Life-cycle simulations of shallow frontal waves and the impact of deformation strain

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SUMMARY

The life cycle of shallow frontal waves and the impact of deformation strain on their development is investigated using the idealized version of the Met Office non-hydrostatic Unified Model which includes the same physics and dynamics as the operational forecast model. Frontal-wave development occurs in two stages: first, a deformation strain is applied to a front and a positive potential-vorticity (PV) strip forms, generated by latent-heat release in the frontal updraught; second, as the deformation strain is reduced the PV strip breaks up into individual anomalies. The circulations associated with the PV anomalies cause shallow frontal waves to form. The structure of the simulated frontal waves is consistent with the conceptual model of a frontal cyclone. Deeper frontal waves are simulated if the stability of the atmosphere is reduced.

Deformation strain rates of different strengths are applied to the PV strip to determine whether a deformation strain threshold exists above which frontal-wave development is suppressed. An objective method of frontal-wave activity is defined and frontal-wave development was found to be suppressed by deformation strain rates \( \geq 0.4 \times 10^{-5} \text{ s}^{-1} \). This value compares well with observed deformation strain-rate thresholds and the analytical solution for the minimum deformation strain rate needed to suppress barotropic frontal-wave development. The deformation strain-rate threshold is dependent on the strength of the PV strip with strong PV strips able to overcome stronger deformation strain rates (leading to frontal-wave development) than weaker PV strips.

KEYWORDS: Barotropic growth Potential-vorticity strip Secondary cyclogenesis

1. INTRODUCTION

In the midlatitudes waves are observed forming along the fronts of pre-existing synoptic-scale weather systems. These ‘frontal waves’ can develop explosively into frontal cyclones, deepening by over 24 mb in 24 hours. Examples of explosive frontal-wave development include the ‘October storm’ of 1987 (Hoskins and Berrisford 1988; Shutts 1990) and the Christmas 1999 storms, Lothar and Martin (Pearce et al. 2001). These storms caused flooding and severe wind damage in the UK and western Europe. The small scales and rapid development of frontal waves make their development particularly hard to forecast. However, not all frontal waves develop into frontal cyclones with ‘perhaps 50% of identified waves failing to deepen their pressure significantly’ (Parker 1998).

Observational and theoretical studies have shown that a wide range of processes may be important for frontal-wave growth. These processes include frontal shear (Joly and Thorpe 1991; Chaboureau and Thorpe 1999), large-scale strain (Dritschel et al. 1991; Bishop and Thorpe 1994a,b; Renfrew et al. 1997; Rivals et al. 1998; Chaboureau and Thorpe 1999), latent-heat release (Hoskins and Berrisford 1988; Shutts 1990; Joly and Thorpe 1991; Reed et al. 1993; Plant et al. 2003; Ahmadi-Givi et al. 2003) and boundary-layer friction (Adamson et al. 2006). However, the reasons why some frontal waves develop whilst others do not are still not fully understood nor are the mechanisms by which this development occurs. In this paper we concentrate on the impact that deformation strain has on barotropic frontal-wave development using the idealized version of the Met Office non-hydrostatic Unified Model.

Eliassen and Kleinschmidt (1957) observed that positive potential-vorticity (PV) anomalies are often observed at low levels in newly formed frontal cyclones. Positive
PV anomalies are also observed forming in strips along the trailing cold fronts of extratropical cyclones. These positive PV strips are caused by latent-heat release in the moist ascent ahead of the cold front. Positive PV anomalies in frontal ascent regions have been simulated in the horizontal strain deformation model of Thorpe and Emanuel (1985) and the horizontal shear deformation models of Emanuel et al. (1987) and Joly (1989). Thorpe and Emanuel (1985) applied a deformation strain field to a two-dimensional front in their moist model and found that latent-heat release produced a strip of PV aligned along the front. They proposed that frontal waves may be caused by the PV strip being unstable to small-scale perturbations.

Instabilities arising from strips of anomalous PV such as those found at fronts may be understood in terms of the interaction of counter-propagating Rossby edge waves. A two-dimensional flow is unstable if the gradient of PV changes sign within the domain (Rayleigh 1880; Charney and Stern 1962). This instability is known as barotropic instability. Joly and Thorpe (1990) modelled the instability of a strip of positive PV ahead of a cold front (representing the PV generated by latent-heat release in the frontal updraught) using a two-dimensional dry model. They showed that their PV strip was unstable to perturbations with wavelengths smaller than 1000 km and that the fastest growing modes had growth rates with e-folding times of about a day, a similar horizontal scale and growth rate to observed frontal waves. Schar and Davies (1990) found similar results when they applied perturbations to a surface warm band ahead of a cold front. Further work by Malardel et al. (1993) extended the work of Joly and Thorpe (1990) and examined nonlinear frontal-wave growth. They found that initially frontal-wave growth is barotropic but that saturation of modes occurred, resulting in weak frontal waves, unless the initial perturbation was deep. In that case baroclinic growth can occur provided there is sufficient baroclinicity. In this paper we investigate the barotropic stage of frontal-wave growth.

Not all of the PV strips that form along fronts become unstable and form frontal waves. One process that has been shown to inhibit frontal-wave growth is deformation strain. Dritschel et al. (1991) and Bishop and Thorpe (1994a,b) examined the effect of a deformation strain acting on a low-level PV strip. Dritschel et al. (1991) found that, for a two-dimensional vorticity strip, linear wave growth was suppressed by a deformation strain greater than 1/4 of the vorticity of the strip. Edge wave interactions on a PV strip were solved analytically by Bishop and Thorpe (1994a,b). They found that the deformation strain acts to increase the intensity of the PV strip but at the same time acts to flatten any frontal waves by compressing them and extending them in the cross- and along-front directions, respectively. However, if at a later stage the deformation strain weakens, then frontal waves can begin to develop. An analytic expression for the minimum deformation strain rate needed to suppress barotropic frontal-wave development was found (discussed further in section 5(d)). Thus it is necessary for the deformation strain to be reduced below a threshold value for barotropic frontal-wave development to occur. The effect of strain on a shallow warm band instability has been analysed by Juckes (1995). In this quasi-two-dimensional system, strain cannot suppress instability indefinitely because, under the influence of strain, the vorticity of the strip increases. Juckes (1995) resolves the development of the primary vortices and smaller secondary vortices.

Observational case-studies have also been carried out to test the hypothesis that frontal-wave development is influenced by the strength of the deformation strain acting on the front. Renfrew et al. (1997) analysed several frontal cyclones observed in the North Atlantic. They found that for small or decreasing deformation strain rates there was rapid intensification of the frontal cyclones and that strong deformation strain rates
can act to suppress frontal-wave growth. Other observational studies by Rivals et al. (1998) and Chaboureau and Thorpe (1999) found that non-developing frontal waves are subject to high values of deformation strain whilst developing frontal waves are subject to low values of deformation strain. This is in agreement with the theoretical results of Bishop and Thorpe (1994a,b).

Another process that has been shown to inhibit frontal-wave growth is frontal shear. Joly and Thorpe (1991) performed a numerical study of normal modes for a front undergoing shear frontogenesis. They found that strong shear is detrimental to frontal-wave growth. Chaboureau and Thorpe (1999) also investigated the effect of shear on the growth of the frontal waves that occurred in the FASTEX campaign. They found that some of the developing cases were associated with strong shear, in disagreement with the Joly and Thorpe (1991) results. However, the shear values were rather small compared with those used by Joly and Thorpe (1991). Thus they concluded that the role of frontal shear is less discriminating than the action of strain. In this paper we concentrate on the impact that deformation strain has on barotropic frontal-wave development as it appears to have a larger impact than frontal shear.

A conceptual model for barotropic frontal-wave development, based on the papers of Thorpe and Emanuel (1985), Joly and Thorpe (1990) and Bishop and Thorpe (1994a,b), is shown in Fig. 1. This development occurs in two stages:

(i) Formation of PV strip: A deformation strain field acting on a front causes the horizontal temperature gradient to tighten. This leads to ascent (line convection) on the warm side of the front. As the moist air parcels ascend they cool and the water vapour condenses releasing latent heat. Steady heating leads to the creation of a positive PV strip, aligned along the front (Fig. 1(a)).

(ii) Break-up of PV strip: If the deformation strain field acting on the front reduces, the PV strip breaks up into individual PV anomalies with associated cyclonic circulations. The circulations extend towards the surface and cause frontal waves to form along the front (Fig. 1(b)).

The first aim of this study is to model the complete evolution of barotropic frontal-wave development starting with the generation of an along-front PV strip, showing the buckling and roll-up of the PV strip into individual anomalies (which are associated with frontal waves) and ending with the decay of these discrete PV anomalies. This will combine the stages modelled by Thorpe and Emanuel (1985), Joly and Thorpe (1990) and Malardel et al. (1993). The second aim of this study is to test the hypothesis that strong deformation strain can suppress frontal-wave development and, if so, to determine whether a deformation strain threshold exists above which frontal-wave development is suppressed. The dependence on the strength of the PV strip is also examined. This deformation strain threshold will be compared to theoretical and observed values. Explosive and deep development requires interaction with an upper-level jet through baroclinic instability. We do not attempt to model this stage of the development. The numerical model used is the idealized Met Office non-hydrostatic Unified Model. This model includes the same physics and dynamics as the operational forecast model and so the results should be applicable to operational forecasts of the development of frontal waves. This is an advantage over the previous idealized numerical modelling studies of Thorpe and Emanuel (1985), Joly and Thorpe (1990) and Malardel et al. (1993).

The paper is structured as follows. The model set-up, initial conditions and boundary conditions are described in section 2. The application of the deformation strain and method of measuring frontal-wave activity are described in section 3. Section 4 contains
Figure 1. A conceptual model for barotropic frontal-wave development. (a) First stage of frontal-wave development—formation of PV strip, (b) second stage of frontal-wave development—break-up of PV strip. Contours are isotherms, shading is positive PV anomaly. Thin arrows represent the deformation strain flow, thick arrows represent vertical ascent and dashed arrows represent horizontal circulations.

the results describing the evolution of a positive PV strip and the effect of changing the stability of the atmosphere to frontal-wave structure. The deformation strain-rate threshold for frontal-wave development, its dependence on the strength of the PV strip and comparisons with theory and observations are described in section 5. The conclusions are given in section 6.

2. THE MODEL

The idealized version of the Met Office non-hydrostatic Unified Model (version 5.3) is used. This model solves non-hydrostatic, deep-atmosphere dynamics using a semi-implicit, semi-Lagrangian numerical scheme (Cullen 1993). The model includes a comprehensive set of parametrizations, including those for the boundary layer (Lock et al. 2000), mixed-phase cloud microphysics (Wilson and Ballard 1999) and convection (Gregory and Rowntree 1990). The radiation scheme is excluded as radiation is unlikely to be important on the time-scales relevant to this study (i.e. of the order of a day). Thus the idealized version of the model includes the same physics and dynamics as
the operational forecast model but with an idealized set-up. The model runs on a rotated latitude–longitude horizontal grid with Arakawa C staggering, and a terrain-following hybrid-height vertical coordinate with Charney–Philips staggering. A domain with horizontal grid spacing 0.11° (approximately 12 km) was used with 480 × 120 points in the horizontal. The vertical grid spacing increases with height from the surface to the top of the model at 16 km. The model was run with 64 vertical levels, this gives a grid spacing of between 200 and 300 m in the mid-troposphere with higher resolution near the surface in the boundary layer. The model is on an $f$-plane with a Coriolis parameter appropriate for the latitude 55°N ($f = 1.19 \times 10^{-4}$ s$^{-1}$). The lower boundary is flat and entirely over the sea with a uniform sea surface temperature of 15°C.

(a) Initial conditions

The initial profiles of temperature and moisture are idealized but representative of a frontal system. The temperature profile consists of a moist adiabat up to the tropopause at 10 km and an isothermal profile in the stratosphere to the top of the model. The specific humidity is defined such that the relative humidity is 70% in the initial profile. After the initial temperature field has been initialized an upper-level jet is added to the initial model state. The jet consists of a westerly wind increasing linearly with height from zero at the surface to a maximum of 60 m s$^{-1}$ at the tropopause (10 km) and decreasing linearly to zero at 16 km. In the meridional direction the jet strength is defined by a cosine function (from $-\pi$ to $+\pi$) with a width of 800 km; beyond the central $\pm 400$ km the jet strength is zero. The strength and horizontal and vertical structure of the upper-level jet were chosen to represent typical values found in a frontal system. The meridional temperature gradient is calculated from the thermal-wind relationship and is added to the model temperature field. This enhances the relative humidity on the cold side of the front and reduces it on the warm side. The meridional variation in surface fluxes arising from the constant sea surface temperature leads to reduced static stability on the cold side of the front and enhanced static stability on the warm side. However, sensitivity studies (see section 4(b)) have shown that changes in the atmospheric static stability do not affect the horizontal scale or deformation strain-rate threshold of the simulated frontal waves.

(b) Boundary conditions

The model is forced from the boundaries by calculating the strength of the deformation strain and the corresponding pressure at each time step. The fields fixed at the boundaries include potential temperature, specific humidity, density, pressure, and zonal, meridional and vertical winds. Fixing the boundary fields causes inconsistencies at the boundaries as these fields are free to evolve within the domain. This problem is overcome by making the domain large enough that frontal-wave development occurs away from the transition zone of the interior fields to the boundary values. Sensitivity studies showed that frontal-wave development was not influenced by inconsistencies at the fixed lateral boundaries.

3. Methodology

(a) Application of deformation strain

A deformation strain field was chosen that is irrotational and non-divergent and can be represented by the stream function,

$$\psi = -\alpha xy,$$
where $\alpha$ is a constant, which leads to a wind field defined by

$$u_d = \alpha x, \quad v_d = -\alpha y,$$

where $u_d$ is the horizontal wind component in the $x$-direction (zonal) and $v_d$ is the horizontal wind component in the $y$-direction (meridional). The magnitude of the deformation field is constant from the surface to 5 km and then reduces linearly with height to the tropopause at 10 km; above the tropopause the deformation wind is zero. The deformation wind field is added to the jet wind field in the initial conditions and forced from the lateral boundaries.

Experiments have been performed in which a constant deformation strain rate of $3 \times 10^{-5}$ s$^{-1}$ is applied to the front for the first 13 hours of the model run and then reduced to a lower value at a constant rate. This deformation strain rate is approximately twice the maximum value found in observational case-studies (Renfrew et al. 1997; Rivals et al. 1998; Chaboureau and Thorpe 1999) and results in high wind speeds at the east and west boundaries of the domain. The strong deformation strain rate was applied to enhance the growth rate of a PV strip along the front. When the PV strip had formed and reached an average magnitude of approximately 1.9 PVU$^*$ (after 13 hours) the deformation strain rate was reduced at a constant rate to a lower value, yielding realistic wind speeds at the east and west boundaries. The strength of the PV strip after 13 hours is typical of that observed along fronts. Results from the simulations in which the deformation strain rate was reduced to a lower value ranging from 0 to $0.8 \times 10^{-5}$ s$^{-1}$ are presented in this paper. The deformation strain-rate reduction was driven by the lateral boundaries. Ideally the deformation strain rate would be reduced instantaneously so that the strength of the PV strip after the strain rate reduction would be identical for each of the model runs with different strain rates. However, reducing the strength of the deformation strain rate instantaneously was found to produce too much of a shock to the model, leading it to develop unrealistically for a short period of time. Thus, the deformation strain rate was reduced at a constant rate over periods ranging from 2 hours (when the deformation strain rate was reduced to zero), to 1.47 hours (when the deformation strain rate was reduced to $0.8 \times 10^{-5}$ s$^{-1}$). The difference in the strength of the average PV of the strip at 900 mb (the typical level of maximum PV) for these two model runs after the deformation strain rate had been reduced was 0.03 PVU. This is small compared to the average PV of the strips at this time of 2.2 PVU. The rapid reduction of this deformation strain is artificial and was performed to enable comparison between different runs. In the real world the decrease in deformation strain occurs over perhaps 6–12 hours (Chaboureau and Thorpe 1999). The environmental deformation field is a result of the synoptic features around the front and the reduction in deformation strain is a result of the decay of the primary frontal cyclone or a change in the synoptic environment.

(b) Measurement of frontal-wave activity

An objective method of measuring frontal-wave activity is described in this section. Hewson (1997) identifies frontal waves as points on a front characterized by a local maximum, in the along-front direction, in the relative vorticity of the cross-front geostrophic wind at 1 km. This is denoted by $\xi_{xfg}$, where subscript $xfg$ stands for cross-front geostrophic wind. The cross-front direction is defined as the direction in which the rate of change of the wet-bulb potential temperature $\theta_w$ at 900 mb is a maximum, in the

$^*$ 1 PVU $= 10^{-6}$ K m$^2$kg$^{-1}$s$^{-1}$.
direction of cold air. This parameter is also used to measure frontal-wave activity in this idealized modelling study.

High values of $\xi_{xfg}$ cause the $\theta_w$ contours to buckle which is an indication of frontal-wave development. In order to quantify the amount of buckling of the $\theta_w$ contours the magnitude of $\xi_{xfg}$ is calculated along the 286 K $\theta_w$ contour. The 286 K $\theta_w$ contour is chosen as the maximum values of $\xi_{xfg}$ occur along this contour. As frontal waves have scales of approximately 200–1000 km, a filter is applied to remove very long- and short-wavelength features. The filtered data is then averaged over the length of the 286 K $\theta_w$ contour (denoted $\xi_{xfg}$) to determine the amount of buckling. Thus high average values of $\xi_{xfg}$ indicate large amounts of buckling of the 286 K $\theta_w$ contour and hence frontal-wave development. An example of the application of this method is given in section 5(a).

4. EVOLUTION OF THE PV STRIP

(a) Simulation of frontal waves

This section describes an experiment carried out to simulate the formation of a PV strip aligned along the front. The subsequent evolution of the PV strip and the formation of frontal waves is also simulated. In this experiment a constant deformation strain rate of $3 \times 10^{-5}$ s$^{-1}$ was applied for the first 13 hours of the model run. The deformation strain rate was then reduced to zero over the next two hours and remained at zero for the remainder of the model run. From 0–13 hours the high deformation strain rate acted to compress the front in the meridional direction. This frontogenetic process resulted in the formation of a PV strip, orientated in a zonal direction positioned along the warm side of the front, see Fig. 2. The PV strip had a width of approximately 40 km and a depth of approximately 5 km and was generated as a result of latent-heat release caused by moist air ascending at the front. Figures 3, 5 and 6 show the evolution of PV, $\theta_w$ and $\xi_{xfg}$ fields in the region in which frontal-wave development occurs, shown by the black box in Fig. 2.

Figure 3 (left-hand side) shows the evolution of the PV strip >1.0 PVU for the first 33 hours of the model run. At 6 hours (Fig. 3(a)) there is no PV >1.0 PVU present as the vertical circulation caused by the deformation strain was weak at that stage. The corresponding $\theta_w$ at 900 mb (Fig. 3(b)) shows that the $\theta_w$ contours are parallel to the
Figure 3. Left column: evolution of PV at 900 mb, contours every 2 PVU, first contour is 1 PVU. Right column: evolution of $\theta_w$ at 900 mb, contours every 1 K, shaded above 286 K. Results are for the simulation in which the strain rate is reduced from $3 \times 10^{-5}$ s$^{-1}$ to zero between 13 and 15 hours.
x-direction and are undisturbed. At 9 hours a PV strip has begun to form along the warm side of the front (Fig. 3(c)) and the corresponding $\theta_w$ gradient has tightened along most of the front (Fig. 3(d)). The PV strip continues to increase in magnitude as the vertical motion strengthens (Fig. 3(e)). At 13 hours the deformation field begins to reduce and reaches zero at 15 hours. Figure 3(f) shows that the front is still two-dimensional at this time as there is no temperature variation in the zonal direction. However, three hours later the PV strip has begun to buckle (Fig. 3(g)) and local maxima of PV have formed along the PV strip. The PV strip reaches its localized maximum magnitude of 9.5 PVU at this time. The corresponding $\theta_w$ contours show signs of small-scale disturbances (Fig. 3(h)). From 21–24 hours the PV strip has larger-scale perturbations (Fig. 3(i)) and wave features are observed in the $\theta_w$ contours (Fig. 3(j)). At 27 hours the PV strip has broken up into individual anomalies (Fig. 3(k)), each of which is associated with a wave feature on the 286 K $\theta_w$ contour (Fig. 3(l)) and hence can be described as a frontal wave. The individual PV anomalies develop from the local maxima of PV seen in Figs. 3(e) and (g). These individual anomalies are then advected towards the right-hand side of the domain by the upper-level jet, gradually decaying away (not shown). The frontal waves in this simulation have a wavelength of approximately 300 km and are shallow features, with PV anomalies less than 4 km deep.

The complete evolution of barotropic frontal-wave development has been simulated. A PV strip is generated aligned along the front as a result of applying a strong deformation strain. As the deformation strain is removed the PV strip begins to buckle and eventually rolls up into individual anomalies. These anomalies are associated with frontal wave-like features in the $\theta_w$ contours. The PV anomalies are then advected by the upper-level jet and begin to decay. An example of the structure of these simulated frontal waves is shown in Fig. 4 for the frontal wave highlighted in Fig. 5(d). The surface pressure and temperature at 1000 mb is shown in Fig. 4(a), with a simplified cloud analysis (relative humidity $>90\%$ at one or more model levels between 0.5 and 3.5 km) superimposed. Isentropic surface analysis carried out on this frontal wave, Fig. 4(b), reveals a flow of warm moist air, the warm conveyor belt, ascending up over the warm front to form part of the frontal cloud band. A second cooler moist flow, the cold conveyor belt, flows rearwards relative to the system. Part of the cold conveyor belt (CCB 1) wraps cyclonically around the low centre and forms the lower part of the cloud head. Another part of the cold conveyor belt (CCB 2) bends anticyclonically rising to form the upper part of the cloud head. Typical surface wind speeds are between 5 and 10 m s$^{-1}$.

The structure of the air flows within the frontal waves compares well with the classical conceptual model of a frontal cyclone (Browning and Roberts 1994).

(b) Sensitivity to atmospheric static stability

The simulated frontal waves in section 4(a) have a horizontal scale of approximately 300 km and a depth of less than 4 km. The horizontal scale of these frontal waves is on the lower limit of that seen in observations of frontal cyclones (ranging from 250–1000 km), and the depth is also shallower than is often observed (frontal waves can extend up to the tropopause, $\approx 9$ km). This may be because there is no upper-level feature for the low-level frontal waves to interact with and/or because the atmosphere is too stable for deep convection to occur.

Experiments have been carried out in which the stability of the atmosphere was varied to determine whether or not deeper frontal-wave development will occur when the atmosphere is less stable. The idealized initial conditions were set up such that the initial temperature profile had constant Brunt–Väisälä frequency in the troposphere and an isothermal profile in the stratosphere to the top of the model. Experiments were
Figure 4. (a) Surface pressure (solid lines) and temperature at 1000 mb (dashed lines) after 33 hours, (b) visualization of the 3D pattern of airflow, obtained from relative-flow analyses, on two isentropic surfaces. Thick solid and dashed streamlines show saturated flows along the 14 and 13 °C θ_e-surfaces, respectively. Dashed (14 °C) and solid (13 °C) lines show heights of these two surfaces at 1 km intervals. Shading represents relative humidity >90% at one or more levels between 0.5 and 3.5 km.
Figure 5. As Fig. 3 but for varying deformation strain rates at 33 hours. The box indicates the region shown in Fig. 4.
performed in which the Brunt–Väisälä frequency was set to $0.9 \times 10^{-2}$, $1.1 \times 10^{-2}$ and $1.3 \times 10^{-2}$ s$^{-1}$. For comparison, the Brunt–Väisälä frequency of the initial temperature profile used for the experiments in section 4(a) is approximately $1.1 \times 10^{-2}$ s$^{-1}$ in the lowest 3 km.

It was found that, for the run in which the Brunt–Väisälä frequency was $0.9 \times 10^{-2}$ s$^{-1}$, the depth of the simulated frontal waves was 5.2 km. When the stability of the atmosphere was increased (Brunt–Väisälä frequency was $1.1 \times 10^{-2}$ s$^{-1}$) the depth of the simulated frontal waves was 3.0 km, and finally when the Brunt–Väisälä frequency was $1.3 \times 10^{-2}$ s$^{-1}$ the depth of the simulated frontal waves was only 2.0 km. Thus decreasing the stability of the atmosphere increases the depth of the simulated frontal waves. The horizontal scale of the simulated frontal waves was not affected by variations in the Brunt–Väisälä frequency. These frontal waves do not interact with the upper levels and thus seem to be developing by barotropic instability only.

5. Deformation strain-rate threshold

(a) Subjective measure of frontal-wave activity

In this section experiments are described in which deformation strain rates of different strengths continue to act on the PV strip once it has formed. A constant deformation strain rate of $3.0 \times 10^{-5}$ s$^{-1}$ was applied for the first 13 hours of the model run. The deformation strain rate was then reduced to $0.0, 0.2 \times 10^{-5}, 0.4 \times 10^{-5}, 0.6 \times 10^{-5}, \text{or } 0.8 \times 10^{-5}$ s$^{-1}$ at a constant rate.

Figure 5 shows the PV at 900 mb (left-hand side) and the $\theta_w$ at 900 mb (right-hand side) for each of the model runs at 33 hours. The PV and $\theta_w$ fields are compared after 33 hours because by this time any developing frontal waves have reached their mature phase and will show up as ‘s’-shaped features in the $\theta_w$ contours. It can be seen that as the deformation strain rate increases the break-up of the PV strip into individual PV anomalies decreases and the strength of the individual PV anomalies decreases. For stronger deformation strain rates the PV anomalies are more confined in the meridional direction and extended in the zonal direction and have weaker magnitudes. This is also seen in the corresponding $\theta_w$ fields: the frontal-wave features are smoothed out and elongated in the zonal direction. The individual anomalies are also advected further towards the east by the stronger low-level deformation field. These results suggest that frontal-wave development is inhibited by strong deformation strain rates.

(b) Objective measure of frontal-wave activity

The objective measure of frontal-wave activity described in section 3(b) is applied in this section to quantify the findings of section 5(a) that frontal wave development is inhibited by strong deformation strain rates. Figure 6 shows $\xi_{xfg}$ along the 286 K $\theta_w$ contour at 9, 15 and 33 hours and the corresponding Fourier-transformed data for the model run in which the deformation strain field is reduced to zero. At 9 hours (i.e. before the deformation strain rate is reduced) the magnitude of $\xi_{xfg}$ is small along the 286 K $\theta_w$ contour (Fig. 6(a)). The associated amplitude spectrum generated by a Fourier transform, Fig. 6(b), shows small amplitudes at grid-length scales (24 km). The amplitude then rises as the wavelength increases with a maximum amplitude occurring for wavelengths of 1000 km. At 15 hours, when the deformation strain rate has been reduced to zero, small-scale positive and negative $\xi_{xfg}$ anomalies occur along the length of the 286 K contour. The spectrum at 15 hours shows larger amplitudes at all wavelengths, in particular there is increased amplitudes at wavelengths from 24 to
150 km (Fig. 6(d)). At 33 hours, when the frontal waves are reaching their mature stage, the $\xi_{xfg}$ features are larger in scale, Fig. 6(e), and can be associated with individual features on the 286 K $\theta_w$ contour (see Fig. 5(b)). The corresponding spectrum, Fig. 6(f), shows a reduction in amplitude at wavelengths < 150 km compared to Fig. 6(d) and a peak in the amplitude for wavelengths between 200 and 300 km. Applying a filter (192–1024 km) to the Fourier-transformed data removes long- and short-wavelength features that dominate the $\xi_{xfg}$ field from 13–20 hours. These features create a large $\xi_{xfg}$ value but are not representative of frontal-wave development.

Figure 7 shows the evolution of $\xi_{xfg}$ for the runs in which different deformation strain rates are applied to the PV strip. For the first 13 hours $\xi_{xfg}$ is the same for each model run. The deformation strain rate is then reduced from $3.0 \times 10^{-5}$ s$^{-1}$ to a lower deformation strain rate ($0.0, 0.2 \times 10^{-5}, 0.4 \times 10^{-5}, 0.6 \times 10^{-5},$ or $0.8 \times 10^{-5}$ s$^{-1}$). This causes $\xi_{xfg}$ to vary for each experiment. All of the model runs have a maximum $\xi_{xfg}$ occurring at 14 hours. This maximum $\xi_{xfg}$ is a result of the many, small-scale disturbances that form along the PV strip due to the sudden reduction in the deformation strain field (Fig. 6(c)) and is not representative of frontal-wave development; $\xi_{xfg}$ then reduces sharply for the next 4 hours. For the run in which the deformation strain rate is reduced to zero, $\xi_{xfg}$ increases again after 18 hours and then reduces gradually. This second maximum in $\xi_{xfg}$ is associated with frontal-wave development (Figs. 3(j) and (l)). When the deformation strain rate is reduced to $0.2 \times 10^{-5}$ s$^{-1}$, $\xi_{xfg}$ also increases between 23 and 35 hours and then reduces gradually. When the deformation strain rate is greater than $0.2 \times 10^{-5}$ s$^{-1}$, $\xi_{xfg}$ shows no significant increases between 18 and 42 hours, with $\xi_{xfg}$ eventually reaching values
Figure 7. Evolution of the average vorticity of the cross-front geostrophic wind, \( \xi_{xfg} \), for varying deformation strain rates. Experiments in which the deformation strain rate was reduced to 0, 0.2 \( \times 10^{-5} \), 0.4 \( \times 10^{-5} \), 0.6 \( \times 10^{-5} \), and 0.8 \( \times 10^{-5} \) s\(^{-1}\). Time is measured from the start of the simulation.

of approximately 0.3 \( \times 10^{-5} \) s\(^{-1}\) (similar to the value of \( \xi_{xfg} \) at the start of the model runs). Thus frontal-wave development occurs when the deformation strain rate is less than 0.4 \( \times 10^{-5} \) s\(^{-1}\) implying that the deformation strain-rate threshold is between 0.2 \( \times 10^{-5} \) s\(^{-1}\) and 0.4 \( \times 10^{-5} \) s\(^{-1}\). This agrees well with the subjective measure of frontal-wave activity in section 5(a).

A sensitivity study was carried out on the upper and lower bounds of the filter to see how sensitive the deformation strain threshold was to changes in their values. Varying the upper bound from 384 to 1536 km increases \( \xi_{xfg} \) between 13 and 18 hours for each of the model runs. Varying the lower bound of the filter from 192 to 24 km also increases \( \xi_{xfg} \) between 13 and 18 hours. Thus including short- and long-wavelength features increases \( \xi_{xfg} \) significantly between these times, as shown in Fig. 6(d) for the case in which the deformation strain is reduced to zero. However, the overall conclusions of frontal-wave development after 18 hours are not affected by the limits of the Fourier filter.

(c) Sensitivity to strength of vorticity strip

Bishop and Thorpe (1994b) showed that the minimum deformation strain needed to suppress barotropic frontal-wave development is determined by the strength of the vorticity strip. Stronger vorticity strips are capable of overcoming stronger deformation strain rates than weaker vorticity strips. Thus experiments were performed in which PV strips of varying strengths were subject to a deformation strain rate of 0.2 \( \times 10^{-5} \) s\(^{-1}\) to determine whether the deformation strain threshold found in section 5(b) changes if the strength of the PV strip changes. The PV strips of varying strengths were achieved...
by applying a deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ to the front for differing lengths of time. The time taken to reduce the deformation strain rate from $3.0 \times 10^{-5} \text{ s}^{-1}$ to $0.2 \times 10^{-5} \text{ s}^{-1}$ was constant at 1.87 hours.

Figure 8 shows $\xi_{xfg}$ for the experiments in which a deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ was applied for 10, 11, 12 and 13 hours. The average strength of the PV strips at the corresponding times was 1.4, 1.5, 1.7 and 1.9 PVU, respectively. When a deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ is applied for longer, the maximum $\xi_{xfg}$ increases. For weaker vorticity strips (deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ applied for 10 hours), $\xi_{xfg}$ remains at a value of approximately $3.0 \times 10^{-5} \text{ s}^{-1}$ from 7 hours after the deformation strain rate has been reduced. For stronger vorticity strips (deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ applied for 11 hours), $\xi_{xfg}$ does not go below this value until 14 hours after the deformation strain rate has been reduced. For even stronger vorticity strips (deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ applied for 12 and 13 hours), $\xi_{xfg}$ never falls below $3.0 \times 10^{-5} \text{ s}^{-1}$. There is also some increase in $\xi_{xfg}$ between 6 and 18 hours after the deformation strain rate has been reduced. Thus there is frontal-wave development for the cases in which the relative vorticity strip is strong (deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ applied for 12 and 13 hours) and no frontal-wave development for the case in which the vorticity strip is weakest (deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ applied for 10 hours). It is not clear whether there is frontal-wave development in the case in which a deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ is applied for 11 hours. Thus a deformation strain rate of $0.2 \times 10^{-5} \text{ s}^{-1}$ is sufficient to suppress all frontal-wave development along the weakest relative-vorticity strip.
Figure 9. Evolution of the theoretical minimum strain rate, $\alpha_{\text{min}}$, for varying deformation strain rates. Experiments in which the deformation strain rate was reduced to $0.0\times10^{-5}$, $0.2\times10^{-5}$, $0.4\times10^{-5}$, $0.6\times10^{-5}$ and $0.8\times10^{-5}$ s$^{-1}$. Time is measured from the start of the simulation.

(d) Theoretical deformation strain-rate threshold

Using the Bishop and Thorpe (1994a,b) analytic solution it is possible to calculate the theoretical minimum strain rate $\alpha_{\text{min}}$ necessary to suppress all barotropic frontal-wave growth:

$$\alpha_{\text{min}} = \frac{f}{4} \left( \frac{\bar{\xi}_f - \bar{\xi}_a}{\bar{\xi}_f + \bar{\xi}_a + 2f} \right) e^{-2\mu},$$

where $f$ is the Coriolis parameter, $\bar{\xi}_f$ is the average relative vorticity of the front, $\bar{\xi}_a$ is the ambient average relative vorticity and $2\mu$ is the non-dimensional wave number of the disturbance (product of the wave number and the width of the strip). Taking $\bar{\xi}_a$ to be zero and setting $2\mu = 0$ (i.e. making the assumption that the wavelength is large compared to the width of the PV strip), the evolution of $\alpha_{\text{min}}$ was calculated for each model run using the averaged Fourier-filtered (192–1024 km) relative vorticity along the 286 K $\theta$ contour as $\bar{\xi}_f$. The assumption that the wavelength of the frontal waves is large compared to the width of the PV strip was used to calculate $\alpha_{\text{min}}$ as it is impossible to determine the wavelength of individual disturbances along the front early on in their development. Thus, the $\alpha_{\text{min}}$ calculated gives an upper bound on the minimum strain rate necessary to suppress all barotropic frontal-wave growth.

Figure 9 shows the evolution of the theoretical minimum strain rate $\alpha_{\text{min}}$ for the runs in which deformation strain rates of $0.0\times10^{-5}$, $0.2\times10^{-5}$, $0.4\times10^{-5}$, $0.6\times10^{-5}$ and $0.8\times10^{-5}$ s$^{-1}$ were applied to the PV strip. For the first 13 hours $\alpha_{\text{min}}$ is the same for each model run. All of the model runs have a maximum $\alpha_{\text{min}}$ occurring after 16 or
Figure 10. Evolution of the theoretical minimum strain rate, \( \alpha_{\text{min}} \), for varying strengths of vorticity strip. Experiments in which a deformation strain rate of \( 3.0 \times 10^{-5} \) s\(^{-1} \) was applied for 10, 11, 12 and 13 hours. Time is measured from the time at which a deformation strain rate of \( 0.2 \times 10^{-5} \) s\(^{-1} \) is applied.

17 hours when \( \xi_f \) reaches its maximum value. After this time \( \xi_f \) in each model run is very similar giving similar values of \( \alpha_{\text{min}} \). Although \( \xi_f \) evolves with time throughout the model runs, it changes by a similar amount for each run. Thus, the deformation strain-rate threshold found in section 5(b) does not need to be normalized (to take into account the differing strengths of the relative-vorticity strips) as \( \alpha_{\text{min}} \) is similar for each run.

Figure 9 shows that \( \alpha_{\text{min}} \) is between \( 0.4 \times 10^{-5} \) and \( 0.6 \times 10^{-5} \) s\(^{-1} \) from approximately 20–35 hours. This is the period during which frontal-wave development occurs (see Fig. 7). When the applied deformation strain rate is below \( \alpha_{\text{min}} \) frontal-wave development is predicted by theory to occur, and when the applied deformation strain rate is above \( \alpha_{\text{min}} \) frontal-wave development is predicted to be suppressed. Thus the theoretical minimum strain-rate threshold is between \( 0.4 \times 10^{-5} \) and \( 0.6 \times 10^{-5} \) s\(^{-1} \). This theoretical value is greater than the deformation strain-rate threshold found using \( \xi_{\text{xf}} \) to determine development in section 5(b). This may be because the wavelength of the disturbances are not large compared to the width of the PV strip, thus \( 2\mu \neq 0 \) and hence \( \alpha_{\text{min}} \) is reduced. In this simulation the width of the PV strip is approximately 40 km and the frontal waves have a wavelength of approximately 300 km. Thus the non-dimensional wave number of a frontal wave \( \mu = 0.13 \), which leads to \( \alpha_{\text{min}} \) values 23% less than previously calculated. Also the effects of diffusion are not taken into account in the theoretical calculation.

Figure 10 shows \( \alpha_{\text{min}} \) for the experiments in which a deformation strain rate of \( 3.0 \times 10^{-5} \) s\(^{-1} \) was applied for 10, 11, 12 and 13 hours. For each run the peak vorticity of the strip, and hence the maximum value of \( \alpha_{\text{min}} \), occurs at the time at which the strain rate begins to be reduced to \( 0.2 \times 10^{-5} \) s\(^{-1} \). After this time \( \alpha_{\text{min}} \) reduces rapidly for each
model run over a period of 7 hours. From 5 hours after the deformation strain rate has been reduced, the strength of the vorticity strip is different for each model run. Figure 10 shows that, when a deformation strain rate of $3.0 \times 10^{-5} \text{ s}^{-1}$ is applied to the front for 10 hours, a theoretical minimum strain rate $\alpha_{\text{min}}$ of $0.27 \times 10^{-5} \text{ s}^{-1}$ is sufficient to completely suppress frontal-wave development. Again, this is slightly higher than the applied value of $0.2 \times 10^{-5} \text{ s}^{-1}$ which was found to suppress frontal-wave development for the run using $\xi_{xfg}$ in section 5(c).

Renfrew et al. (1997) found for their observed cases that the deformation strain threshold was in the range $0.5 \times 10^{-5}$ to $1.2 \times 10^{-5} \text{ s}^{-1}$ depending on the strength of the vorticity strip and, for a front with typical strength relative vorticity of $1.0 \times 10^{-4} \text{ s}^{-1}$, a critical threshold of around $0.8 \times 10^{-5} \text{ s}^{-1}$ fitted the observations. Chaboureau and Thorpe (1999) showed frontal-wave development occurred for FASTEX∗ cases in which the deformation strain rate was $\leq 0.6 \times 10^{-5} \text{ s}^{-1}$. A direct comparison cannot be made with the deformation strain thresholds found in the observational studies of Renfrew et al. (1997) and Chaboureau and Thorpe (1999) because in this experiment we are looking at a number of waves along a vorticity strip and not at individual frontal-wave cases. However, the observed deformation strain-rate thresholds are consistent with our results.

6. CONCLUSIONS

An idealized modelling experiment has been performed to simulate the life cycle of barotropic frontal waves and to investigate the impact of deformation strain on their development. The model used was the idealized version of the Met Office non-hydrostatic Unified Model (v5.3). The initial conditions consisted of a meridional temperature gradient in thermal wind balance with an upper-level zonal jet. A constant low-level deformation field was then applied to the front in order to develop a potential-vorticity strip. When the potential-vorticity strip had developed, the deformation strain rate was reduced to different strengths and frontal-wave development determined.

Barotropic frontal-wave development was found to occur in two stages. First, a deformation strain field acting on the front acts to tighten the horizontal temperature gradient. Ascent occurs on the warm side of the front. As moist air parcels ascend they cool and water vapour condenses releasing latent heat. Steady heating leads to the creation of a positive PV strip aligned along the front. Second, as the deformation strain field acting on the front is reduced the PV strip breaks up into individual PV anomalies with associated cyclonic circulations. The circulations extend towards the surface and cause frontal waves to form along the front. Analysis of these frontal waves shows a similar structure to the conceptual model of a frontal cyclone, although they are shallower than typically observed frontal cyclones. Deeper frontal waves can be simulated if the stability of the atmosphere is reduced.

Experiments were performed to determine whether a deformation strain threshold exists above which barotropic frontal-wave development is suppressed. A comparison was made of frontal-wave development along a front on which deformation strain rates of different strengths were acting. Frontal-wave activity was measured using the vorticity of the cross-front geostrophic wind at 900 mb averaged along a $\theta_w$ contour. Barotropic frontal-wave development was suppressed by deformation strain rates $\geq 0.4 \times 10^{-5} \text{ s}^{-1}$, for a strip with average potential vorticity of 2.2 PVU. This value is slightly less than that predicted by the analytical solution of Bishop and

* Fronts and Atlantic Storm Tracks EXperiment.
Thorpe (1994a,b). This may be because we have assumed the wavelength of the frontal waves is infinite and the effects of diffusion are neglected in the theoretical calculation. The deformation strain-rate threshold is dependent on the strength of the relative-vorticity strip with stronger PV strips able to overcome stronger deformation strain rates leading to frontal-wave development. Since these simulations are run using a numerical model with the same physics and dynamics as the Met Office forecast model, our findings show that an accurate representation of the environmental flow field is important for an accurate forecast of frontal-wave development.

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