Unimodality of wave amplitude in the Northern Hemisphere

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ABSTRACT

A non-parametric statistic for local wave amplitude of the 500hPa geopotential height field is introduced. Here it is used to detect the existence, or otherwise, of multimodality in its distribution function. The empirical distribution function for the full 1960–2000 period is close to a Weibull distribution with shape parameters between 2 and 3 and with richer structure near the ends of the N. Hemisphere storm tracks. There is substantial interdecadal variability, but no consistent signature of multimodality or bimodality in the local wave amplitude.

The zonally averaged wave amplitude, akin to the more usual wave amplitude index, follows a normal distribution. This is consistent with the central limit theorem which applies to the construction of the wave amplitude index. For the period 1960–1970 we do find apparent bimodality. However, the different amplitudes are realized at different longitudes, so there is no bimodality at any location.

As a corollary, it is found that many commonly used statistics to detect multimodality in atmospheric fields potentially satisfy the assumptions underlying the central limit theorem and therefore can only show approximately normal distributions. We conclude that these techniques may therefore be suboptimal to detect multimodality.
1. Introduction

The idea that the atmosphere could have multiple equilibria is attractive on many levels. It may explain the apparent occurrence of weather regimes. It would also mirror low-order nonlinear models which have been used as didactic analogs for more complex behaviour in the real atmosphere. The idea of multiple equilibria in the atmosphere was given prominence by the seminal Charney and DeVore (1979) paper.

Authors have been looking for statistical evidence for such multiple equilibria in the atmosphere for many years (e.g., Hansen and Sutera, 1986; Mo and Ghil, 1988; Kimoto and Ghil, 1993; Corti et al., 1999; Monahan et al., 2001, Christiansen, 2005a). However, almost with equal regularity others have pointed out a lack of statistical significance in these results (Nitsche et al., 1994; Stephenson et al., 2004; Christiansen, 2005b). The problems have been confounded by parameter dependence of the significance testing and of the construction of the statistic. The discussion does not seem to be settled either way. Whether the atmosphere has multiple equilibria currently appears to be more an article of faith than an observed reality.

The elegance of low-order nonlinear models as didactic analogs (e.g., Palmer, 1999) has probably increased our desire to find multiple equilibria in the real atmosphere. However, these low-order models are by construction not realistic. Also the multiple equilibria in the Charney and DeVore model were found to occur in unrealistic parameter regimes (Tung and Rosenthal, 1985). Other simple models of multiple equilibria inspired on nonlinear oscillators (e.g., Malguzzi and Speranza, 1981) have always involved many idealizations, although, perhaps surprisingly, the multiple equilibria in the more realistic barotropic model of Ambaum and Verkley (1995) can be mapped on response curves of nonlinear oscillators (Ambaum, 1997).

It may be argued\(^2\) that there is a distinction between weather regimes, as exemplified by Baur’s Großwetterlagen, and circulation regimes, as measured, for example, by Rossby’s and Namias’ index cycles. The weather regimes as experienced by the synoptician may be bimodal or

\(^2\)D. Straus (2006), personal communication.
multimodal only quite locally in both time and space. It is then not clear that a long term
global statistic would usefully reflect the various regimes. However, if a dynamical model
would underly the existence of such weather regimes they should normally be robust and
observable if a suitable statistic was chosen. Secular climate trends and decadal variability
would confound such a picture because statistics would perhaps never be stable. With the
caveat of statistical stability, we should be looking for local measures that are as robust as
possible for secular trends and decadal variability.

In this paper we construct and analyze a local wave amplitude statistic that is non-parametric.
This will alleviate the problems with parameter dependence of distribution functions in earlier
studies. We find that there is no strong indication for bimodality or multimodality in the
500hPa geopotential height amplitude or the highly related wave amplitude index (Hansen
and Sutera, 1986). We also find that the bimodality observed in Hansen and Sutera’s wave
amplitude index is most likely not a reflection of true bimodality at any location. Integrated
statistics, such as the wave amplitude index or principal component time series, are found to
be suffering from the fast convergence to Gaussian statistics of the central limit theorem which
makes such indices suboptimal to detect potential bimodality.

2. Local wave amplitude

For any periodic function \( f(x) \) of \( x \) we can define a local wave amplitude \( A(x) \) as:

\[
A^2 = f^2 + (f^H)^2,
\]

where \( f^H \) is the spatial Hilbert transform of \( f \) (e.g., Von Storch and Zwiers, 1999). The Hilbert
transform is most easily defined in the spectral domain: the spatial Fourier transform \( \hat{f} \) of \( f \)
and its Hilbert transform \( \hat{f}^H \) are related through:

\[
\hat{f}^H = i \text{sgn}(k) \hat{f},
\]

with \( \text{sgn}(k) \) the sign of \( k \). In other words, to find the Hilbert transform of a periodic function,
each Fourier component is shifted by a quarter wavelength so that all sines become cosines and

\[
\sim 2 \sim
\]
all cosines become minus sines. Some more properties of Hilbert transforms can be found in the aforementioned textbook. An application of the Hilbert transform in atmospheric dynamics can be found in Ambaum and Athanasiadis (2006), where it plays a central role in the dynamics of Rossby edge waves and where the wave amplitude, as defined above, was shown to be locally conserved for linearized surface quasi-geostrophic dynamics. A direct application of the Hilbert transform to wave amplitude diagnostics can be found in Zimin et al. (2003).

The above definition of wave amplitude has some desirable properties which make it a very natural choice. A monochromatic wave \( A \cos(kx + \phi) \) is found to have a constant wave amplitude \( A \). For superpositions of monochromatic waves the wave amplitude accurately describes the wave envelope, as can be easily checked using simple test functions. For example, let us choose \( f = \cos(k+\epsilon)x + \cos(k-\epsilon)x \) with \( \epsilon \ll k \). This can be rewritten as \( f = 2 \cos kx \cos \epsilon x \), a carrier wave of wavenumber \( k \) modulated with an amplitude of \( 2 \cos \epsilon x \). The Hilbert transform of \( f \) is \( f^H = - \sin(k+\epsilon)x - \sin(k-\epsilon)x \) and we find that \( f^2 + (f^H)^2 = (2 \cos \epsilon x)^2 \), as required.

We will use this local wave amplitude definition to provide an amplitude for 500hPa geopotential height that is a function of longitude. The advantage of a longitude dependent local amplitude above a zonal average measure, such as Hansen and Sutera’s (1986) wave amplitude index, is that synoptic experience shows that most interesting large scale waves have the form of localized blocks, usually over the Atlantic and Pacific basins. The local wave amplitude allows isolation of a consistent definition of wave amplitude over those areas. Another advantage is that our local wave amplitude is independent of the phase of the carrier wave under consideration. So if the internal structure of the block (i.e. the phase of the carrier wave) moves slightly or the block evolves under incident mobile systems, the local wave amplitude remains constant, contrary to measures dependent on prescribed patterns (e.g. empirical orthogonal functions.) This reflects the synoptic experience that the area under consideration is still experiencing high wave amplitude even though the exact phase might vary because of smaller scale development. A third advantage is that the local wave amplitude does not contain any
parameters. So no wave bands are chosen which could give rise to parameter dependency of its statistics.

The usual method to find a longitude dependent measure of 500hPa geopotential height variation is to average the field over a particular latitude belt. This has some obvious disadvantages. A good example may be the transition from a very zonal flow to a strongly blocked flow. For the zonal flow the wave amplitude of interest is around the subtropical jet latitudes while for the blocked flow the waves of interest are further north, generally away from the zonal mean jet(s). To capture all cases we would need to choose a very wide latitude belt, which gives rise to very washed out statistics. If a narrower band is chosen, the statistics become dependent on the boundary choices, as explored in Christiansen (2005a).

To overcome this problem we devise a flow-dependent latitude weighting of the geopotential height $Z$. The chosen weighting $w(\phi)$ is proportional (the constant of proportionality is such that the latitudinal integral over $w$ equals one) to the total variance of $Z$ along the latitude circle:

$$w(\phi) \propto \int_0^{2\pi} (Z(\lambda, \phi) - \langle Z(\phi) \rangle)^2 \cos(\phi) d\lambda,$$  \hspace{1cm} (3)

where $\langle Z \rangle$ is the zonal average of $Z$. This way the local wave amplitude is always evaluated at the latitudes where the waves are. The seasonal variation in jet location is automatically accounted for, as well as decadal variability and secular trends in jet latitudes. Another advantage is that this weighting does not have any parameters which may influence the statistics. So, a latitudinal weighting as in Eq. 3 is defined and a longitudinally varying geopotential anomaly $f(\lambda)$ is found as:

$$f(\lambda) = \int_0^{\pi/2} (Z(\lambda, \phi) - \langle Z(\phi) \rangle) w(\phi) d\phi.$$  \hspace{1cm} (4)

This geopotential anomaly is then used to calculate the local wave amplitude from Eq. 1. An example can be found in Fig. 1 where a strong Atlantic and Pacific block are clearly visible in the geopotential height field. The latitudinal weighting picks out the latitudes of the blocks and the resulting wave amplitudes pick out the longitudinal envelopes of the blocking waves with regions of relatively modest wave amplitude in between.
In summary, we have defined a non-parametric local wave amplitude which generalizes the usual wave amplitude index by providing a consistent wave amplitude as a function of longitude. Because our local wave amplitude can be evaluated at a longitude of interest, independent of the phase of the carrier wave, and because it always takes into account the latitudes of interest, we expect this index to show real bimodality or multimodality if it is there in the data. This is examined in detail in Section 3. In section 4 the relationship between our local wave amplitude and Hansen and Sutera’s wave amplitude index will be discussed.

3. Statistics of local wave amplitude

In this section we show some of the main statistics for the local wave amplitude, defined in the previous section. The data we used are extracted from the ERA-40 archive (Källberg et al., 2005), enhanced with operational analyses, of 6 hourly data for the four decades between the years 1960 and 2000. The data were low-pass filtered with a cut-off frequency of 10 days—we used a Lanczos filter with length of 79 data points (20 days). The cut-off frequency was chosen to exclude synoptic activity from our amplitude index. A five day cut-off would perhaps be a more traditional choice, but in work with P. I. Athanasiadis, as yet to be published, we show that variability is dominated by synoptic mobile systems at frequencies higher than 10 days. The 10 day cut-off is therefore a conservative choice to exclude these. We have re-calculated the key statistics presented in Fig. 2 with a five day cut-off filter and found very similar results. The filtered data were further limited to the months NDJFM, the meteorological winter period extended with adjacent months from the transitional seasons. As argued in the previous section, our local wave amplitude follows natural variations in latitudinal wave location, so there was no need to de-seasonalize or de-trend the data.

In Fig. 2 we present the empirical probability distribution function of the local wave amplitude as a function of longitude for all four decades. The overall structure shows peaks in average wave amplitude around 30W and 140E, the mid-Atlantic and west-Pacific respectively. For the mid-Atlantic this maximum amplitude presumably is associated with the strong variability of

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the Atlantic jet, also associated with the North Atlantic Oscillation. The west-Pacific maximum is likely associated with the strong climatological trough over that area, which also registers as a high wave amplitude. The distribution function peaks at lower amplitude (and higher probabilities) around 90W and 60E. Although the distribution function has rich structure there is no obvious strong sign of local bimodality.

It is not clear a priori what theoretical distribution function the amplitude has at each longitude. If the two contributions $f$ and $f^H$ to the amplitude in Eq. 1 were locally independent and had a Gaussian distribution with mean zero, standard deviation $\sigma$, then the amplitude would follow a Rayleigh distribution, which is the same as Weibull distribution with shape parameter $k = 2$ and scale parameter $\sigma$. In general, the Weibull distribution is defined as:

$$g(A; \sigma, k) = \frac{1}{\sigma} \left( \frac{A}{\sigma} \right)^{k-1} \exp \left( -\left( \frac{A}{\sigma} \right)^k \right),$$

(5)

with $\sigma$ the scale parameter of the distribution and $k$ the shape parameter. Given that there is (a weak) a priori argument that the amplitude could follow a Rayleigh distribution it makes sense to fit the empirical data to its generalization, a Weibull distribution.

Examples of such fits to a Weibull distribution are plotted in Fig. 3. These fits show a range of shape parameters (1.8–3.0) and scale parameters. The longitudes are chosen to represent extremes of shape or scale parameter (see also Fig. 4). Longitudes around the Greenwich meridian (top right panel, Fig. 3) and 150W (not shown) are unusual in that the amplitude distribution has fairly substantial deviations from the fitted Weibull distributions and could be candidates for multimodal distributions —see below. For all other longitudes, the fit to the Weibull distribution is remarkably good.

Figure 4 shows the shape and scale parameters for all longitudes of the fitted Weibull distributions. The standard errors have not been plotted as they are less than 1 percent for both parameters at all longitudes. The scale parameters clearly follow the dependency of wave amplitude evident in Fig. 2. Perhaps surprisingly, the shape parameter is strongly correlated with the scale parameter. The interpretation is that for locations with high wave amplitudes
the distribution does not simply scale up, it in fact moves away from the zero amplitude case (there are relatively few low amplitude cases). It can also be seen that the Rayleigh distribution (shape parameter of 2) is a fairly good model for the amplitude distribution for the longitudes of low average wave amplitude.

Figure 5 shows the empirical and fitted amplitude distributions at the Greenwich meridian for the four separate decades between 1960 and 2000. It is clear that the amplitude distribution has substantial inter-decadal variability. Although this statistic is not directly comparable to Hansen and Sutera’s wave amplitude index (see Section 4 for a detailed discussion), as in Christiansen’s 2005 study, we find that 1990–2000 is the most perturbed decade, with a high shape parameter and high scale parameter for the fitted Weibull distribution. Especially in the period 1970–1990 the empirical distribution appears remarkably broad which, in synoptic experience over those two decades, would certainly be experienced as a peculiar distribution: the near extremes are about as likely as the mean. Subtracting a smooth fitted distribution from the empirical distribution in these decades would give a bimodal structure.

4. Wave amplitude index and central limit theorem

We can define a wave amplitude index (WAI) by integrating the squared amplitude $A$ over longitude:

$$WAI^2 = \int_0^{2\pi} A(\lambda)^2 \, d\lambda = \int_0^{2\pi} (f^2 + (f^H)^2) \, d\lambda = 2 \int_0^{2\pi} f^2 \, d\lambda$$  \hspace{1cm} (6)

By Parceval’s theorem the zonal integral of $f^2$ equals the sum over the squared amplitudes of its Fourier components. Therefore, this definition becomes equivalent to Hansen and Sutera’s WAI if the signal was prefiltered to zonal wave-numbers 2–4 and if the zonal anomaly $f$ was defined over a fixed latitude belt. Following Eq. 2, the squared amplitudes of the Fourier components of the Hilbert transform are the same, so that the total variance of the Hilbert transform is the same as the total variance of the original function, leading to the last equality in Eq. 6. From our definition of the WAI it also becomes clear that our local wave amplitude generalizes the WAI in that besides amplitude information, it shows the relevant phase information as well.
(i.e. at what longitudes waves interfere constructively to produce high local amplitudes.)

Figure 6 shows the empirical distribution with a fitted normal distribution for our WAI defined over all four decades. A Weibull distribution did not fit the data —see below. The fitted normal distribution has a mean amplitude of 141(0.2)m and a standard deviation of 34(0.2)m. It is clear that the normal distribution fits the data very well. The empirical distribution has some structure near the peak but even if it were statistically significant, there is no important multimodality.

Figure 7 shows our WAI over the different decades. Over all decades the Gaussian fit is good except for the 1960–1970 period. The clear bimodality for this decade was also observed by Hansen and Sutera (1986) and Christiansen (2005a). Quite apart from possible data issues for the pre-satellite era, this bimodality of the WAI turns out to be misleading. This becomes clear from Fig. 8, the empirical distribution of wave amplitude at each longitude for the 1960–1970 period. This figure is analogous to Fig. 7 with the phase information of each wave component retained. Note that no smoothing has been attempted so most of the small scale structure in Fig. 8 is sampling noise. However, it now becomes clear that the high amplitudes in Fig. 7 mainly occur at longitudes around 130E, while the low amplitudes occur around 40E and 90W. There is no single longitude where the wave amplitude has a bimodal distribution as seen in the 1960–1970 WAI. So the bimodality of the 1960–1970 WAI does not represent local bimodality of the wave amplitude, it just represents the fact that different longitudes have on average either high or low wave amplitude with, in this decade, relatively few longitudes having an intermediate wave amplitude.

Next we turn our attention to why the WAI appears to closely follow a normal distribution while the individual contributions are accurately modeled with Weibull distributions. Firstly we determine how much the integrands in the WAI, Eq. 6, are independent. A good measure of how many independent data are to be found in the amplitude function is based on the autocorrelation. If the amplitudes $A(\lambda, t)$ are perfectly correlated between different longitudes, $r(A(\lambda_1), A(\lambda_2)) = 1$, then we have only one degree of freedom. If for each longitude combi-
nation for which \( \lambda_1 \neq \lambda_2 \) we have \( r(A(\lambda_1), A(\lambda_2)) = 0 \) then the number of degrees of freedom is (at least) the number of gridpoints in the longitude direction. For intermediate values of the autocorrelation scale we can estimate the number of degrees of freedom \( M \) as

\[
\frac{1}{M} = \left\langle \frac{1}{N} \sum_{i=1}^{N} r(A(\lambda_j), A(\lambda_i)) \right\rangle
\]

(7)

where \( N \) is the number of gridpoints and \( \langle \ldots \rangle \) denotes an average over longitudes \( \lambda_j \). Using this measure we find around 3.6 degrees of freedom (the number of degrees of freedom is usually thought of as an integer, but in many applications there is no problem in generalizing this to a fractional number), with the spatial decay scale of the autocorrelation of the wave amplitude at about 50 degrees of longitude. So our WAI index is made up as the sum of 3.6 independent variables. The set of empirical distributions at each longitude does satisfy the Lindeberg conditions on the central limit theorem for variable distributions (e.g. Feller, 1968). This then means that our WAI by definition has to follow a normal distribution. Clearly, with only 3.6 degrees of freedom the limit to the normal distribution is not complete. However, the convergence to the normal distribution is generally very swift (also evidenced by how far the empirical distribution functions in Figs. 6 and 7 are from a Weibull distribution) so our WAI is bound to be close to normally distributed.

The application of the central limit theorem to our WAI clearly has profound consequences for its use in detection of multimodality: even if the atmosphere would have multimodality at particular (or all) longitudes, if the amplitude between longitudes decorrelates sufficiently fast, the central limit theorem implies that the WAI index will be normally distributed. Under most circumstances the WAI is unsuitable to detect multimodality. Any observed multimodality will be the result of sampling errors or unstable statistics.

5. Summary and discussion

A non-parametric measure of local (in longitude) wave amplitude has been introduced. A consistent amplitude is defined at each longitude using spatial Hilbert transforms to extract the
envelope of a carrier wave. The carrier wave is produced as a latitudinally weighted geopotential height anomaly at 500hPa. The latitudinal weighting is proportional to the wave variance at each latitude.

The local wave amplitude appears to follow a Weibull distribution at the vast majority of longitudes with shape parameters between about 2 and 3 and scale parameters between 100 and 140m. Note that these scales are higher than found in previous studies because we included the variance of zonal wavenumbers higher than 4 and because for each individual field we give the highest weighting to the latitudes of the largest wave amplitude. The shape parameter turns out to be high where the amplitude is high indicating that in regions of high average amplitudes, the low amplitudes are relatively underrepresented. At the tail ends of the Pacific and Atlantic storm-tracks the empirical distributions are relatively furthest away from the fitted Weibull distributions, although indications of local multimodality or bimodality are not convincing.

The empirical distributions have substantial decadal variability. For example, in the 1970–1990 period the wave amplitude at the Greenwich meridian has a very flat distribution between amplitudes of about 50m and 250m. Although this is largely averaged out over the four decade period, it clearly represents a substantial period with unusual statistics. During this twenty year period there is an unexpectedly high probability to find either relatively high or low amplitudes. It is a subject of future study to determine whether this is the result of weather regime residence times. Any conceptual model for this result should be able to explain why the other two decades apparently do not show these unusual statistics.

The local amplitude can be integrated over longitude to provide a hemispheric wave amplitude index (WAI), similar to Hansen and Sutera’s (1986) WAI. It is shown that for the full data period under consideration the WAI is very close to a normal distribution. Again, there is substantial decadal sampling variability with the 1960–1970 period showing a strongly bimodal distribution. However, it is shown that the high and low amplitude states of this distribution are in fact realized at different longitudes, thus showing no real bimodality in that decade.
With the WAI as the integral of local wave amplitudes, the central limit theorem can be applied to it, insofar as different longitudes are independent. It is shown that the local wave amplitude in fact contains 3 to 4 independent degrees of freedom. This means that the limit to the normal distribution is not complete. However, the convergence is strong enough to generally prevent bimodality to show up, even if bimodality was very clear in the local data and if the data record were long enough to prevent sampling errors. It is also expected that the time filtering of the data increases the longitudinal autocorrelation scale and so reduces the effective number of degrees of freedom. Unfiltered data are expected to show an even more Gaussian WAI.

Methods of looking at distributions in two dimensional projections of the atmospheric phase space usually employ empirical orthogonal functions or some other spatial pattern to span the two dimensional projection space. Such methods are also set to suffer from the strong convergence of the central limit theorem, something which to the author’s knowledge has not been pointed out before. For example, an empirical orthogonal function is a spatial pattern which generally covers uncorrelated locations (e.g. Ambaum et al, 2002). This means that the corresponding time series is a weighted sum of independent variables (although probably only few independent variables) and by the central limit theorem will have Gaussian statistics. Under those circumstances it will by definition be impossible to see more structure in the combined empirical distribution function other than a bivariate Gaussian distribution. Clearly, if it can be shown that any principal component time series truly represents a single atmospheric degree of freedom (this would for example be more likely for stratospheric EOFs, see e.g. Christiansen, 2003) then it will be possible to find non-Gaussian structure, if the underlying atmospheric data were to be non-Gaussian.

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Figure captions

FIGURE 1. Contourmap: low-pass filtered geopotential height at 500hPa for 00Z 7 Feb 1994 (contour interval 50m.) Lower left panel: weight as a function of latitude. Top panel: wave amplitude (in meters) as a function of longitude.

FIGURE 2. Probability distribution function of the local wave amplitude of geopotential height at 500hPa as a function of longitude. At each longitude the distribution function is normalized (i.e. integrates to one over amplitude.) Four decades (1960–2000) of NDJFM low-pass filtered data are included.

FIGURE 3. Plots of Weibull distribution (thick line) fitted to the empirical distribution of wave amplitude (impulses) at four selected longitudes. Four decades (1960–2000) of NDJFM low-pass filtered data are included.

FIGURE 4. Shape parameter (left axis) and scale parameter (right axis) of the Weibull distribution fitted to the empirical distribution of Fig. 2. The thin line at shape parameter 2 corresponds to the Rayleigh distribution. Four decades (1960–2000) of NDJFM low-pass filtered data are included.

FIGURE 5. Plots of Weibull distribution (thick line) fitted to the empirical distribution of wave amplitude (impulses) for the four separate decades at the Greenwich meridian.

FIGURE 6. The WAI as defined in Eq. 6 for the period 1960–2000 (impulses) with a fitted normal distribution (thick line).

FIGURE 7. The WAI as defined in Eq. 6 for the four different decades (impulses) with the fitted normal distributions (thick lines).

FIGURE 8. Probability distribution function of the local wave amplitude of geopotential height at 500hPa as a function of longitude. At each longitude the distribution function is normalized (i.e. integrates to one over amplitude.) The decade 1960–1970 of NDJFM low-pass filtered data is used.
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