and the vertical component of relative vorticity ζ:

$$f\mathbf{k}\cdot\nabla\times\mathbf{G} = h\left[\underbrace{2Df\zeta}_{\text{Term 2A}} + \underbrace{f\mathbf{u}\cdot\nabla\zeta}_{\text{Term 2B}} + \underbrace{f\left(v\frac{\partial D}{\partial x} - u\frac{\partial D}{\partial y}\right)}_{\text{Term 2C}}\right] \qquad (6)$$

The product of divergence, planetary vorticity, and relative vorticity is found in Term 2A; this product is not found elsewhere in the divergence, NBE or vorticity equations. Term 2B is proportional to the horizontal advection of relative vorticity. Term 2C is proportional to the vertical component of the cross-product of the vector velocity with the horizontal gradient of divergence.

Term 3 can also be re-expressed as:

$$\frac{g}{2}\frac{\partial}{\partial t}\nabla^2(h-h_0)^2 = g\nabla^2\left[(h-h_0)\frac{\partial(h-h_0)}{\partial t}\right] =$$

$$g\left[\underbrace{\frac{\partial h}{\partial t}\nabla^2 h}_{\text{Term3A}} + \underbrace{(h-h_0)\left(\nabla^2\frac{\partial h}{\partial t}\right)}_{\text{Term3B}} + \underbrace{2\left(\frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial t\partial x} + \frac{\partial h}{\partial y}\frac{\partial^2 h}{\partial t\partial y}\right)}_{\text{Term3C}}\right] \qquad (7)$$

In Term 3A, the Laplacian of height appears, reminiscent of the Laplacian of geopotential found in the divergence and NBE equations.

3. SCALE ANALYSIS

Following Haltiner and Williams (1980, Ch. 3-2), a simple scale analysis of the Lighthill-Ford radiation subterms in (5), (6) and (7) can be performed for synoptic-scale mid-latitude flows with small Rossby number ($Ro \ll 1$), i.e. background conditions which in our experience are representative of many clear-air turbulence outbreaks.

Assuming velocity and length scales of *U* and *L*; an advective time scale *T*; and the ratio of divergent and rotational components of the velocity scales as *Ro*, then

$$T \sim \frac{L}{U},\ \zeta \sim \frac{U}{L},\ D \sim Ro\frac{U}{L} \qquad (8)$$

In addition, by invoking the definition

$$f \equiv \frac{1}{Ro}\frac{U}{L} \qquad (9)$$

we can scale Terms 1 and 2 with respect to (nonzero) *Ro*:

$$\frac{\partial}{\partial t}(\nabla\cdot\mathbf{G}) \sim \frac{U^3}{L^2}\left[\underbrace{Ro^2}_{\text{Term 1A}} \quad \underbrace{Ro}_{\text{Term 1B}} \quad \underbrace{1}_{\text{Term 1C}}\right] \qquad (10)$$

$$f\mathbf{k}\cdot\nabla\times\mathbf{G} \sim \frac{U^3}{L^2}\left[\underbrace{1}_{\text{Term 2A}} \quad \underbrace{Ro^{-1}}_{\text{Term 2B}} \quad \underbrace{1}_{\text{Term 2C}}\right] \qquad (11)$$

To leading order for $Ro \ll 1$, therefore,

$$\frac{\text{Term 1}}{\text{Term 2}} \propto \frac{\text{Term 1C}}{\text{Term 2B}} \sim Ro \qquad (12)$$

This result agrees with Williams et al.'s (2005) comment that the ratio of Terms 1 and 2 should scale as the bulk Rossby number. We have, however, extended their results and identified the leading-order components within Terms 1 and 2 which lead to that scaling. We also confirm for low *Ro* their speculation that only Term

2 might be retained for "many practical purposes." Additionally, Medvedev and Gavrilov's (1995) analysis emphasized the equivalent of our Term 2B for inertia-gravity wave forcing by quasi-geostrophic motions.

To scale Term 3, we rewrite it in terms of Reznik et al.'s (2001) nonlinearity parameter λ:

$$\frac{g}{2}\frac{\partial}{\partial t}\nabla^2\left(h-h_0\right)^2 \sim \frac{U^3}{L^2}\lambda^2 \qquad (13)$$

Furthermore, for standard quasi-geostrophic conditions, Reznik et al. note that λ scales as *Ro*. Therefore for our analysis:

$$\frac{g}{2}\frac{\partial}{\partial t}\nabla^2\left(h-h_0\right)^2 \sim \frac{U^3}{L^2}Ro^2 \qquad (14)$$

Thus, for small *Ro*, Term 3 would be similar in magnitude to O(Ro^2), i.e. three orders of magnitude smaller than Term 2B. This scaling is broadly consistent with the quasi-geostrophic model results of Williams et al. (2005, Fig. 7), who found Term 2 to be a factor of 40 or more larger than Terms 1and 3; and with Sugimoto et al. ("Spontaneous gravity wave generation from unsteady rotational flows in the shallow water system on an *f*-plane and a rotating sphere," Spontaneous Imbalance Workshop presentation, August 2006), who found Term 3 negligible for large *Ro*.

To summarize, our scale analysis of the Lighthill-Ford radiation terms identifies the following:

Leading-order term: Term 2B (relative vorticity advection)
Second-order terms: Term 1C, Term 2A, Term 2C
Higher-order terms: Term 1A, Term 1B, Term 3

Therefore, we should expect to find that regions of large advection of relative vorticity should be a dominant source of spontaneous gravity wave generation. Medvedev and Gavrilov (1995, Eq. 23) also identified advection of relative vorticity as the source term for inertia-gravity waves in their extension of Lighthill's theory.

The Jacobian term, divergence-vorticity product, and cross-product of velocity with the gradient of divergence may also play non-negligible roles for situations in which $Ro < 1$ but not $Ro \ll 1$. As a result, we also retain these terms. Medvedev and Gavrilov (1995, Eq. 18) identified a source term similar to Terms 1A and 1C as the most important for the generation of mesoscale waves.

4. INTERPRETATION

Term 2 of the Lighthill-Ford radiation term is essentially identical to the numerator of the advective Rossby number used by Uccellini et al. (1984) to diagnose unbalanced flow. CAT diagnostics based on the advective Rossby number, such as the inertial-advective wind and the Lagrangian Rossby number (see Zhang et al. 2000) have been used with some success in CAT forecasting. Since the leading-order term of our scale analysis results from Term 2, this provides theoretical justification for the use of inertial-advective CAT predictors and suggests their success may be related to Lighthill-Ford gravity wave generation.

The leading-order Term 2B has been previously identified as a CAT forecasting diagnostic. Shapiro (1978) related gradients of potential vorticity to CAT; Kaplan et al. (2005) created a CAT predictor related to gradients of relative vorticity. A long-standing rule-of-thumb CAT forecasting technique in the aviation meteorology community has been to identify regions of strong negative absolute vorticity advection (e.g., Appendix A, item p of Sharman et al. 2006). However, during the past four decades deformation and vertical shear, rather than horizontal vorticity advection, have been emphasized in CAT forecasting techniques. Interestingly, deformation and divergence tendency can be related to Term 1C, raising the possibility that some portion of the success of deformation-based and divergence-tendency-based CAT diagnostics may also be tied to Lighthill-Ford processes.

Another second-order subterm, Term 2A, possesses a unique characteristic that was proposed in an earlier paper on CAT. Knox (1997), noting Sparks et al.'s (1977) observations of frequent CAT in both strongly anticyclonic and strongly cyclonic flows, suggested that a parabolic nonlinear relationship between CAT and absolute vorticity might exist, with a minimum at intermediate values of absolute vorticity. The absolute value of Term 2A has this relationship with absolute vorticity.

In summary, Lighthill-Ford theory can be related to several existing CAT forecasting techniques of the past forty years, which were often inspired by empirical approaches.

5. IMPLEMENTATION OF THEORY

We hypothesize that gravity waves spontaneously emitted according to Lighthill-Ford theory relate to clear-air turbulence felt by aircraft in the following manner. First, the gravity wave acts upon the environment and destabilizes it. If the environment is close to being dynamically unstable with respect to Kelvin-Helmholtz instability (i.e., the environment has small Richardson number *Ri*), then the gravity wave causes *Ri* to be reduced locally to less than 0.25 and turbulence ensues (Miles and Howard 1964; Dutton and Panofsky 1970). Therefore, not only Lighthill-Ford forcing but also the environmental *Ri* must be considered in the production of turbulence. In this way, even weak gravity waves may initiate turbulence.

The intensity of the turbulence is not addressed by either of these two quantities. A separate quantity, turbulent kinetic energy (TKE) dissipation, is the only known quantitative approach that is correlated with aircraft turbulence intensity (McCann 1999; also see “eddy dissipation rate” in Cornman et al. 1995).

For these reasons, we pursue application of Lighthill-Ford theory as a CAT forecasting diagnostic using the TKE approach of McCann (2001). In this paper, McCann outlined a simple first-order turbulence closure, ingredients-based CAT forecast technique and presented a procedure for combining the ingredients. We summarize this procedure below.

To reiterate, the guiding assumption in McCann (2001) is that gravity waves locally modify the environmental *Ri* which can then trigger CAT via Kelvin-Helmholtz instability. [This theory fundamentally differs from the Mancuso and Endlich (1966) approach in that gravity waves do not “degenerate” into turbulence. Their theory implies that the turbulence cascade begins at the mesoscale while McCann’s implies a much smaller scale.] Because the modified *Ri* fluctuates within a gravity wave, only portions of the wave are turbulent. The maxima of the two sources of production of gravity wave-enhanced turbulent kinetic energy may be estimated as

$$\varepsilon_{buoy} = K_h(\hat{a}-1)N^2 \qquad (15)$$

and

$$\varepsilon_{wshr} = K_m\left(\frac{\partial \mathbf{V}}{\partial z}\right)^2\left(1+\hat{a}\sqrt{Ri}\right)^2 \qquad (16)$$

In (15) and (16), ε_{buoy} and ε_{wshr} are the gravity wave modified TKE dissipation due to buoyancy and wind shear, respectively, K_h and K_m are the eddy thermal diffusivity and the eddy viscosity, respectively, and **V** is the vector horizontal wind. The ratio, K_m / K_h , is a turbulent Prandtl number; the closer this ratio is to 0.25, the less intermittent the turbulence. The eddy viscosity is empirically determined so that the resulting TKE dissipation estimates the eddy dissipation rate of actual aircraft (Cornman et al. 1995). The eddy thermal diffusivity, K_h = 4 K_m . The Brunt-Väisälä frequency squared is $N^2 = \frac{g}{\Theta}\frac{\partial \Theta}{\partial z}$, in which Θ the potential temperature.

A key parameter in (15) and (16) is the non-dimensional amplitude,

$$\hat{a} = Na / |\mathbf{V} - \mathbf{c}| \qquad (17)$$

where *a* is the actual wave amplitude and **c** the wave phase velocity; it is an inverse Froude number. The non-dimensional amplitude denominator is the Doppler-adjusted wind velocity (Dunkerton 1997).

Maximum positive TKE production from buoyancy arises when $\hat{a}$ > 1 and from wind shear when

$$\hat{a} > 2 - Ri^{-1/2} \qquad (18)$$

The method assumes that in the typical forecast time the TKE production eventually cascades into molecular TKE dissipation through the inertial subrange felt by the aircraft. At any one moment an aircraft may feel less than the maximum TKE dissipation because of its position within the gravity wave.

Any type of gravity wave forcing may be implemented, but the method requires knowledge of the wave amplitudes and phase velocities which, for the most part, are unknown. Due to a lack of consensus about the scaling properties of inertia-gravity wave amplitudes in theories and atmospheric observations, we seek guidance from laboratory experiments (Williams et al. 2007, submitted to *J. Atmos. Sci.*). The inertia-gravity waves generated in these experiments have an amplitude which scales linearly with the Rossby number. Noting that the Lighthill-Ford source term varies as Ro^2 we deduce that, if atmospheric inertia-gravity waves behave like those in the laboratory, then their amplitude must be proportional to the square root of the leading-order and second-order terms in the Lighthill-Ford source term:

$$\hat{a}^2 \propto f\mathbf{u}\cdot\nabla\zeta + 2Df\zeta - f\mathbf{k}\cdot\mathbf{u}\times\nabla D - 2\frac{\partial}{\partial t}J(u,v) \qquad (19)$$

We assume constant mean wave properties in (19).

The proportionality constant in (19) was determined empirically by matching distributions of pilot reports of turbulence in strong CAT outbreaks prior to 2005-06 with the patterns of TKE dissipation that fit the best. This constant was then held fixed for later analyses.

In addition, it should be noted that $\hat{a}$ is also inversely proportional to the square root of density. This allows us to compute $\hat{a}$ (and thus TKE dissipation) at all levels down to the ground. The Lighthill-Ford forcing is stronger higher in the atmosphere.

In the case of Term 1C, we compute the instantaneous time derivative of the Jacobian as

$$\frac{\partial}{\partial t}J(u,v) = J(\frac{\partial u}{\partial t}, v) + J(u, \frac{\partial v}{\partial t}) \qquad (20)$$

with the time derivatives in (19) calculated from the equation of motion via

$$\frac{\partial u}{\partial t} = -\frac{\partial \Phi}{\partial y} - fv - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}$$
$$\frac{\partial v}{\partial t} = \frac{\partial \Phi}{\partial x} - fu - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} \tag{21}$$

where Φ is the geopotential height.

In the companion paper (Part II; McCann et al. 2008 ARAM Conference), we demonstrate and discuss the results of this approach for both short-term and seasonal analyses of CAT occurrence. The results suggest that this approach has the potential to provide major improvements in CAT forecasting accuracy.

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