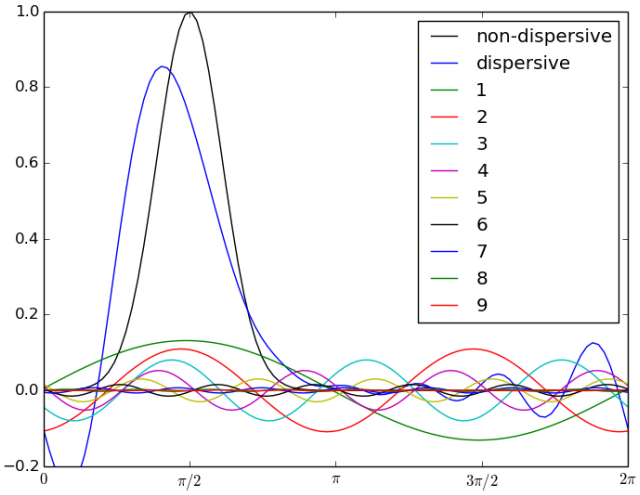
Waves and Dispersion

Hilary Weller <h.weller@reading.ac.uk>

13th October 2015

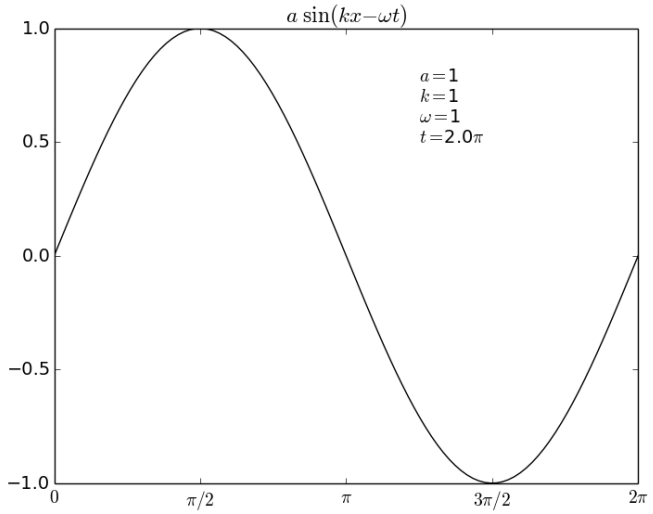


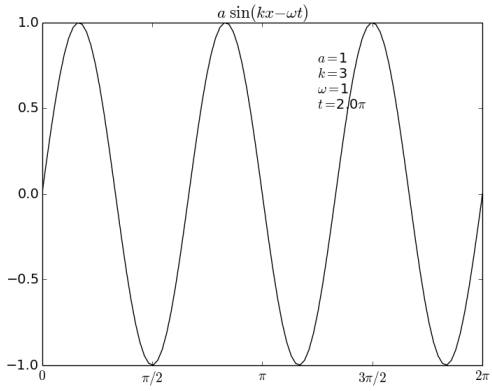
1 Some Background on Waves

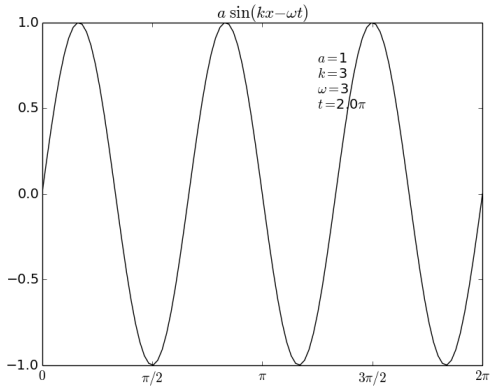
A travelling wave can be described by the equation

$$y = a\sin(kx - \omega t) \tag{1}$$

where $y(x,t)$ is the height of the wave at position x, time t
a is the amplitude of the wave
k is the wavenumber (number of whole waves between 0 and 2π)
ω is the angular wave frequency – the number of complete oscillations in time 2π at a fixed point



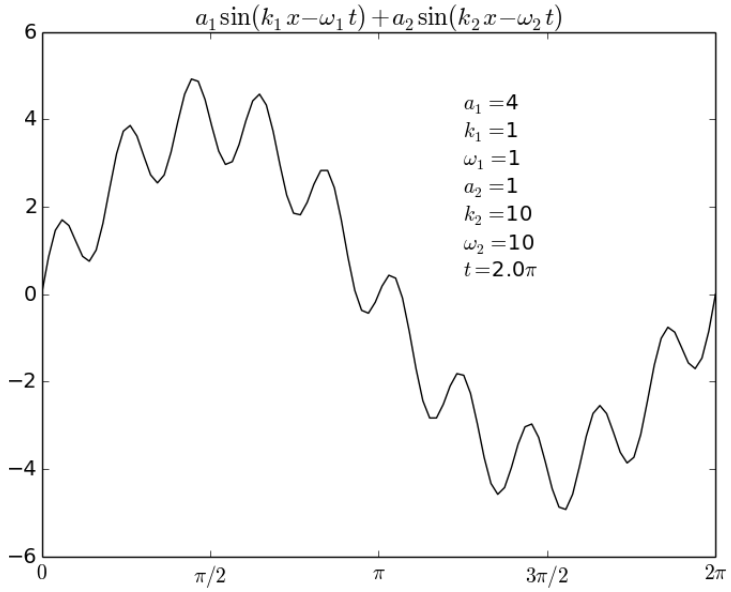




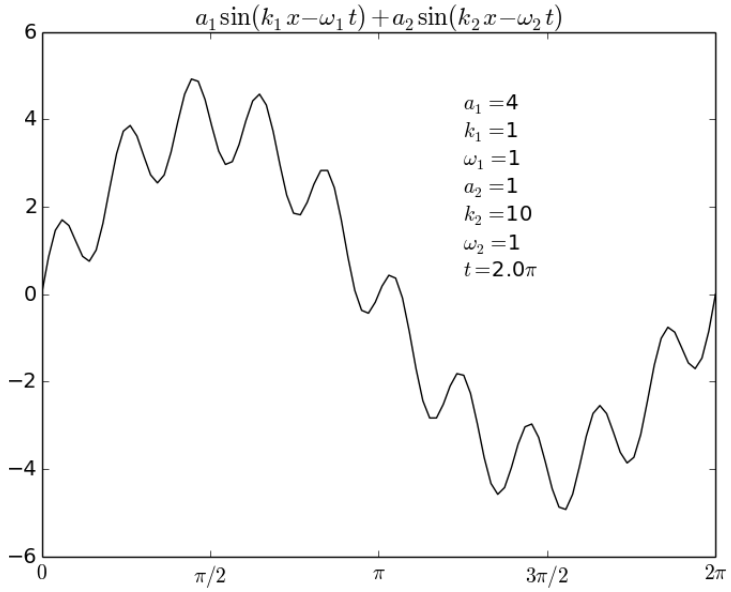
Exercise:
Write down an expression for the wave length, λ, and the wave speed, u in terms of k and ω

2 Dispersion

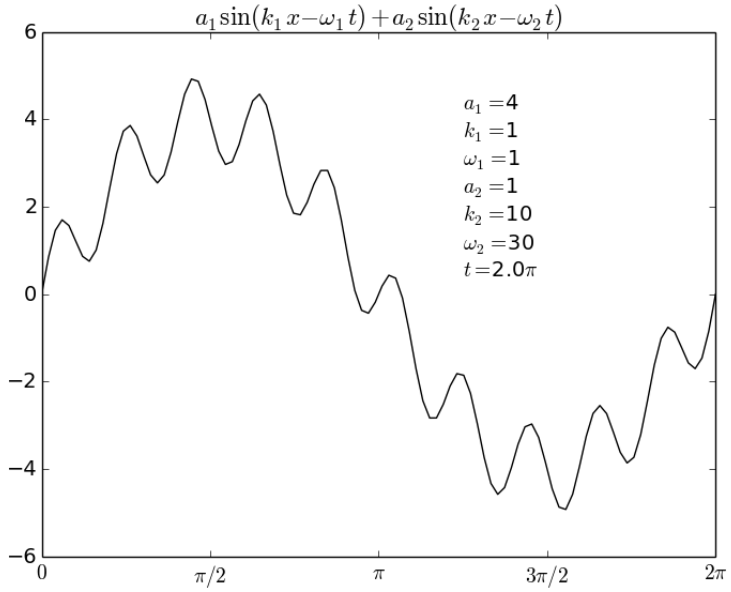
Dispersion occurs when waves of different frequencies propagate at different speeds.



- $u = \dfrac{\omega_1}{k_1} = \dfrac{\omega_2}{k_2}$ so wave of both wavelengths propagate at the same speed
- No dispersion occurs – the function does not change shape

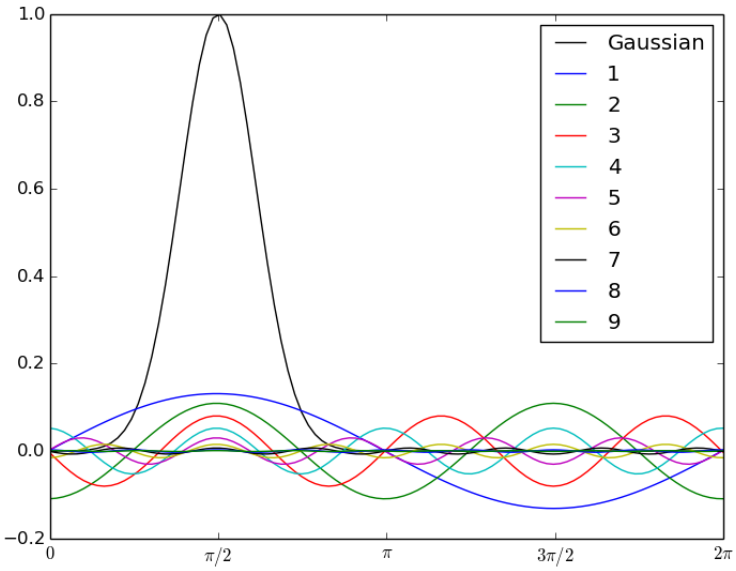


- $u_1 = \dfrac{\omega_1}{k_1} = 10\dfrac{\omega_2}{k_2}$ so the short wavelength waves are much slower than the long waves
- Dispersion occurs – the function changes shape

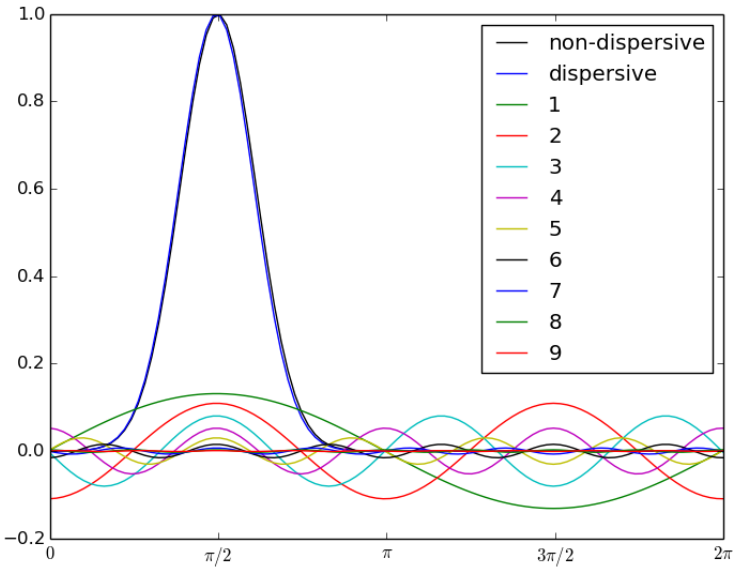


- $u_1 = \dfrac{\omega_1}{k_1} = \dfrac{1}{3}\dfrac{\omega_2}{k_2}$ so the short wavelength waves travel more quickly
- Dispersion occurs – the function changes shape

For a function to propagate without changing shape, all of the Fourier modes must propagate at the same speed:



Propagation of a Gaussian of high wavenumber waves propagate more slowly



3 Some Excellent Animations and Videos which demonstrate Dispersion

- Some description and animations from Dr. Dan Russell, Grad. Prog. Acoustics, Penn State:
  `http://www.acs.psu.edu/drussell/Demos/Dispersion/dispersion.html`
- A video of the wake of a motor-boat from JNHeyman (41 seconds):
  `https://www.youtube.com/watch?v=lWi_KpBy8kU`
- A video of ripples in a pond with descriptions of the dispersion (2:28)
  `https://www.youtube.com/watch?v=dESm6VjfSNs`