For the linear stability analysis of split-explicit methods we consider the linear system of equations

\[ \frac{\partial y}{\partial t} = -c_s y + L y \]

with \( c_s \gg 1 \). Using central differences for the fast part and the third-order upwind scheme for the advection part together with the von Neumann stability ansatz leads to the simplified ODE

\[ \frac{d^2 y}{dt^2} + L y = N y \]

with

\[ L = \frac{1}{\Delta t^2} \left( 6 \tan(\varepsilon) \Delta x / h \right) \begin{pmatrix} 1 & \varepsilon \\ -\varepsilon & 1 \end{pmatrix} \]

\[ N = \frac{1}{\Delta t^2} \left( \cos(4\varepsilon) \Delta x / k - 4 \cos(2\varepsilon) \Delta x / k + 3 + i(4 \sin(4\varepsilon) \Delta x / k) + 8 \sin(2\varepsilon) \Delta x / k \right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

where \( L \) represents the fast part and \( N \) the slow modes. The eigenvalues of the fast part are purely imaginary while the eigenvalues of the slow part have small real parts.

**Application to the Compressible Euler Equations**

We applied the split-explicit methods to the 2D compressible Euler equations in flux form. We use a finite volume spatial discretisation on an Arakawa C grid, so the winds are defined on the cell edges while all scalar variables are defined at the cell centers. We implemented the model in MATLAB with block-static adaptive grids and cut cells for the representation of topography. The fast modes can be integrated with forward-backward Euler, Størmer-Verlet and with the trapezoidal rule. The equations are:

\[ \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( V_x f(x,y,t) \right) - \frac{\partial}{\partial y} \left( V_y f(x,y,t) \right) \]

\[ \frac{\partial V_x}{\partial t} = \frac{\partial}{\partial x} \left( u f(x,y,t) \right) - \frac{\partial}{\partial y} \left( v f(x,y,t) \right) \]

\[ \frac{\partial V_y}{\partial t} = \frac{\partial}{\partial x} \left( v f(x,y,t) \right) - \frac{\partial}{\partial y} \left( u f(x,y,t) \right) \]

Here \( u \) and \( v \) are the horizontal and vertical winds, \( p \) is the density, \( \rho \) the pressure, \( \theta \) the potential temperature and \( g \) the acceleration of gravity. The prognostic variables are \( u, v, p, \rho \) and \( \theta \), the pressure \( p \) is given diagnostically by the equation of state.

**Results**

We apply the methods to 2 standard tests: For the rising bubble the initial atmosphere is adiabatically stratified with the exception of a perturbation of the potential temperature which is up to 2 degrees warmer than the surrounding air and causes the bubble to rise. For the density current test the bubble is up to 15 degrees colder than the surrounding air so that it will sink and after crashing the ground several eddies will form. To make the tests more stringent a horizontal background wind (from the left) of 20m/s \(^{-1}\) is added. For the rising bubble test the horizontal resolution is 125m and the third-order upwind scheme is used while for the density current test the resolution is 200m and the fifth-order upwind scheme is used. The maximal (horizontal) wind speeds that occur are 28m/s \(^{-1}\) for the first test respectively 50m/s \(^{-1}\) for the second test. Because the maximal CFL numbers of the advection schemes are 1.7 (third-order) respectively 1.4 (fifth-order) the time step restrictions from advection are \( \Delta t \leq 7.5 \Delta t_{\text{step}} \) (rising bubble) respectively \( \Delta t \leq 5.6 \Delta t_{\text{step}} \) (density current).

**Bibliography**

