Statistical test of frequency of SSWs

To compare the frequency of SSWs in a GCM run with the frequency of SSWs in the reanalysis data, consider each winter in the reanalysis or model run to be a separate, independent observation of the frequency of major warming events per winter. Thus, for example, in the NCEP/NCAR dataset we have 45 observations of the frequency of events per winter, 23 observations with no events, 17 with one event and 5 with two events.

First, calculate the sample mean frequency ($\overline{x}$) of SSWs per winter season. This is the expected value of the 45 observations in the NCEP/NCAR reanalysis

$$\overline{x} = E[X] = \sum_x xPr\{X = x\}. \tag{1}$$

In Eq. 1 x represents an observed frequency of SSWs per winter and $Pr\{X = x\}$ is the probability with which that frequency is observed. The sample variance of the frequency of warming events s^2 is calculated in a similar fashion using

$$s^2 = \sum_x (x - \overline{x})^2 Pr\{X = x\}. \tag{2}$$

Second calculate the sample standard error, e, using the sample variance and the number of SSWs N.

$$e = \frac{\sqrt{s^2}}{\sqrt{N}}. \tag{3}$$

Finally, the values of $\overline{x}$ and e are used to construct a t-test which compares the mean frequency of SSWs in the reanalysis and your GCM dataset. The null hypothesis of this test is: *the mean frequency of SSWs in the GCM and NCEP/NCAR reanalysis is equal.* The test is two-sided because there is no a priori reason to expect that the difference between the means should be positive or negative. A t-statistic comparing the expected frequency of each model run and the NCEP/NCAR reanalysis is calculated in the standard way (p.122, Wilks, 'Statistical Methods in the Atmospheric Sciences')

$$t = \frac{\overline{x}_r - \overline{x}_m}{[e_r^2 + e_m^2]^{1/2}}. \tag{4}$$

The r subscript denotes reanalysis statistics and the m subscript denotes model statistics. The t statistic is then compared to critical values of a t-distribution with degrees of freedom calculated using the expression,

$$df = \frac{(e_r^2 + e_m^2)^2}{(e_r^2)^2/(N_r - 1) + (e_m^2)^2/(N_m - 1)} \tag{5}$$