Internal inertio-gravity waves in the laboratory: Mechanisms, properties, and impacts

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Abstract

This paper describes laboratory observations of internal inertio-gravity waves (also referred to as inertia-gravity waves) emitted from balanced fluid flow. In a rotating two-layer annulus experiment, the wavelength of the inertia-gravity waves is very close to the deformation radius. Their amplitude varies linearly with Rossby number in the range 0.05-0.14, at constant Burger number (or rotational Froude number). This linear scaling challenges the notion, suggested by several dynamical theories, that inertia-gravity waves generated by balanced motion will be exponentially small. It is estimated that the balanced flow leaks roughly 1% of its energy each rotation period into the inertia-gravity waves at the peak of their generation. The findings of this study imply an inevitable emission of inertia-gravity waves at Rossby numbers similar to those of the large-scale atmospheric and oceanic flow. Extrapolation of the results suggests that inertia-gravity waves might make a significant contribution to the energy budgets of the atmosphere and ocean.

1. Introduction

Inertia-gravity waves are observed ubiquitously throughout the stratified parts of the atmosphere (e.g., Eckermann and Vincent 1993; Sato et al. 1997; Dalin et al. 2004) and ocean (e.g., Thorpe 2005). Orthodox mechanisms for inertia-gravity wave generation include dynamical instability (e.g., Kelvin-Helmholtz shear instability; Chandrasekhar 1961), which is a known source of atmospheric gravity waves (Fritts 1982, 1984). Another possible mechanism is the interaction between the flow and a physical obstruction (e.g., generation in the wake of a ship; Lighthill 1978), which is the mechanism by which mountains generate atmospheric gravity waves (Hines 1989). Direct forcing of the ocean by the atmosphere is a known source of oceanic gravity waves (Wunsch and Ferrari 2004). Finally, inertia-gravity waves are also radiated during the geostrophic adjustment of a hypothetical fluid (Rossby 1938), in which geostrophic balance is approached from an unbalanced initial condition.

Despite the above insights, our understanding of the sources of inertia-gravity waves remains incomplete. In particular, Ford (1994) showed that even balanced flows may undergo a generalized adjustment that is accompanied by the emission of inertia-gravity waves. By “balance” we refer here in a generic way to a dynamical relationship that allows the fluid state to be diagnosed through potential vorticity inversion. In contrast to geostrophic adjustment, the Ford (1994) mechanism involves (weak) departures from balance that arise spontaneously as the flow evolves (Ford et al. 2000; Viúdez and Dritschel 2006), and not those that arise from ageostrophic initial conditions. It is proving extraordinarily difficult to determine whether or not this mechanism is a significant source of inertia-gravity waves in real geophysical flows, however (McIntyre 2001). This hinders the development of parameterizations of the waves in general circulation models (Kim et al. 2003).

2. The rotating laboratory annulus

Here, we study the spontaneous generation of inertia-gravity waves the rotating two-layer annulus apparatus used by Lovegrove et al. (1999, 2000) and Williams et al. (2003, 2005). The annulus has an inner sidewall of radius 62.5 mm, an outer sidewall of radius 125.0 mm, and a total depth of 250.0 mm. The annulus gap width is thus 62.5 mm, and the two immiscible fluid layers have equal resting depths of 125.0 mm. The base and lid are both horizontal and the annulus is mounted on a turntable. The base and sidewalls rotate about the axis of symmetry, and the lid (in contact with the upper layer) rotates relative to the base and sidewalls. For all of the experiments described in this paper, there is a super-rotation of the lid and working fluids relative to the turntable.

The upper layer is water and the lower layer is a mixture of limonene and CFC-113. The lower-layer constituent liquids are both practically insoluble in water and are mixed in such proportions that the composite density (1003 kg/m^3) is slightly greater than that of water (997 kg/m^3). Importantly, the lower layer has a large optical activity due to the limonene. Therefore, when the system is illuminated from below with white light and viewed from above through crossed polaroids, there is a relationship between the colour perceived and the height of the internal interface. This is the effect responsible for the colour gradients in the images, which are captured by a video camera viewing the annulus from above. This flow visualization technique was proposed by Hart and Kittelman (1986). It is a non-invasive method for visualizing interface perturbations with high resolution in the vertical coordinate (see following paragraph), the horizontal coordinates (in our case, 0.5 mm), and time (in our case, 0.04 s).

Williams et al. (2004) calibrated the experiment by deriving the quantitative relationship between hue (a measure of the dominant wavelength) and the height of the internal interface. Given an image captured by the camera, the two-dimensional hue field can be calculated from the digitized red, green, and blue pixel information and then projected onto the calibration curve to obtain an interface height map.

However, during the present study it was discovered that the pixel jitter noise in the blue channel is much larger than in the red and green channels. This noise limits the vertical resolution of the retrieved interface height measurements, which is a key consideration because we wish to be able to detect small-amplitude inertia-gravity waves. For the purposes of the present study, we therefore disregard the blue channel and modify the Williams et al. (2004) calibration curve to use only the red and green channels. This strategy reduces the error in retrieved interface height measurements from 1 mm (as reported by Williams et al. 2004) to 0.3 mm, and it will be used throughout this paper.

3. Experimental results

A series of experiments was performed using the apparatus described in the previous section. A photograph from one of the experiments is shown in Figure 1. Two trains of internal inertia-gravity waves at the internal interface are clearly visible in the troughs of the large-scale flow pattern. The wavelength of the inertia-gravity waves is very close to the deformation radius. Full details are provided by Williams et al. (2008).

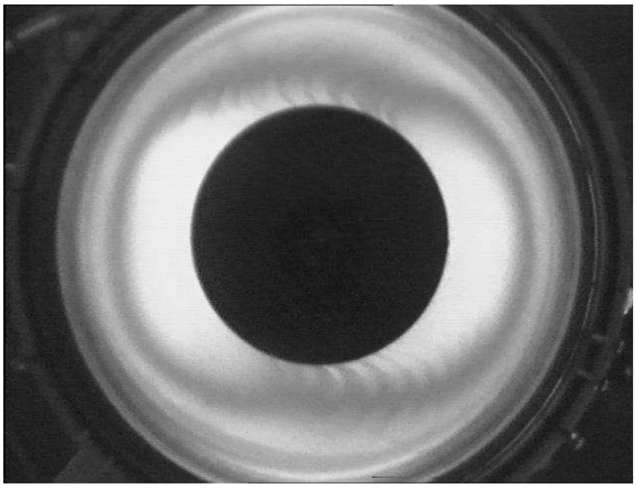

Figure 1: The view of the rotating annulus from above, showing trains of inertia-gravity waves in the troughs of the large-scale flow pattern. A colour movie showing the evolving flow can be viewed at http://www.youtube.com/RotatingAnnulus.

The mechanism by which the laboratory inertia-gravity waves are generated is of interest. Because the waves continue to be generated long after the start of the experiments, generation by initial-condition adjustment can be ruled out. Because the measurement technique used is non-invasive, generation by a physical obstruction can also be ruled out. Using data from a quasi-geostrophic numerical model of the large-scale flow, five different indicators of inertia-gravity wave generation have been examined and will now be discussed.

The shear instability indicators, including the Richardson number, are inconsistent with the generation observed in the laboratory. Ford (1994) derived an indicator for the spontaneous emission of inertia-gravity waves in rotating shallow water. Spontaneous emission here refers to wave excitation in which nonlinear interactions of the basic state energize linear wave modes with only weak back-reaction on the basic state itself. Ford's indicator was found to be qualitatively consistent with the observed laboratory inertia-gravity wave generation. This laboratory experiment thus provides the first opportunity to test, in a real fluid, theoretical predictions about spontaneous inertia-gravity wave generation.

From careful measurements, the amplitude of the emitted inertia-gravity waves is found to vary linearly with Rossby number, at constant Burger number (or rotational Froude number). This linear scaling challenges the notion, suggested by several dynamical theories, that inertia-gravity waves generated by balanced motion will be exponentially small. Furthermore, it is estimated that the balanced flow leaks roughly 1% of its energy each rotation period into the inertia-gravity waves, at the peak of their generation.

These findings may have important consequences for geophysical fluid flows. Our laboratory observations imply an inevitable emission of inertia-gravity waves at Rossby numbers similar to those of the large-scale atmospheric and oceanic flow. The appreciable energy leak from the balanced laboratory motions suggests that the geophysical analogue of our laboratory emission might be a significant source of inertia-gravity waves in the atmosphere and ocean. Furthermore, the linear Rossby number scaling suggests that the inertia-gravity waves might be far more energized than predicted by at least some theories.

4. Discussion

It is worth remarking that the systematic emission of inertia-gravity waves has not been observed in other similar laboratory experiments, even though they have been used for decades to investigate baroclinic waves. We speculate that this is because our flow visualization technique allows a much higher spatiotemporal resolution than has been achieved in previous studies. Also, on the rare occasions that inertia-gravity waves have been reported previously [e.g., Read (1992), although the amplitudes were an order of magnitude smaller than in our study], they were measured by probes in the fluid and it was natural to attribute their generation to an interaction between the flow and the probes. Unusually for rotating laboratory experiments, the visualization technique that we use is non-invasive, ruling out this possibility.

Our observation that the inertia-gravity wave amplitude is only linearly dependent on Rossby number, and not exponentially dependent, might be due to dissipative processes. In these experiments, the inertia-gravity waves could be affected, or possibly even controlled, by viscosity and surface tension. Thus, we are not able to rule out the possibility that the inertia-gravity waves are fully saturated over the range of Rossby numbers we have been able to access, and that their amplitude is determined more by the dissipation than by the forcing. If this were the case, then our estimate of the energy flux into the inertia-gravity waves must be considered a lower bound. We plan future experiments to investigate this possibility, if practicable.

Inertia-gravity wave generation may depend upon the Burger number (or its reciprocal, the rotational Froude number) as well as the Rossby number. Unfortunately, for two independent reasons, it is virtually impossible to study the dependence upon the former using the present laboratory apparatus. First, over the part of parameter space at which inertia-gravity waves coexist with a regular baroclinic wave, the range of possible Froude numbers at constant Rossby number is much smaller than the range of possible Rossby numbers at constant Froude number. Second, the interface becomes steeply sloped at high Froude numbers, and the fluid begins to access interface heights outside the range of the calibration curve, so that we can no longer infer interface heights (and hence inertia-gravity wave amplitudes) from the colour laboratory images.

We acknowledge that the extrapolation of our results to geophysical flows is crude and provisional. Although the laboratory fluid and geophysical fluids share similar Rossby numbers, they have different Burger numbers, geometries, stratifications, and aspect ratios. Nevertheless, this is the first time the emission of inertia-gravity waves has been studied in this way in a real fluid rather than a numerical model or theoretical analysis. Our study therefore makes a vital contribution to the debate about the role of inertia-gravity waves in geophysical flows.

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