

Available Potential Energy and Exergy in Stratified Fluids

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Abstract

Lorenz's theory of available potential energy (APE) remains the main framework for studying the atmospheric and oceanic energy cycles. Because the APE generation rate is the volume integral of a thermodynamic efficiency times the local diabatic heating/cooling rate, APE theory is often regarded as an extension of the theory of heat engines. Available energetics in classical thermodynamics, however, usually relies on the concept of exergy, and is usually measured relative to a reference state maximising entropy at constant energy, whereas APE's reference state minimises potential energy at constant entropy. This review seeks to shed light on the two concepts; it covers local formulations of available energetics, alternative views of the dynamics/thermodynamics coupling, APE theory and the second law, APE production/dissipation, extensions to binary fluids, mean/eddy decompositions, APE in incompressible fluids, APE and irreversible turbulent mixing, and the role of mechanical forcing on APE production.

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1 List of acronyms

- APE: Available Potential Energy
- AIE: Available Internal Energy
- KE: Kinetic Energy
- GPE: Gravitational Potential Energy
- IE: Internal energy
- AMOC: Atlantic Meridional Overturning Circulation

2 List of key terms

- **Available Potential Energy:** The difference in potential energy between the actual state and Lorenz's reference state.
- **Available Potential Energy density:** The locally-defined sign positive definite energy quantity whose volume integral yields Lorenz's globally-defined APE.
- **Actual state:** The actual state of the system considered (atmosphere, oceans, or any stratified fluid).
- **Lorenz's Reference state:** The state of minimum potential energy in an isentropic re-arrangement of mass. It may not be unique in binary fluids.
- **Exergy:** The part of the internal energy measuring the departure of the system considered from thermodynamic and mechanical equilibrium.
- **Extended exergy:** Terminology introduced by (**author?**) Kucharski 1997 to refer to the APE density from an exergy viewpoint.
- **Pseudo-Energy:** Terminology introduced by (**author?**) Shepherd 1993

to refer to the Available potential energy density from a Hamiltonian viewpoint.

- **Dynamics/Thermodynamics coupling:** Generic term used to refer to the interactions between mechanical energy (KE+GPE) and thermodynamic internal energy or enthalpy.
- **Horizontal convection:** The boundary-driven circulation resulting from differential heating/cooling applied on a surface of constant geopotential.

3 Introduction

The concept of available potential energy (APE) was first introduced in the atmospheric context by **(author?)** Margules 1905 (who termed it “available kinetic energy”) and subsequently by **(author?)** Lorenz 1955. Margules and Lorenz sought to understand the source of energy for storms and the nature of the processes maintaining the global large-scale atmospheric circulation against dissipation respectively. They both recognised that only a fraction of the total potential energy (the sum of the internal and gravitational potential energies) is actually available for conversion into kinetic energy. They defined APE as the difference of potential energy between the actual state and the reference state minimising potential energy in an isentropic re-arrangement of mass, as illustrated in Fig. 1. **(author?)** Dutton and Johnson 1967, Van Mieghem 1973, Dutton 1986, Wiin-Nielsen and Chen have reviewed and discussed extensively APE and atmospheric energetics.

For a hydrostatic atmosphere, APE is commonly expressed as:

$$\text{APE} := \int_V [h - h_R] dm \quad (1)$$

(Pauluis 2007), where h and h_R are the specific enthalpies of the actual and reference state respectively, while $dm = \rho dV$ is the mass of an elementary air parcel. Treating the atmosphere as a dry gas leads to the following two-components energy cycle:

$$\frac{dK}{dt} = \mathbf{C}(\mathbf{A}, \mathbf{K}) - D, \quad (2)$$

$$\frac{dA}{dt} = -C(A, K) + \underbrace{\int_V \left(\frac{T - T_r}{T} \right) \dot{Q} dm}_G, \quad (3)$$

where K is the volume-integrated kinetic energy, A is the total APE, $C(A, K)$ is the energy conversion between APE and kinetic energy (KE), D is the volume-integrated viscous dissipation, and T_R the temperature of the parcel in the reference state. A key feature of Eq. (3) is the APE generation term G having the form of a volume integral of a thermodynamic efficiency-like factor $(T - T_R)/T$ times the local diabatic heating/cooling rate \dot{Q} , which is reminiscent of the celebrated Carnot formula (Carnot 1824), and suggests a link with the classical theory of heat engines.

This link is not entirely clear, however, because the concept of APE differs from the concept of available energy that had historically been introduced earlier as part of the development of heat engines by (author?) Gibbs 1873, Gibbs 1875 among others, now commonly referred to as “exergy” after (Rant 1956). Indeed, exergy usually measures the available thermodynamic energy arising from the departure of the system considered from thermodynamic (and mechanical) equilibrium, and hence from a reference state maximising entropy at constant energy. In contrast, the reference state entering Margules/Lorenz APE minimises potential energy at constant entropy, which is fundamentally different. The apparent disagreement between Lorenz/Margules APE theory and the classical thermodynamic concept of exergy prompted a number of studies argu-

ing that available energetics in the atmosphere should be based on an isothermal reference state, presumably the state of maximum entropy, e.g., (author?) Livesey and Dutton 1976, Dutton 1973, Pearce 1978, Blackburn 1983, Marquet 1991, Karlsson 1990, Bannon 2005. As shown by many authors, the use of exergy-based available energetics leads to relatively straightforward formulations of local available energetics, the lack of which in Lorenz's APE theory had impeded its application to the study of energetics in regional domains. The study by (author?) Andrews 1981, which for the first time showed how Lorenz's globally defined APE could in fact be derived from a local APE density, marked a breakthrough in the field, allowing (author?) Marquet 1995, Kucharski 1997, Kucharski 2001 to greatly clarify the links between APE and exergy. This review surveys the main conceptual issues underlying various theories of available energetics in stratified fluids, with a number of illustrations drawn from some recent applications.

4 Thermodynamic and fluid dynamics views of available energy

4.1 Useful work, exergy, and the convexity of internal energy

Understanding the general principles controlling the amount of useful work that can be produced by devices exchanging heat and work with an environment of much larger dimensions, thus often idealised as an isothermal reservoir of uniform temperature T_0 and **pressure** P_0 , is one of the main concerns of classical thermodynamics. Owing to its importance in a considerable number of fields of physics, the issue has given rise to a large body of literature. Useful reviews and discussions of the main results are contained in (author?) Keenan 1951, Haywood 1974, Gaggioli 1998, while (author?) Karlsson 1990, Marquet 1991 also review historical developments. It seems now relatively well understood

that the main thermodynamic property underlying available energy or exergy is the convexity of the specific internal energy $e = e(\eta, v)$ in the (specific entropy η , specific volume v) space. Convexity ensures that for any convex function $f(\mathbf{x})$, the function $f_{ex}(\mathbf{x})$:

$$f_{ex}(\mathbf{x}) := f(\mathbf{x}) - f(\mathbf{x}_0) - \nabla_{\mathbf{x}}f(\mathbf{x}_0) \cdot (\mathbf{x} - \mathbf{x}_0) \approx \frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \mathbf{H}(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0), \quad (4)$$

is sign positive definite, where $\mathbf{H}(\mathbf{x}_0)$ is the Hessian matrix of the second derivatives at $\mathbf{x} = \mathbf{x}_0$. Applying such a construction to internal energy, whose total differential is $de = Td\eta - Pd v$, with T and P the temperature and pressure respectively, defines the *exergy* of internal energy:

$$e_{ex} := e - e_0 - T_0(\eta - \eta_0) + P_0(v - v_0). \quad (5)$$

The associated geometrical construction is illustrated in Fig. 2. The positive definite character of e_{ex} is easily established by decomposing Eq. (5) into a “work” and “heat” components $e_{ex} = e_{ex}^{work} + e_{ex}^{heat}$,

$$e_{ex}^{work} := h - h(\eta, P_0) + (P_0 - P)v = - \int_{P_0}^P \int_P^{P'} \frac{1}{\rho^2 c_s^2} dP'' dP' \quad (6)$$

$$e_{ex}^{heat} := h(\eta, P_0) - h(\eta_0, P_0) - T_0(\eta - \eta_0) = \int_{\eta_0}^{\eta} \int_{\eta_0}^{\eta'} \frac{T}{c_p} d\eta'' d\eta' \quad (7)$$

where c_s^2 is the squared speed of sound and c_p is the specific heat capacity **at constant pressure**. In the limit of small pressure and entropy departures from P_0 and η_0 , e_{ex} approximates to:

$$e_{ex} \approx \frac{(P - P_0)^2}{2\rho_0^2 c_0^2} + \frac{T_0(\eta - \eta_0)^2}{2c_{p0}} \quad (8)$$

which clearly demonstrates that the exergy of internal energy resides in the departure of the system from mechanical and thermodynamical equilibrium. In the thermodynamics literature, the quadratic quantity Eq. (8) is often called the

thermodynamic length, and plays an important role in thermodynamic optimisation theory, e.g., **(author?)** Salamon and Berry 1983, Crooks 2007. The *anergy* is defined as the difference between internal energy and exergy:

$$a := e - e_{ex} = e_0 + T_0(\eta - \eta_0) - P_0(v - v_0). \quad (9)$$

The above decomposes internal energy as the sum of the anergy a and of the sign positive definite work and heat exergies $e_{ex}^{work} + e_{ex}^{heat}$. These three different quantities have a well-defined local meaning, even for temporally and spatially varying T_0 and P_0 , and can therefore be used to describe local energetics. Although the most common choice in classical thermodynamics is to use constant T_0 and P_0 , many authors have proposed to use a hydrostatically balanced P_0 for applying the concept to meteorology, e.g., **(author?)** Fortak 1998, Bannon 2005. Instead of using a constant T_0 , some authors have suggested that T_0 should be the temperature of the isothermal state corresponding to a state of maximum entropy, e.g., **(author?)** Dutton 1973, Livesey and Dutton 1976. In that case, exergy measures the distance to a state in both thermodynamic and mechanical equilibrium, with $T_0 = T_0(t)$ a function of time and $P_0 = P_0(z, t)$ is a function of both z and t . The corresponding local formulation of energetics is as follows:

$$\rho \frac{D}{Dt} \left[\frac{\mathbf{v}^2}{2} + e_{ex} \right] + \nabla \cdot [(P - P_0)\mathbf{v}] = \rho G_0 + \rho \mathbf{v} \cdot \mathbf{F}_{\mathbf{v}}, \quad (10)$$

$$\rho \frac{D}{Dt} [a + \Phi] + \nabla \cdot (P_0 \mathbf{v}) = \rho T \frac{D\eta}{Dt} - \rho G_0, \quad (11)$$

where the generation term is given by:

$$\rho G_0 = \left(\frac{T - T_0}{T} \right) \dot{Q} + \rho(\eta_0 - \eta) \frac{dT_0}{dt} + \left(1 - \frac{\rho}{\rho_0} \right) \frac{\partial P_0}{\partial t}, \quad (12)$$

where $\dot{Q} = \rho T D\eta/Dt$ is the local total diabatic heating/cooling rate. In writing the above equations we assume that the momentum and kinetic energy equations

take the following form:

$$\rho \frac{D\mathbf{v}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{v} + \nabla P = -\rho \nabla \Phi + \rho \mathbf{F}_{\mathbf{v}}, \quad (13)$$

$$\rho \frac{D}{Dt} \frac{\mathbf{v}^2}{2} + \nabla \cdot (P\mathbf{v}) = \underbrace{\rho P \frac{Dv}{Dt} - \rho \frac{D\Phi}{Dt}}_{C(e_{ex}, e_k)} + \rho \mathbf{v} \cdot \mathbf{F}_{\mathbf{v}}, \quad (14)$$

where $\mathbf{v} = (u, v, w)$ is the three-dimensional velocity field, $\boldsymbol{\Omega}$ is Earth rotation vector, Φ is the geopotential, and $\mathbf{F}_{\mathbf{v}}$ a representation of the viscous force. The term $C(e_{ex}, e_k)$ represents the conversion between exergy and kinetic energy.

The quantities $\mathbf{v}^2/2 + e_{ex}$ and $a + \Phi$ obey separate conservation laws in absence of diabatic/viscous effects, but become coupled through the generation term G_0 (Eq. (12)) when diabatic/viscous effects are retained. When T_0 and P_0 are chosen to be time-independent, G_0 reduces to a Carnot-like thermodynamic production term, with a local thermodynamic efficiency $(T - T_0)/T$. When P_0 is chosen to be spatially uniform, e_{ex}^{work} and e_{ex}^{heat} are decoupled in absence of diabatic/viscous effects, but the use of a z -dependent $P_0(z, t)$ couples heat and work even for purely adiabatic/inviscid motions, making such decomposition less meaningful. Such a coupling can be avoided for $P_0 = P_0(\eta)$ a function of entropy alone, but this would be at the expenses of satisfying hydrostatic balance. Equations similar to Eqs. (10) and (11) have been discussed in great details by **(author?)** Fortak 1998, who discussed how to define exergies and anergies for the most common thermodynamic potentials. See also **(author?)** Bannon 2005 for a generalisation of exergy to a multi-component fluids, with an application to a moist atmosphere, where P_0 also satisfies hydrostatic balance.

4.2 Extended ex-ergy and APE density for stratified fluids

Although the above exergy and anergy can obviously be used for describing local energetics, significant departure from thermodynamic equilibrium may however leave a large fraction of the exergy untapped and effectively unavailable. Likewise, the local thermodynamic efficiency $(T - T_0)/T$ may significantly overestimate the local production of kinetic energy due to diabatic heating/cooling, since a thermodynamic efficiency $(T - T_R)/T$ constructed from the z -dependent background temperature $T_R(z)$ would be on average smaller. Simply replacing the constant or time dependent T_0 by a z -dependent $T_R(z, t)$ in Eq. (5) is not really satisfactory, however, because as shown by **(author?)** Kucharski 1997, Kucharski 2001, this couples $\mathbf{v}^2/2 + e_{ex}$ and $a + \Phi$ even for purely adiabatic motions. **(author?)** Kucharski 1997, Kucharski 2001 show that in order to avoid such a coupling, the anergy and exergy have to be generalised as follows:

$$a := e_r + Q(\eta, S) - Q(\eta_R, S_R) - P_R(v - v_R), \quad (15)$$

$$e_{ex} := e - a = e - e_R - Q(\eta, S) + Q(\eta_R, S_R) + P_R(v - v_R), \quad (16)$$

where $Q(\eta, S)$ is a function of the materially conserved variables η and S to be discussed shortly, where S can either represent salinity or the total water mixing ratio, depending on the context (Kucharski's results were derived for a mono-component fluid, we extend them here for a binary fluid). It is clear that Eqs. (15) and (16) encompass the previous definitions (9) and (5), which are recovered in the particular case where $Q(\eta, S) = Q(\eta) = T_0\eta$.

An explicit construction of the function $Q(\eta, S)$ entering Eqs. (15) and (16) was first discussed by **(author?)** Andrews 1981 in the case of a mono-component fluid in absence of diabatic and viscous effects. Considering a hydrostatically-

balanced reference state with z -dependent reference entropy and pressure profiles $\eta_R(z)$ and $P_R(z)$, such that $d\eta_R/dz > 0$ everywhere, allows one to use entropy as a vertical coordinate and thus regard $P_R = P_R(\eta_R)$ as a function of specific entropy. In that case, it is possible to express the reference temperature profile as $T_R(z) = T(\eta_R(z), P_R(z)) = T(\eta_R, P_R(\eta_R)) = \tilde{T}(\eta_R)$, and to construct the quantity $Q(\eta)$ from:

$$Q(\eta) - Q(\eta_R) = \int_{\eta_R}^{\eta} T_R(\eta', P_R(\eta')) d\eta' = \int_{\eta_R}^{\eta} \tilde{T}(\eta') d\eta'. \quad (17)$$

(**author?**) Andrews 1981 was the first to show that in the case where the reference state corresponds to that introduced by Margules and Lorenz, the quantity e_{ex} (16) can be regarded as the local counterpart of Lorenz's globally-defined APE, which he called *APE density*. (**author?**) Shepherd 1993 recovered the same quantity from a Hamiltonian approach, and called it the *pseudo-energy*, whereas (**author?**) Kucharski 1997 called it the *extended exergy*. For Q defined by Eq. (17), the sign positive definite character of e_{ex} can be established from the decomposition $e_{ex} = e_{ex}^{work} + e_{ex}^{heat}$ in terms of the following pressure and entropy components:

$$e_{ex}^{work} := h - h(\eta, P_R) + (P_R - P)v = - \int_{P_R}^P \int_P^{P'} \frac{1}{\rho^2 c_s^2} dP'' dP' \quad (18)$$

$$e_{ex}^{heat} := h(\eta, P_R) - h(\eta_R, P_R) - \int_{\eta_R}^{\eta} \tilde{T}(\eta') d\eta' = \int_{\eta_R}^{\eta} \int_{\eta'}^{\eta_R} \Gamma(\eta', P_R(\eta'')) \frac{dP_R}{d\eta''}(\eta'') d\eta'' d\eta' \quad (19)$$

where $\Gamma = \partial^2 h / \partial \eta \partial P = \alpha T / (\rho c_p)$ is the adiabatic lapse rate, where α is the thermal expansion coefficient. The linearised expression of the pressure component is the same as for classical exergy. The linearised expression for the entropy component of extended exergy becomes:

$$e_{ex}^{heat} \approx -\Gamma_R \frac{dP_R}{d\eta_R} \frac{(\eta - \eta_R)^2}{2} = -\Gamma_R \frac{dP_R}{dz} \frac{(\eta - \eta_R)^2}{2\partial\eta_R/\partial z} = \frac{1}{2} N_R^2 \zeta^2, \quad (20)$$

where $N_R^2 = \rho_R g \Gamma_R \partial \eta_R / \partial z$, using the fact that the reference pressure is in hydrostatic balance, $dP_R/dz = -\rho_R g$, and using the fact that by construction of the reference state $\eta(\mathbf{x}, t) = \eta_R(z_R, t)$, with $\zeta = z - z_R$ being the **vertical parcel displacement from its reference position**.

In the general case of a binary fluid where the reference state can be altered by diabatic effects, the various reference profiles must be regarded as functions of time as well. While the density and pressure $\rho_R = \rho_R(z, t)$ and $P_R = P_R(z, t)$ must remain functions of z and t and in hydrostatic balance at all times, it seems possible for the reference entropy and S profiles to possess horizontal variations provided that they are density compensated, i.e., satisfy $\rho_R(\eta_R(\mathbf{x}, t), S_R(\mathbf{x}, t), P_R(z, t)) = \rho_R(z, t)$. In that case, the reference pressure can also be regarded as $P_R = P_R(z, t) = P_R(\eta_R, S_R, t)$ a function of η_R , S_R and time. In the most general case, therefore, $Q(\eta, S, t)$ becomes a function of entropy, S , and time, and is to be obtained from integrating the following differential relation: $dQ = \tilde{T}d\eta + \tilde{\mu}dS$, where μ is the relative chemical potential, $\tilde{T}(\eta, S, t) = T(\eta, S, P_R(\eta, S, t))$ and $\tilde{\mu}(\eta, S, t) = \mu(\eta, S, P_R(\eta, S, t))$. The corresponding evolution equations for $\mathbf{v}^2/2 + e_{ex}$ and $a + \Phi$ therefore become:

$$\rho \frac{D}{Dt} \left[\frac{\mathbf{v}^2}{2} + e_{ex} \right] + \nabla \cdot [(P - P_R)\mathbf{v}] = \rho G_{ex} + \rho \mathbf{v} \cdot \mathbf{F}_v, \quad (21)$$

$$\rho \frac{D}{Dt} [a + \Phi] + \nabla \cdot (P_R \mathbf{v}) = \rho \left[T \frac{D\eta}{Dt} + \mu \frac{DS}{Dt} \right] - \rho G_{ex}, \quad (22)$$

where the local APE generation term is given by:

$$\rho G_{ex} = \rho \left\{ \left(T - \tilde{T} \right) \frac{D\eta}{Dt} + (\mu - \tilde{\mu}) \frac{DS}{Dt} + \frac{\partial(Q_R - Q)}{\partial t} \right\} + \left(1 - \frac{\rho}{\rho_R} \right) \frac{\partial P_R}{\partial t}. \quad (23)$$

The expressions Eqs. (21), (22) and (23) mimic the corresponding expressions for the classical exergy Eqs. (10), (11) and (12). Our approach to a binary fluid is somewhat more general than (**author?**) Bannon 2004's, which only consid-

ered z -dependent $\eta_R(z)$ and $S_R(z)$ reference profiles. The expression is valid for non-hydrostatic motions. These equations show that in absence of diabatic and viscous effects, $a + \Phi$ and $\mathbf{v}^2/2 + e_{ex}$ are individually conservative quantities. It is important to note that locally, the generation term (23) possesses a number of additional terms, namely the pressure and Q terms, that are lacking in **(author?)** Pauluis 2007's global APE approach. **(author?)** Scotti et al. 2006 and **(author?)** Molemaker and McWilliams 2010 provide indications, in the context of Boussinesq fluids, that these additional terms may occasionally dominate and therefore be key to understand local kinetic energy production.

(author?) Kucharski and Thorpe 2000 used the extended exergy to describe the energy cycle of an idealised baroclinic wave. Fig. 3 shows that in general, the distribution of extended exergy or APE density differs significantly from that of classical exergy, and likewise for the corresponding Carnot-like thermodynamic efficiency factors. The difference in thermodynamic efficiency is most dramatically illustrated in the oceanic case; indeed, **(author?)** Tailleux 2010 shows that while the classical exergy predicts a net production of exergy $O(90 \text{ TW})$ due to the surface buoyancy fluxes, this number reduces to $O(0.5 \text{ TW})$ when APE is used, which is about two orders of magnitude smaller, a considerable difference!

4.3 Alternative views on the dynamics/thermodynamics coupling

Although APE theory emphasises the construction of a positive **definite** potential energy reservoir, the description of the atmospheric and oceanic energy cycles is however concerned with the magnitude and sign of the energy conversions, rather than with the sign of the reservoirs. From that viewpoint, the construction of Lorenz's APE is significant only to the extent that it helps anticipate the sign

and magnitude of the kinetic to potential energy conversion. To shed light on the issue, we review a number of approaches providing an alternative view of the thermodynamics/dynamics coupling that take as their starting point the following local evolution equation for the kinetic energy:

$$\rho \frac{D}{Dt} \frac{\mathbf{v}^2}{2} = -\rho \frac{D\Phi}{Dt} - \mathbf{v} \cdot \nabla P + \rho \mathbf{v} \cdot \mathbf{F}_v. \quad (24)$$

Instead of writing down the pressure work as $\mathbf{v} \cdot \nabla P = \nabla \cdot (P\mathbf{v}) - \rho P Dv/Dt$ in order to make the link with internal energy, the alternative is to regard the pressure work as a conversion with the specific enthalpy h , e.g., Dutton (1992), whose total differential is: $dh = Td\eta + \mu dS + vdP$, so that:

$$\mathbf{v} \cdot \nabla P = \frac{DP}{Dt} - \frac{\partial P}{\partial t} = \rho \left[\frac{Dh}{Dt} - T \frac{D\eta}{Dt} - \mu \frac{DS}{Dt} \right] - \frac{\partial P}{\partial t}. \quad (25)$$

This leads to a description of local energetics that takes the generic form:

$$\rho \frac{De_k}{Dt} = -C(e_k, e_h) - C(e_k, e_g) + \rho \mathbf{v} \cdot \mathbf{F}_v, \quad (26)$$

$$\rho \frac{De_h}{Dt} = C(e_k, e_h) + \rho G_h, \quad (27)$$

$$\rho \frac{De_g}{Dt} = C(e_k, e_g), \quad (28)$$

where $e_k = \mathbf{v}^2/2$, $e_g = \Phi$, while the energy conversions are $C(e_k, e_h) = \mathbf{v} \cdot \nabla P$ and $C(e_k, e_g) = \rho D\Phi/Dt$. If e_h is taken as the specific enthalpy, the generation term G_h is given by:

$$\rho G_h = \rho \left[T \frac{D\eta}{Dt} + \mu \frac{DS}{Dt} \right] + \frac{\partial P}{\partial t} \quad (29)$$

A simple analysis of the structure of Eqs. (27-28) reveals, however, that it is possible to obtain alternative formulations by defining $e_h^* = e_h - Q(\eta, S)$ where $Q(\eta, S)$ is an arbitrary function of the materially conserved variables η and S , such that:

$$\rho \frac{De_h^*}{Dt} = C(e_k, e_h^*) + G_h^* \quad (30)$$

$$\rho G_h = \rho \left[(T - T^*) \frac{D\eta}{Dt} + (\mu - \mu^*) \frac{DS}{Dt} \right] + \frac{\partial P}{\partial t} \quad (31)$$

by defining $T^* = \partial Q / \partial \eta$ and $\mu^* = \partial Q / \partial S$. Clearly, $C(e_k, e_h) = C(e_k, e_h^*)$, so that the transformation leaves the energy conversion between kinetic energy and the thermodynamic energy unchanged.

4.3.1 EXAMPLE 1: MARQUET (1991)'S AVAILABLE ENTHALPY Motivated by **(author?)** Pearce 1978 and classical exergy, **(author?)** Marquet 1991 introduced the available enthalpy $e_h^* = h - T_0 \eta$ for a dry atmosphere, corresponding to the choice $Q(\eta, S) = T_0 \eta$, with T_0 constant, in which case G_h^* becomes:

$$\rho G_h^* = \rho (T - T_0) \frac{D\eta}{Dt} + \frac{\partial P}{\partial t}. \quad (32)$$

An interesting property of available enthalpy is that for a perfect gas, it naturally splits into a temperature and pressure components $e_h^* = a_T + a_p$, with:

$$a_T = c_p (T - T_0) - c_p T_0 \ln \frac{T}{T_0}, \quad a_p = RT_0 \ln \frac{p}{p_0}$$

where a_T is positive sign definite. Although a_p is not sign definite, it appears nevertheless possible to obtain a meaningful and rigorous decomposition of available enthalpy into mean and eddy components, such as:

$$a_T = \underbrace{c_p (T - T^*) - c_p \ln \frac{T}{T^*}}_{\text{eddy}} + \underbrace{c_p (T^* - T_0) - c_p \ln \frac{T^*}{T_0}}_{\text{mean}},$$

$$a_p = \underbrace{RT_0 \ln \frac{P}{P^*}}_{\text{eddy}} + \underbrace{RT_0 \ln \frac{P^*}{P_0}}_{\text{mean}},$$

if T^* and P^* are taken as mean properties, depending solely on height for instance, defining the eddy parts $T' = T - T^*$ and $P' = P - P^*$. In fact, the properties of a_T and a_p are such that a decomposition in an arbitrary number of subcomponents is possible without introducing any 'interaction' term that normally plagues classical eddy/mean decompositions. This framework was used by

(**author?**) Marquet 2003 to diagnose a baroclinic wave energy cycle, and extensions to a moist atmosphere were discussed by (**author?**) Marquet 1993 and (**author?**) Bannon 2005.

4.3.2 EXAMPLE 2: DYNAMIC/POTENTIAL ENTHALPY DECOMPOSITION Another important construction is based on using the potential enthalpy $Q = h(\eta, S, P_0)$, e.g., (McHall 1990, McDougall 2003), which (**author?**) McDougall 2003 argues in the oceanic context is the most accurate quantity to measure “heat”, where P_0 is a fixed reference pressure. This leads one to define e_h^* as:

$$e_h^* = h - h(\eta, S, P_0) = \int_{P_0}^P v(\eta, S, P') dP', \quad (33)$$

which was termed the *dynamic enthalpy* by (**author?**) Young 2010 and *effective potential energy* by (**author?**) Nycander 2010, and the generation term as:

$$\rho G_h^* = \rho \left[(T - \theta) \frac{D\eta}{Dt} + (\mu - \mu_r) \frac{DS}{Dt} \right] + \frac{\partial P}{\partial t}, \quad (34)$$

where θ is the potential temperature, defined by the implicit relationship $\eta(T, S, P) = \eta(\theta, S, P_0)$ in the oceans, while $\mu_r = \mu(\eta, S, P_0)$. In absence of diabatic effects, when $\partial P/\partial t$ is small enough to be neglected, the standard Bernoulli theorem stating that $B = \mathbf{v}^2/2 + \Phi + \int_{P_0}^P v(\eta, S, P') dP'$ is conserved along streamlines is recovered. In the oceans, because $T = \theta$ and $\mu = \mu_r$ at the ocean surface where $P = P_0$, the volume-integral of G_h^* has the interesting property of lacking any dependence on the surface buoyancy fluxes, and to be entirely controlled by irreversible molecular diffusive processes. Recently, the global budgets of dynamic and potential enthalpy have been shown to play a key role in extending (**author?**) Paparella and Young 2002’s epsilon-theorem to a fully compressible ocean with a general nonlinear equation of state, see (**author?**) Tailleux 2012.

4.3.3 EXAMPLE 3: LINK WITH APE DENSITY APPROACH Finally, an obvious choice motivated by the construction of the APE density is $e_h^* = h - Q(\eta, S)$, with $Q(\eta, S)$ defined from Lorenz's reference state as explained in Section 2.3, for which the generation term becomes:

$$\rho G_h^* = \rho \left[(T - \tilde{T}) \frac{D\eta}{Dt} + (\mu - \tilde{\mu}) \frac{DS}{Dt} \right] + \frac{\partial P}{\partial t}. \quad (35)$$

Interestingly, even though e_h^* is neither sign positive definite nor vanishing for Lorenz reference state, G_h^* is nevertheless very close to G_{ex} (Eq. 23) from which it differs only in the pressure-dependent term. As a result, it has a similar predictive power as G_{ex} with regard to anticipating the local production of kinetic energy by diabatic effects.

4.4 APE density in incompressible Boussinesq fluids

As seen above, the APE density for a compressible stratified fluid appears be rooted in the convexity of internal energy and hence fundamentally a thermodynamic quantity (without being a function of state though). This is intriguing, because atmospheric and oceanic motions are often regarded as well described by the incompressible Boussinesq approximation, for which the globally-defined APE is usually defined only in terms of GPE, viz.,

$$APE = \int_V \rho g(z - z_R(\mathbf{x}, t)) dV. \quad (36)$$

Although it might be tempting to conclude that APE is of a different nature in a Boussinesq fluid, it is important to realize that the nature of a quantity should not depend on the kind of approximations used to describe it. To reassure oneself that APE is indeed a thermodynamic quantity even in Boussinesq fluids, however, one needs to invoke (**author?**) Holliday and McIntyre 1981's results,

which demonstrate that Eq. (36) can actually be regarded as the volume-integral of the following sign positive definite APE density E_a ,

$$E_a(\mathbf{x}, t) = \rho g (z - z_R(\mathbf{x}, t)) + P_R(z, t) - P_R(z_R(\mathbf{x}, t), t), \quad (37)$$

where $P_R(z, t)$ is the reference pressure in hydrostatic balance with the reference density $\rho_R(z, t)$. By using the property that $\rho(z, t) = \rho_R(z_R, t)$, with defines the reference depth $z_R(\mathbf{x}, t)$, Eq. (37) can be shown to be equivalent to:

$$E_a = - \int_0^\zeta g \tilde{\zeta} \rho'_R (z - \tilde{\zeta}) d\tilde{\zeta} \approx \rho_0 N_R^2 \frac{\zeta^2}{2}, \quad (38)$$

where $N_R^2 = -(g/\rho_0)\rho'_R$ is the squared Brunt-Vaisala frequency. Under this form, Eq. (38) is identical to the entropy part of **(author?)** Andrews 1981's APE density, and no longer bears any similarity to the integrand of Eq. (36).

There appears to be a growing interest for using the APE density as a diagnostic tool to get insights into the stratified processes. Thus, **(author?)** Scotti et al. 2006 extended **(author?)** Holliday and McIntyre 1981's framework to allow for an arbitrary reference state and for diabatic effects in the context of the study of internal waves, which was also discussed by **(author?)** Lamb 2007, Lamb 2010, with **(author?)** Kang and Fringer 2010 reviewing a number of approaches to defining a local APE density. See also **(author?)** Rouillet and Klein 2009 and **(author?)** Molemaker and McWilliams 2010 for studies of turbulent stratified flows.

5 Issues in the study of the global atmospheric energy cycle

5.1 The 2-component energy cycle and the second law

The simplest way to describe the atmospheric energy cycle is in terms of the two-component energy cycle given in the introduction. For a steady-state, the

net APE generation rate G must balance the total viscous dissipation D , viz.,

$$G = \int_V \left(\frac{T - T_R}{T} \right) \dot{q} dm = D. \quad (39)$$

Eq. (39) is reminiscent of the classical Carnot formula for a reversible heat engine, if D is identified with the “useful work”. **(author?)** Ozawa et al. 2003 argued that Lorenz’s APE theory is equivalent to the second law of thermodynamics, as if one approximates T_R as constant and neglects the viscous contribution to \dot{q} , then Eq. (39) reduces to:

$$-T_r \int_V \frac{\dot{q}}{T} dm = D \quad (40)$$

since $\int_V \dot{q} dm = 0$ in a steady-state. Eq. (40) is indeed equivalent to the second law, because the entropy budget can be written as:

$$\int_V \frac{\dot{q}}{T} dm + \int_V \frac{\varepsilon_K}{T} dm = 0 \quad (41)$$

provided that \dot{q} excludes viscous heating, and that T_r be defined as the “dissipation” temperature $1/T_r = \int_V (\varepsilon_K/T) dm / \int_V \varepsilon_K dm$. In this review, however, we insist that Lorenz’s use of a z -dependent reference temperature profile T_R is key to his approach, and that the use of a constant T_R pertains to exergy theory, not APE. The connection between APE and the second law is important, as it pertains to the debate about whether the atmosphere obeys some extremum principle, with Lorenz speculating that G is maximised, whereas **(author?)** Paltridge 1975 argues that entropy production is maximised, see **(author?)** Lucarini 2009, Pascale et al. 2011, Pascale et al. 2012 for review and further discussion.

5.2 4-component energy cycle and waves/mean flow interactions

A key issue in the theory of the large-scale atmospheric circulation is to understand the interactions between the large-scale motions and the strong eddying and wave motions making up most of the weather. Owing to the zonal symmetry of the atmosphere, it has been common to approach the problem by splitting the mean flow into a zonal mean and eddy components. Such a decomposition can be applied to Lorenz's energy cycle, which leads to the 4-component energy cycle:

$$\frac{\partial A_Z}{\partial t} = -C(A_Z, K_Z) - C(A_Z, A_E) + G_Z, \quad (42)$$

$$\frac{\partial A_E}{\partial t} = -C(A_E, K_E) + C(A_Z, A_E) + G_E, \quad (43)$$

$$\frac{\partial K_Z}{\partial t} = C(A_Z, K_Z) - C(K_Z, K_E) - D_Z, \quad (44)$$

$$\frac{\partial K_E}{\partial t} = C(A_E, K_E) - C(K_Z, K_E) - D_E, \quad (45)$$

where the suffixes Z and E are used to distinguish between a zonal mean and the eddy part respectively, e.g., (Peixoto and Oort 1992). For observational studies of the atmospheric energy cycle, see (author?) Oort and Peixoto 1974, Oort 1983, Oort et al. 1989, Peixoto and Oort 1974.

Following progress in the understanding of wave/mean flow interactions by (author?) Andrews and McIntyre 1976, Andrews and McIntyre 1978a, Andrews and McIntyre 1978b, for instance, (author?) Plumb 1983 and (author?) Kansawa 1984 proposed to reformulate the above 4-component energy cycle in terms of the Transformed-Eulerian mean theory, which dramatically impacts on the form of the various energy conversions, as does the use of different systems of vertical coordinates, e.g., (author?) Bleck 1985. Although there have been many attempts at reformulating Lorenz's 4-component energy cycle using various wave/mean flow decomposition and vertical coordinates system, e.g., (Hayashi 1987, Iwasaki 2001,

Uno and Iwasaki 2006, Murakami 2011), these remain complex and technical, so that in practice, the appealing simplicity of Lorenz's 4-component energy cycle makes it the framework of choice for analysing observations (Li et al. 2007), or for assessing the performances of climate models (Boer and Lambert 2008).

5.3 Role of moisture and of conditional instability

In the classical description of the atmospheric energy cycle, the atmosphere is usually treated as a dry gas, with moisture being assumed to enter the problem only as an additional diabatic term due to latent heat release. From the viewpoint of APE theory, however, this is not satisfactory, because the stability properties of dry and moist air are fundamentally different. For dry air, one may easily construct Lorenz's reference state by sorting the air parcels by ascending potential temperature θ , which uniquely determines their relative buoyancy. For moist air, however, the relative buoyancies of two moist air parcels with different water mixing ratio depends on the pressure at which they are evaluated. This makes it possible for moist air parcels to be stable for small displacements, but unstable to sufficiently large ones that cause the parcels to condensate and release latent heat, making them more buoyant than their environment. This conditional instability requires for the parcels to overcome some energy barrier, called convective inhibition (CIN), before they can freely ascend from their level of free convection (LFC) to their level of neutral buoyancy (LNB). The positive work released by a moist air parcel as it goes from its LFC to LNB is called the Convective Available Potential Energy (CAPE) (Emmanuel 1994, Renno and Ingersoll 1996). Although they are widely used, the concepts of CIN and CAPE are plagued with many conceptual difficulties; since they are based on the so-called parcel method, their

values may sensitively depend on the particular moist air parcel chosen to do the computation (de la Torre et al. 2004). More importantly, however, they fail to account for the negative work of buoyancy forces associated with subsiding motions induced by the ascent of buoyant moist air parcels of finite mass.

From the viewpoint of APE theory, the presence of moisture makes it possible for a purely barotropic conditionally unstable atmosphere to possess more than one local potential energy minimum in the space of all possible adiabatic rearrangements of the air parcels. (author?) Lorenz 1978, Lorenz 1979 used the term “Moist Available Energy” (MAE) to refer to the vertical component of APE associated with conditional instability. (author?) Randall and Wang 1992 developed a similar concept, which they called Generalised CAPE (GCAPE), which they used to build a new parameterisation of deep cumulus convection in (author?) Wang and Randall 1996. (author?) Emmanuel 1994 derived an interesting relation between GCAPE and CAPE in the case of an atmospheric sounding where all the boundary layer air parcels have the same CAPE. (author?) Tailleux and Grandpeix 2004 investigated the possibility of defining a generalised CIN, and found evidence for some atmospheric soundings of the Southern Great Plains of multiple reference states.

Recently, (author?) Pauluis 2007 sought to provide a rigorous generalisation of Lorenz’s dry APE framework to a moist atmosphere, by treating moist air as a binary fluid in local thermodynamic equilibrium, allowing for reference states to evolve discontinuously from shallow to deep convective states. Previously, (author?) Pauluis and Held 2002a, Pauluis and Held 2002b had discussed how moist processes could contribute to irreversible entropy production in the atmosphere. By estimating the APE generation associated with idealised atmospheric

processes, (author?) Pauluis 2007 concludes that the net APE production from processes contributing positively to APE generation appears to greatly exceed all current estimates for the total viscous dissipation. As a result, irreversible APE dissipation processes must exist to make up for the difference. (author?) Pauluis 2007's framework is an important advance that needs to be further pursued to achieve a better understanding of how the net APE generation rate can be separated into a net APE production and dissipation rates.

6 Available energetics and the ocean energy cycle

6.1 Classical view of the ocean energy cycle

Lorenz APE theory was eventually adapted to the study of the oceanic energy cycle by (author?) Bryan and Lewis 1979 and others, with the two-component energy cycle usually expressed as follows:

$$\frac{dK}{dt} = C(A, K) + G_K - D_K, \quad (46)$$

$$\frac{dA}{dt} = -C(A, K) + G_A - D_A, \quad (47)$$

where G_K denotes the mechanical power input by the wind, and D_K the total viscous dissipation. (author?) Roquet and Wunsch 2011 recently reviewed different ways to compute G_K , which is widely estimated to be $G_K = O(1 \text{ TW})$. Owing to the importance of the surface in the oceans, the APE generation is commonly decomposed into a net APE production term G_A by the surface buoyancy fluxes and a net dissipation term D_A due to turbulent molecular diffusive fluxes of heat and salt. (author?) Oort et al. 1989, Oort et al. 1994 estimated $G_A = O(1.2 \pm 0.7 \text{ TW})$, and concluded that the power input due to the wind and surface buoyancy fluxes were comparable.

6.2 Controversy about the sign of $C(A, K)$

While in the atmosphere, there is no choice but for $C(A, K)$ to be strictly positive in order to balance $D_K > 0$, this is not so in the oceans where $C(A, K)$ can in principle take on both signs. Since in a steady state, one must have $C(A, K) = G_A - D_A$, the sign of $C(A, K)$ depends crucially on the magnitude of the APE dissipation. Postulating D_A to be negligible, (author?) Peixoto and Oort 1992 assumed $C(A, K) \approx G_A > 0$, but this result conflicts with the results of ocean general circulation models (OGCMs) in which $C(A, K)$ is systematically found to be negative if a realistic geometry is used, as first shown by (author?) Toggweiler and Samuels 1998 and (author?) Gnanadesikan et al. 2005 in ocean-only models, and by (author?) Gregory and Tailleux 2011 in a range of fully coupled climate models. (author?) Toggweiler and Samuels 1998 found, however, that $C(A, K) > 0$ in an idealised ocean sector geometry with no ACC.

Physically, the reason why $C(A, K)$ can be negative in the oceans is because APE can also be created adiabatically by the wind, in addition to the usual diabatic creation by surface buoyancy fluxes, as illustrated in Fig. 4. Because a negative $C(A, K)$ implies a negative APE generation rate $G_A - D_A$, many authors have argued that the oceans should not be regarded as a heat engine. (author?) Tailleux 2010 argued, however, that what matters is that the APE production G_A by the surface buoyancy fluxes is large and positive. Indeed, it follows from Fig. 4 that a negative $C(A, K)$ only implies that the adiabatic creation of APE by the wind dominates over the diabatic creation by surface buoyancy fluxes, not that the surface buoyancy fluxes are a negligible power source, as is often believed. To illustrate this point, (author?) Gregory and Tailleux 2011 suggested to diagnose the local vertically-integrated value of $C(A, K)$ as a way to locally discrimi-

nate between predominantly wind-driven regions from predominantly buoyancy-driven ones. In a hydrostatic primitive Boussinesq equation model, the vertically-integrated *APE* to *KE* conversion is given by $-\int \mathbf{u} \cdot \nabla_h P dz$, where \mathbf{u}_h and $\nabla_h P$ are the horizontal velocity and pressure gradient respectively, and is illustrated in Fig. 5 for two different coupled climate models. According to this diagnosis, buoyancy-driven regions are found to be primarily linked with western boundary currents, which play a crucial role for the AMOC (Sijp et al. 2012), and regions traditionally associated with deep water formation, whereas the primary region of large wind power input appears to be over the Antarctic Circumpolar Current (ACC) area, in agreement with theoretical expectations.

6.3 Remaining challenges and puzzles

The ocean energy cycle remains plagued with large uncertainties for a number of practical and fundamental reasons. One potentially important source of error stems from the use of the QG-approximation of APE used by (**author?**) Oort et al. 1994, which has long been known in the atmospheric literature (Dutton and Johnson 1967, Taylor 1979) to potentially seriously underestimate both the APE and APE production rate. Such approximation also neglects the internal energy component AIE of APE, which (**author?**) Reid et al. 1981 had previously found to be negative and estimated to only account for 10-20 percent of the total APE. By using the exact definition of APE, (**author?**) Huang 1998 confirmed that the QG approximation seriously underestimates APE in the oceans, but found that AIE may actually account for up to 40% of the total APE.

In exact APE theory, however, it is important to recognise that owing to the binary character of seawater, the relative densities of the fluid parcels depend on

the particular pressure at which they are evaluated. Several reference states may therefore exist, which are each associated with a local potential energy minimum and separated from each other by some energy barrier. (author?) Huang 2005's computation assumes that APE must be defined for the reference state achieving the absolute potential energy minimum, but it may be more physical to chose a reference state closer to the actual state, since the actual state may not possess sufficient excess energy in practice to overcome one or several of the abovementioned energy barriers. These ideas remain poorly understood, however, and their consequences for our understanding of the APE production and dissipation rates remain to be elucidated.

7 APE and irreversible mixing in turbulent stratified fluids

7.1 Evolution of the reference state and irreversible mixing

Since by construction, the reference state can only be affected by diabatic effects, (author?) Winters et al. 1995 suggested that monitoring its temporal evolution could be used to rigorously quantify irreversible diffusive mixing in turbulent stratified fluids. Thus, for a Boussinesq fluid whose density obeys the simple diffusive law $D_t\rho = \kappa\nabla^2\rho$, with κ the molecular diffusivity, (author?) Winters et al. 1995 show that the background potential energy GPE_r in a closed domain evolves in time according to:

$$\frac{dGPE_r}{dt} = \underbrace{\int_S gz_r \kappa \nabla \rho \cdot \mathbf{n} dS}_{W_{r,buoyancy}} - \underbrace{\int_V \frac{\kappa g \|\nabla \rho\|^2}{\partial \rho_r / \partial z_r} dV}_{W_{r,mixing}}, \quad (48)$$

where $z_r = z_r(\rho, t)$ is the position that a parcel of density ρ would occupy in the reference state. Eq. (48) states that GPE_r is affected by diabatic effects due to surface buoyancy fluxes GPE_r ($W_{r,buoyancy}$) and irreversible turbulent mixing

($W_{r,mixing}$). The form of $W_{r,mixing}$ is interesting, because it provides a rigorous definition of the effective turbulent diapycnal diffusivity as:

$$K_\rho = \kappa \left\langle \frac{\|\nabla\rho\|^2}{(\partial\rho_r/\partial z_r)^2} \right\rangle, \quad (49)$$

with $\langle.\rangle$ an average over constant ρ surfaces, which is similar to widely employed (**author?**) Osborn and Cox 1972 model. Eqs. (48) and (49) provide a far more superior approach for inferring K_ρ in direct numerical simulations of stratified turbulence than that based on inferring it from the highly noisy density flux $\overline{\rho'w'}$ (Staquet 2000, Caulfield and Peltier, Staquet 2001, Peltier and Caulfield 2003). Physically, the temporal evolution of ρ_r is closely related to that of its probability density function, as discussed by (**author?**) Tseng and Ferziger 2001, which can be used to design algorithms for estimating the reference state that are faster than sorting.

The knowledge of the temporal evolution of GPE_r determines that of $APE = GPE - GPE_r$. It is of interest to derive the local form of its evolution from taking the time derivative of (**author?**) Holliday and McIntyre 1981's APE density (Eq. (37):

$$\frac{DE_a}{Dt} = (\rho - \rho_r)gw + \nabla \cdot [\kappa g(z - z_r)\nabla\rho] - \rho_0\varepsilon_P + \mathcal{N.L.T.} \quad (50)$$

where

$$\rho_0\varepsilon_P = -\frac{\kappa g\|\nabla\rho\|^2}{\partial\rho_r/\partial z} + \kappa g\frac{\partial\rho}{\partial z}, \quad (51)$$

$$\mathcal{N.L.T.} = \frac{\partial}{\partial t} [P_r(z, t) - P_r(z_r, t)]. \quad (52)$$

Eq. (51) defines the local APE dissipation rate ε_P , which is consistent with the form traditionally used to compute the dissipation ratio $\varepsilon_P/\varepsilon_K$ in stratified turbulence (Oakey 1982) and the mixing efficiency. It is important to point

out that the exact local APE evolution equation also possesses an additional nonlocal term (52) controlled by turbulent molecular diffusion that is lacking in traditional local energetics. (author?) Scotti et al. 2006 and (author?) Molemaker and McWilliams 2010 suggest that this term may sometimes be positive and larger than $\rho_0 \varepsilon_P$, resulting in turbulent mixing acting locally as a net source of APE. This demonstrates the importance of using the local budget of APE density to identify all possible local sources/sinks of APE, as some of these are filtered out by traditional global APE approaches. (author?) Tailleux 2009 discussed a possible extension of (author?) Winters et al. 1995 to a fully compressible stratified fluid, but further work is needed to extend the framework to a binary fluid for instance.

7.2 Energetics of mechanically-stirred horizontal convection

Although the oceans are nonlinear, oceanographers have nevertheless historically sought to rationalise the large-scale ocean circulation as the superposition of a predominantly horizontal wind-driven circulation and of a buoyancy-driven Atlantic meridional overturning circulation (AMOC). The classical view on the AMOC is that high-latitude cooling and its concomitant deep water formation must be counteracted by the downward diffusion of heat by turbulent diapycnal mixing if a steady-state is to be achieved. However, because turbulent mixing is presumably driven principally by the mechanical stirring due to the wind and tides in the oceans, (author?) Munk and Wunsch 1998 and (author?) Huang 1999, among others, went as far as challenging the validity of regarding the AMOC as buoyancy-driven. This idea received subsequently wide support, as it appeared to agree with a widespread interpretation of (author?)

Sandstrom 1908's theorem that the surface buoyancy fluxes should be regarded as a negligible source of power and hence of mechanical stirring in the oceans, even though this contradicts (author?) Oort et al. 1994's conclusion that the buoyancy power input is comparable to that of the wind. As a result, the hypothesis was formulated that the AMOC and its associated meridional heat transport would be negligible in absence of wind and tides. The laboratory experiments by (author?) Whitehead and Wang 2008 provided experimental support for the idea that mechanical stirring could indeed significantly enhance buoyancy-driven horizontal convection.

The above ideas prompted a renewal of interest in studies of ocean energetics (Wunsch and Ferrari 2004, Kuhlbrodt et al. 2007), which all disregarded $G(APE)$ as the relevant measure of the buoyancy power input; instead, the focus shifted to the *net* APE generation rate $G(APE) - D(APE)$ following (author?) Paparella and Young 2002 (although this was not initially recognised), which (author?) Wang and Huang 2005 estimated to be $O(15 \text{ GW})$, a very small number compared to $G(APE)$. (author?) Tailleux 2010 argued, however, that it is no more valid to regard $G(APE) - D(APE)$ as the buoyancy power input as it is to regard $G(KE) - D(KE)$ as the wind power input. (author?) Hughes et al 2009 and (author?) Tailleux 2009 both insist that $G(APE)$ is the relevant measure of the buoyancy power input in the oceans, and that it is incorrect to infer from (author?) Sandstrom 1908's study that the surface buoyancy fluxes is a negligible source of power. Note also that (author?) Coman et al. 2006 could not reproduce (author?) Sandstrom 1908's results.

(author?) Tailleux and Rouleau 2010 departed from previous studies by pointing out that APE theory naturally accounts for the possibility of a mechanically-

controlled buoyancy-driven AMOC, given that the value of $G(APE)$ depends on the reference state, and hence on any physical processes, such as wind- and tidal-driven turbulent diapycnal mixing, affecting the oceanic stratification. This idea was further made explicit in the context of a wind and buoyancy-driven Boussinesq ocean with a linear equation of state, through the formula:

$$G(APE) = \frac{\gamma}{1 - \gamma} G(KE) + \frac{W_{r,laminar}}{1 - \gamma}, \quad (53)$$

which demonstrates the inter-dependence of the mechanical power inputs by the wind and buoyancy forcing. In Eq. (53), $\gamma = D(APE)/[D(APE) + D(KE)]$ is the bulk mixing efficiency of the oceans, which measures the relative importance of the diffusive dissipation of APE versus to the total dissipation, while $W_{r,laminar}$ can be regarded as the net APE generation rate, which (author?) Wang and Huang 2005 estimated to be $W_{r,laminar} = O(15 \text{ W})$.

(author?) Tailleux and Rouleau 2010 argue that owing to the importance of the wind and tides, the last term in the r.h.s of Eq. (53) is negligible, allowing the formula to be inverted to predict that $\gamma \approx G(APE)/[G(APE) + G(KE)]$. With $G(APE) \approx 0.5 \text{ TW}$, and $G(KE) \approx 1 \text{ TW}$, $\gamma \approx 0.33$, which is compatible with estimates of the mixing efficiency estimated from microstructure measurements, e.g., (Oakey 1982, Osborn 1980). In absence of mechanical forcing, the formula predicts that

$$G(APE) = \frac{W_{r,laminar}}{1 - \gamma}. \quad (54)$$

If γ could be regarded as a universal constant close to 20%, as seems to be often assumed in oceanography, Eq. (54) would strongly support the idea that $G(APE)$ would be very small in the absence of the wind and tides. Theoretical and numerical studies reveal, however, that $\gamma \rightarrow 1$ as the Rayleigh number $R_a \rightarrow +\infty$ (Tailleux 2009, Scotti and White 2011). Thus, even though that $W_{r,laminar}$

may be very small, $G(APE)$ may still be comparatively much larger as $\gamma \rightarrow 1$, suggesting that it appears possible for a strong overturning and heat transport to exist even in the absence of wind and tides.

8 Summary points

1. Lorenz/Margules globally-defined APE theory admits a local formulation in terms of APE density/extended exergy, which represents the natural generalisation of classical thermodynamics theory of available energy to a turbulent stratified fluid.
2. Available energy in stratified fluids appears fundamentally related to the convexity of internal energy in the space of its natural variables. APE measures the distance to a reference state minimising potential energy at constant entropy, whereas exergy measures the distance to a reference state in thermodynamic and mechanical equilibrium.
3. Different approaches to available energetics exist, which mainly differ in the way they separate the part of enthalpy available for conversion into kinetic energy from the generation term for available energy. In the oceans, the surface buoyancy production term predicted by exergy theory is about two orders of magnitude larger than that predicted by APE theory, the latter appearing to be the most realistic.
4. The interpretation of energetics and energy conversions in turbulent stratified fluids depends sensitively on the nature of the system of coordinates employed, whether the Boussinesq approximation is used or not, and on the particular approach used to separate the mean and eddy fields.

5. The APE budget can be used to provide a rigorous quantification of irreversible turbulent mixing effects in stratified fluids.
6. Binary and multi-component fluids may admit multiple reference states associated with local potential energy minima and separated by each other by energy barriers, complicating the use of APE theory for such fluids.
7. In the atmosphere, APE production terms greatly exceed all known estimates of viscous dissipation, requiring large APE dissipation mechanisms believed to be associated with moist processes.
8. In the oceans, the wind and tidal forcing may control the APE production by surface buoyancy fluxes through their control of turbulent mixing and hence of the reference state, whereas the buoyancy forcing may partly control the wind power input through altering surface ocean velocities.

9 Future issues

1. Further work is needed to fully understand how the APE generation rate splits into a net production and dissipation terms in a moist atmosphere.
2. More generally, further work is need to fully understand how to rigorously apply APE theory for binary and multi-component fluids, the outstanding questions being: How to compute the different possible reference states? The energy barriers separating them? The separation between production and dissipation terms?
3. A more systematic investigation and evaluation of the different frameworks for available energetics would be useful to evaluate their predictive power in various typical circumstances.

4. Further work is needed to extend (**author?**) Winters et al. 1995's framework to compressible binary and multi-component fluids, which would potentially help in the understanding of double diffusive processes in the atmosphere and oceans.
5. Further work is needed to fully understand how to physically decompose the mean and eddy parts, and to achieve an unambiguous physical interpretation of the different energy conversions in terms of actual physical processes.
6. Although there has been some progress in extending APE theory to include non-resting states, e.g., (**author?**) Van Mighem 1956, Codoban and Shepherd 2003, Andrews 2006, Codoban and Shepherd 2006, which was beyond the scope of this review, further work appears to be needed to fully understand the general principles allowing to deal with arbitrary background mean flows.

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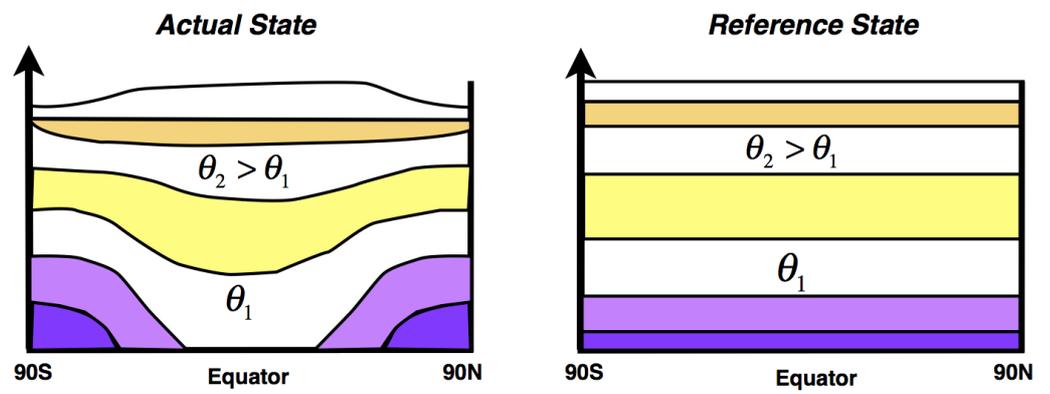


Figure 1: The Lorenz-Margules isentropic re-arrangement of mass for constructing the reference state (right panel) from the actual state (left panel).

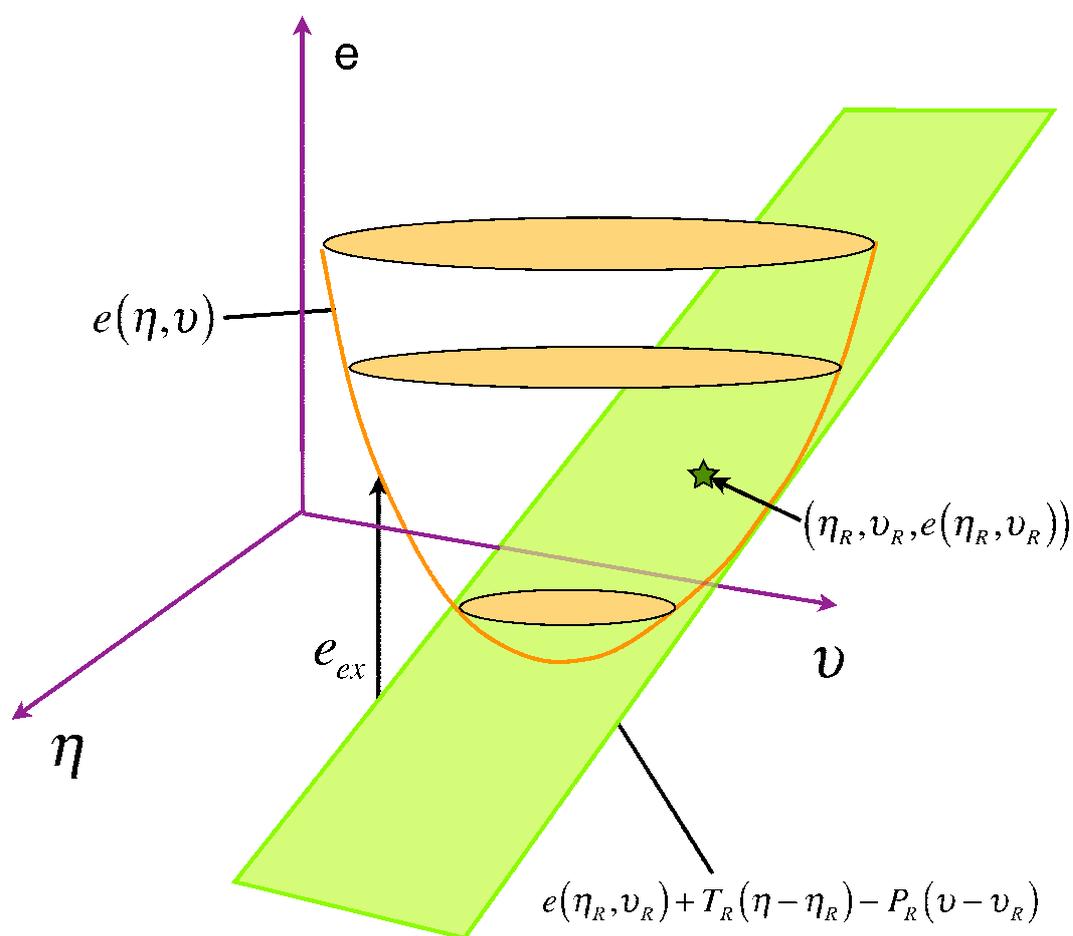


Figure 2: Schematics illustrating the convex character of the internal energy in the specific entropy/volume space. The exergy can be defined as the volume comprised between the internal energy surface and the tangent plane going through the reference value $(\eta_R, v_R, e(\eta_R, v_R))$.

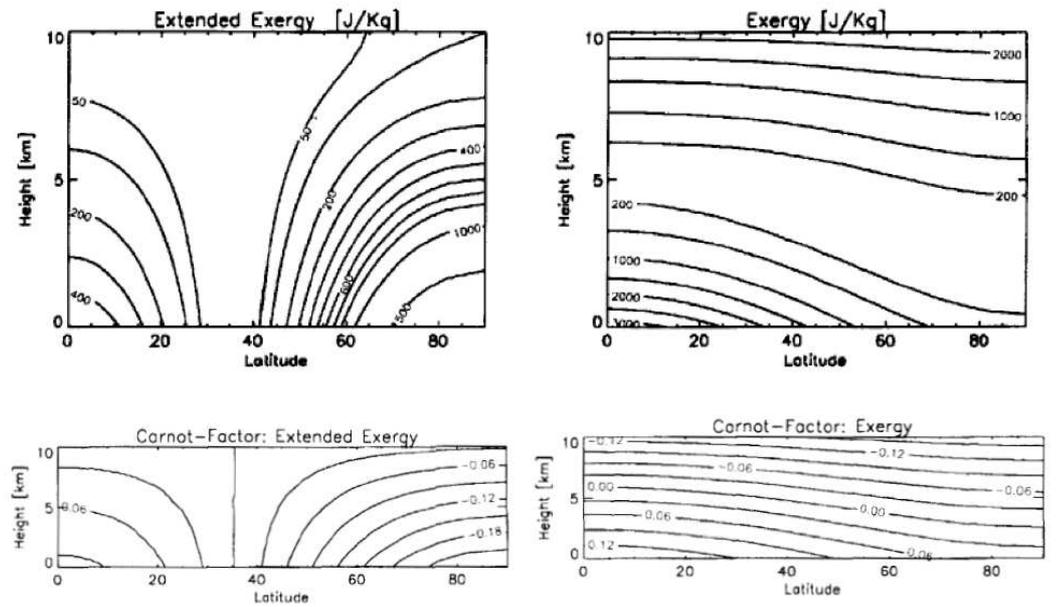


Figure 3: Difference between the extended energy (or APE density, or pseudo-energy) and classical exergy for an idealised atmospheric temperature distribution (top panels), and the corresponding thermodynamic efficiencies (bottom panel). Adapted from (author?) Kucharski 1997

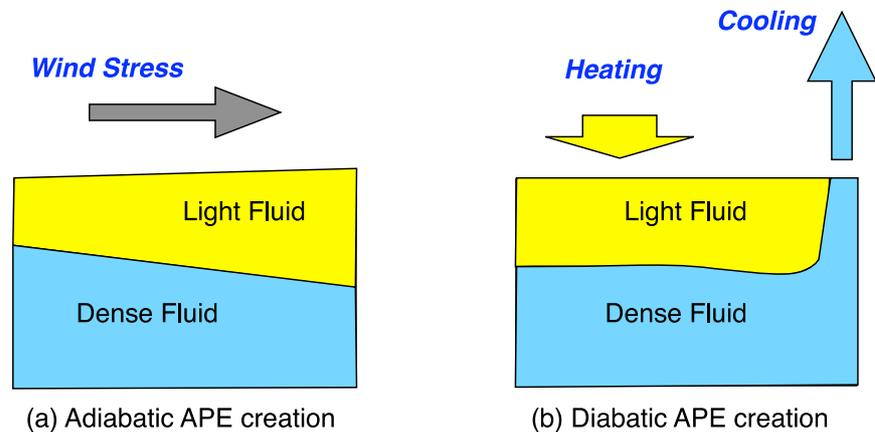


Figure 4: Schematics illustrating the two possible ways that APE can be created in a wind and buoyancy-driven ocean. Adiabatic creation (left panel), diabatic creation (right panel).

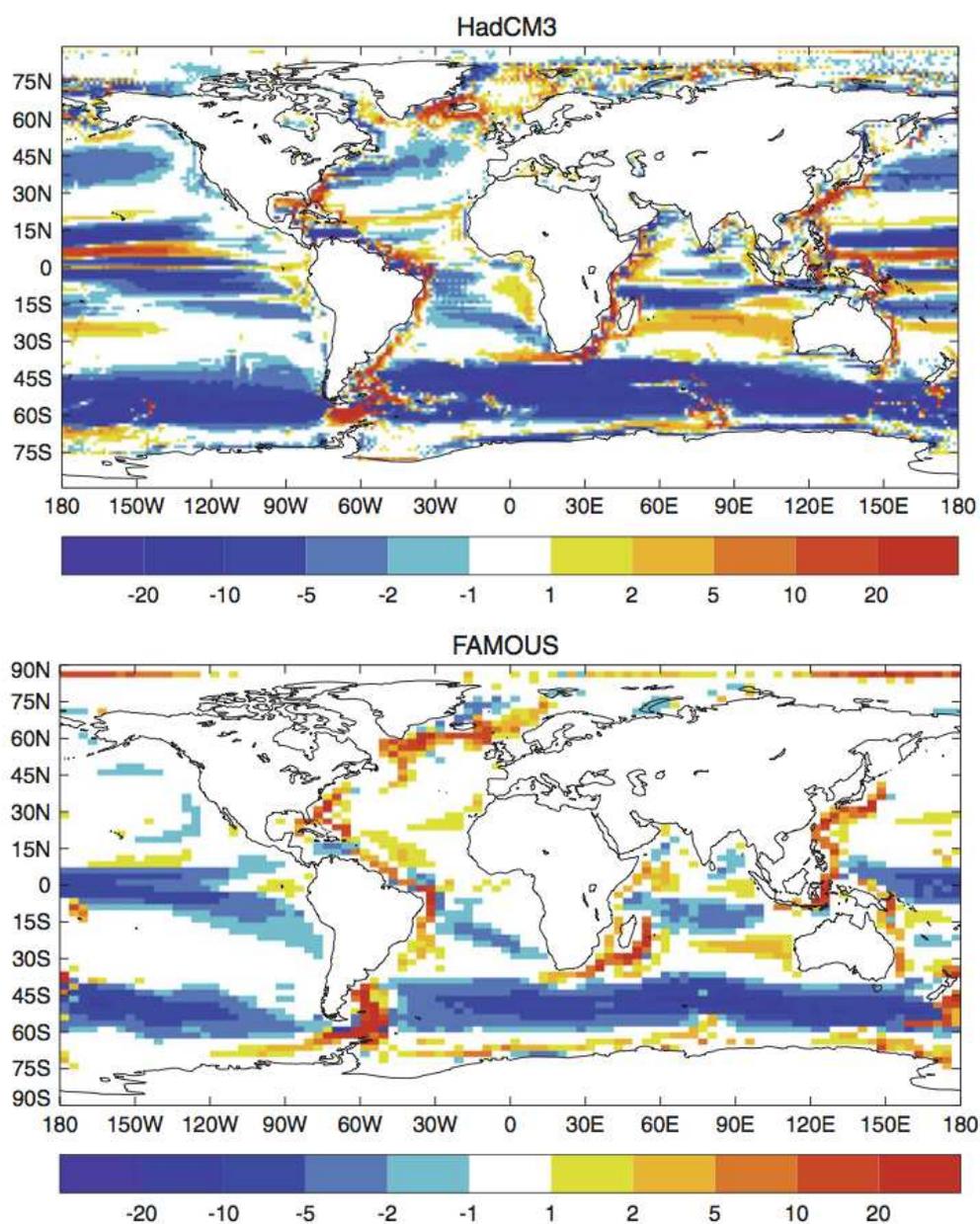


Figure 5: Vertical integral of the local APE to KE conversion $-\int \mathbf{u}_h \cdot \nabla_h P dz$ in $\text{mW}\cdot\text{m}^{-2}$ in the HadCM3 (top) and FAMOUS (bottom) coupled climate models.

Adapted from (author?) Gregory and Tailleux 2011.