Prerequisites and Effects of Multi-Scale and Adaptive Numerical Methods in the Geosciences: A Perspective from Tsunami Simulation

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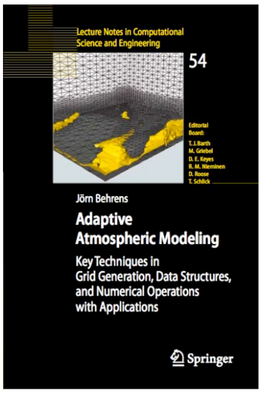
Road Map

In 2006 Book: Road map for 5 years

- Consistent Numerical Methods
- Refinement Criteria
- Sub-Grid Parameterization
- Computational Efficiency
- Realistic and Useful Applications
- ...



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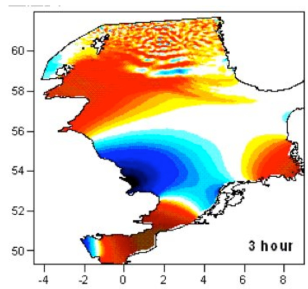
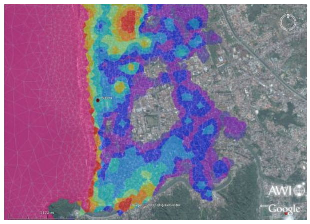
Multiple Scales

Spatial Scales:

- Tides:
  - Ocean-wide/ global range
  - O(10000) km
- Tsunami Wave Propagation:
  - Typical Wave Length
  - O(100) km
- Near-shore waves:
  - Shoaling effect
  - O(1-10) km
- Inundation (Parameterized):
  - No resolution of buildings, etc.
  - O(0.1) km





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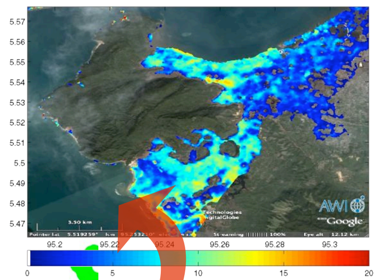
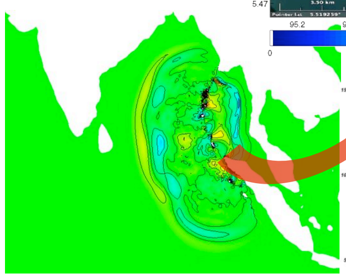
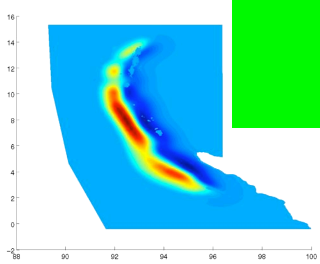
Simulation overview

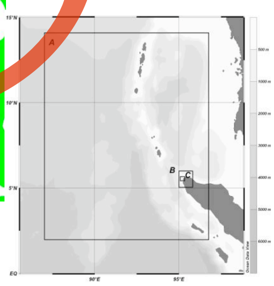
Simulation chain:

- Source
- Propagation
- Inundation









Nested Mesh to bridge the gap

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Simulation overview

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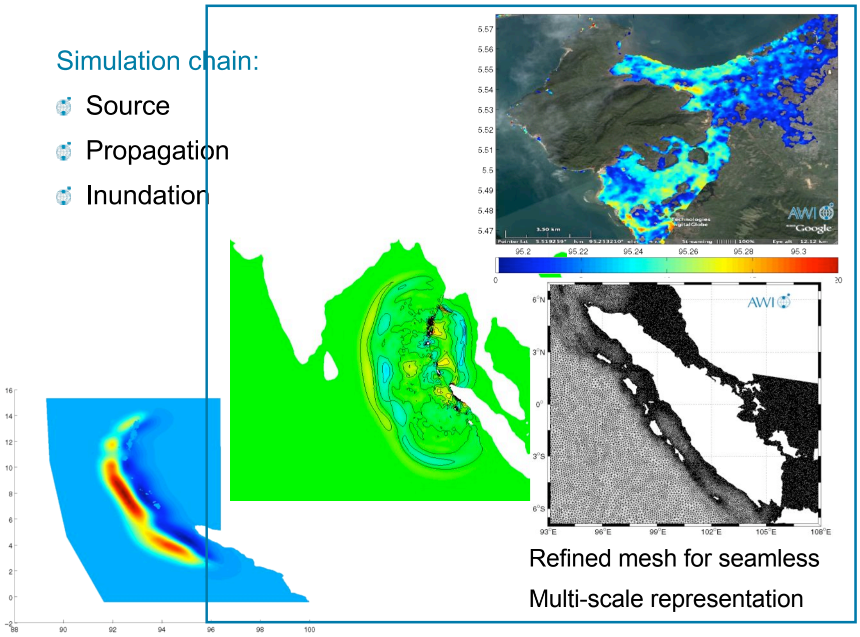
Simulation chain:

- Source
- Propagation
- Inundation



Refined mesh for seamless
Multi-scale representation

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2D shallow water equations

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Shallow water equations used in TsunAWI

Continuity equation

$$\frac{\partial h}{\partial t} + \nabla \cdot (\vec{v}(h+H)) = 0$$

Momentum equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} + \boxed{f \times \vec{v}} + g\nabla h + \boxed{\frac{C_d \vec{v}|\vec{v}|}{\rho(h+H)}} - \boxed{\nabla \cdot (A_h \nabla \vec{v})} = 0$$

Coriolis term

Bottom friction

Viscosity term

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2D shallow water equations

Boundary conditions used in TSUNAWI

Radiation boundary condition (open/liquid boundary)

$$\vec{v} \cdot \vec{n} = \sqrt{\frac{g}{h+H}} h$$

No-slip boundary condition (solid boundary)

$$\vec{v} \cdot \vec{n} = 0$$

Note: inundation boundary conditions according to Lynett/Wu/Liu (2002)

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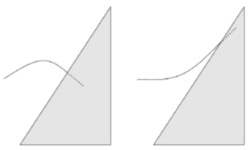
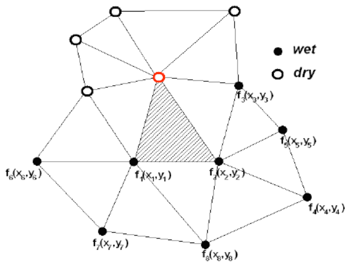
Wetting and Drying

Extrapolation of wave height



Extrapolation based on neigbors



Lynnet, Wu, Liu, 2002

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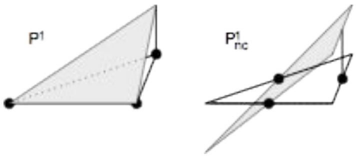
Basis Functions

P_1-P_1^{NC} finite element combination



Conforming linear for h and H:

$$\hat{h}: \quad \hat{h}_1 = 1 - x - y;\ \hat{h}_2 = x;\ \hat{h}_3 = y$$

Non-Conforming linear for v

$$\hat{v}: \quad \hat{v}_1 = 1 - 2y;\ \hat{v}_2 = 2x + 2y - 1;\ \hat{v}_3 = 1 - 2x$$

Exact discrete mass conservation

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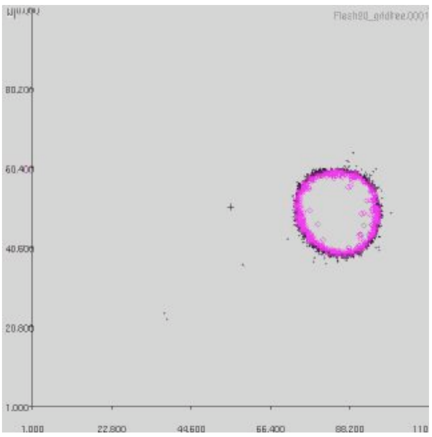
Mesh Independent Schemes

Grid free schemes?



☑ Mesh independent

☒ Not conservative
☒ Not efficient

Iske, J. B., 2001

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Refinement Strategies

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Criteria:

- Heuristic error proxies $\eta = \nabla\Phi$
- Physics-based criteria $\eta = \|\zeta\|_2$
- Mathematical (discretization-) error estimation $\eta \approx \|u - u_h\|$

Time-dependent methods

- Adjoint-based goal oriented methods
- Simplified forward simulation based methods
- Lagrangian methods
- A posteriori error estimation based methods

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Error Estimation

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Gradient based criterion

$$\text{refine if} \quad \nabla h|_\tau > \theta_{\text{ref}} \max(\nabla h|_\tau)$$
$$\theta_{\text{ref}} \text{ threshold value}$$

☑ Simple
☑ Easy to compute

☒ Not robust

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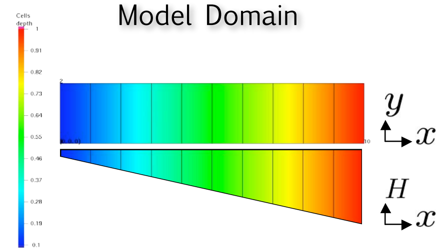
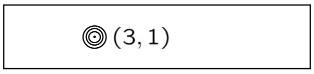
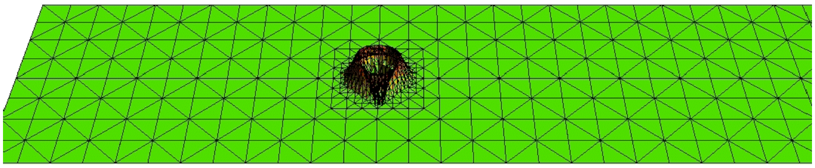
Adaptive mesh refinement

Model Domain



Initial values





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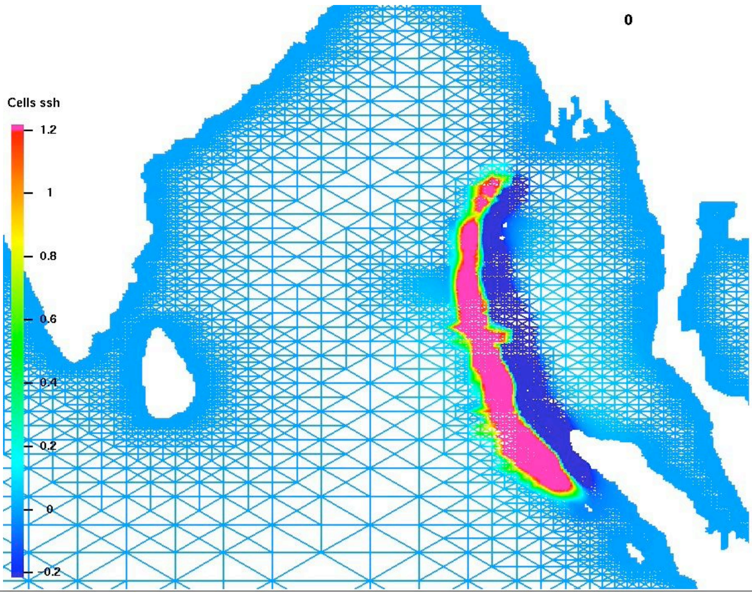
Andaman-Sumatra rupture



Error Estimation II

Instead of

$$\text{refine if} \quad \nabla h|_\tau > \theta_{\text{ref}} \max(\nabla h|_\tau)$$
$$\theta_{\text{ref}} \text{ threshold value}$$

Use

$$\boxed{\begin{aligned} \eta &= \| h|_\tau - \bar{h} \|, \quad \text{with } \bar{h} = \frac{1}{A} \sum_{\text{neighbors } t} h|_t, \\ \text{refine if} \quad \eta &> \theta_{\text{ref}} \bar{\eta}, \quad \text{with } \bar{\eta} \text{ mean error} \\ & \theta_{\text{ref}} \text{ threshold value} \end{aligned}}$$

- ☑ Simple
- ☑ Easy to compute
- ☑ Robust

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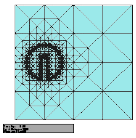
Data Management and Numerics



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Data Management and Parallelization

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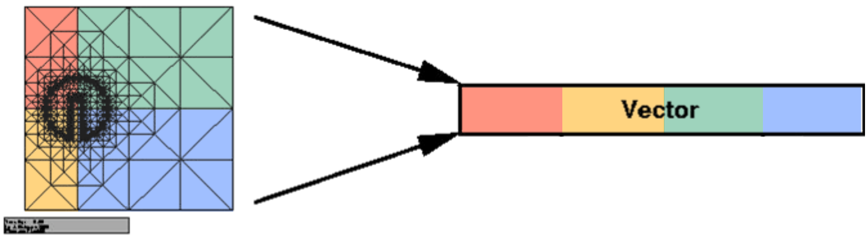

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Grid Ordering

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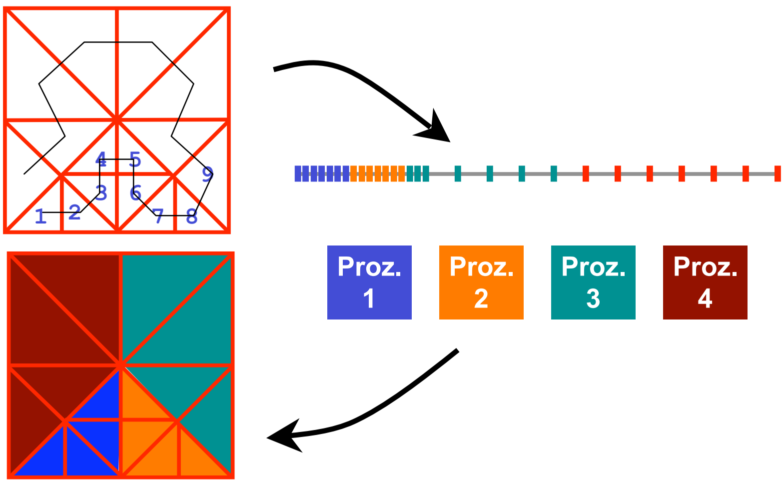
Space-filling curve (Sierpinski)



J.B., J. Zimmermann, 2000

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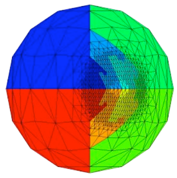
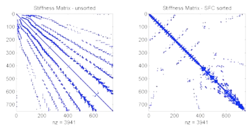
Levels

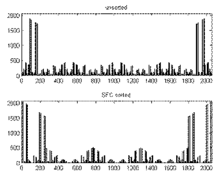
- Grid level: domain decomposition
  → parallelization
  → DD solvers



- System level: matrix ordering
  → sparse storage
  → prevention of fill-in



- Cache level: data layout
  → access optimization



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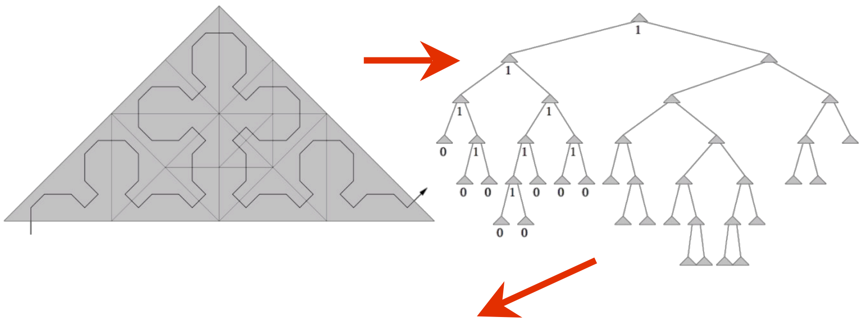
1st Observation: Refinement tree



1110100111000100...

Refinement tree as bitstream

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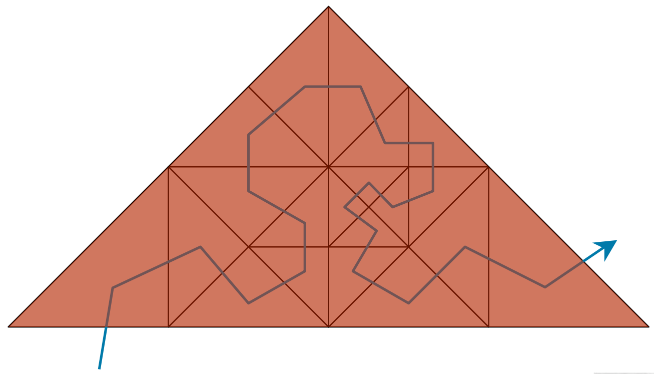
2nd Observation: Element-wise computation



- Depth first traversal induces Sierpinski curve
- Element by element computation

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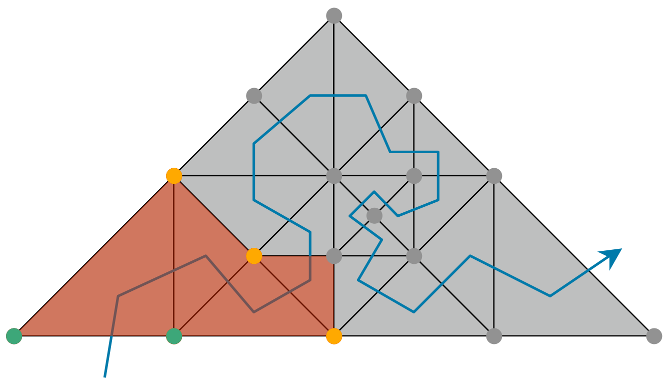
3rd Observation: Nodal computations

- not yet accessed
- just being computed
- waiting for further computation
- computations finished



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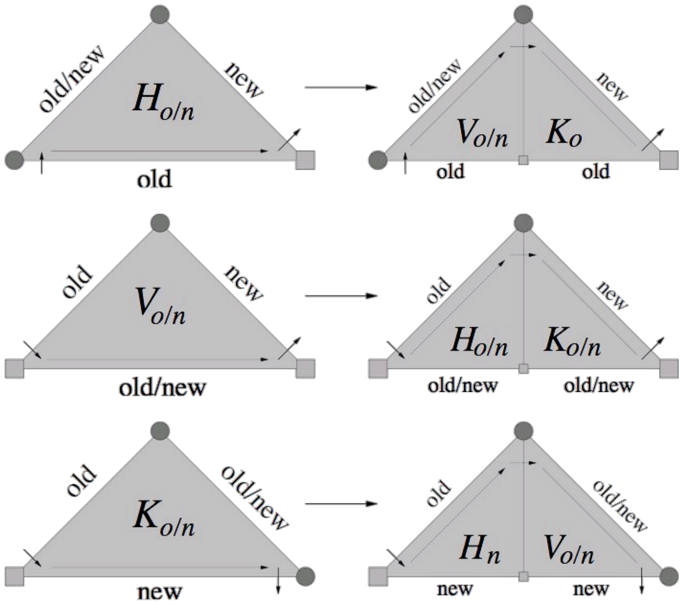
4th Observation: Types of elements



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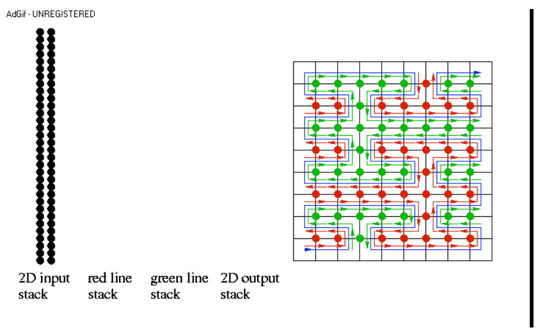
Idea

- Linearize grid structure by depth-first-traversal/Sierpinski curve
- Take element types for computing order
- Use heap data structures for storing intermediate values
- Use input/output stream



Mehl, Zenger (2004)

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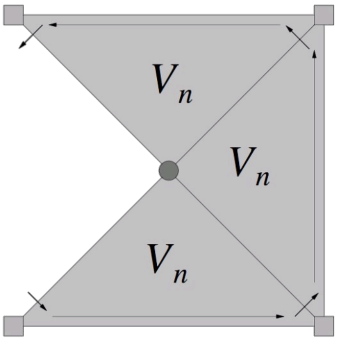
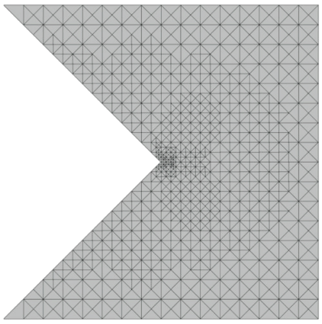
Preliminary result – reentrant corner





- Poisson's eq. with local refinement
- Solver: MG preconditioned CG
- Refinement criterion: hierarchical basis based (Deuflhard)

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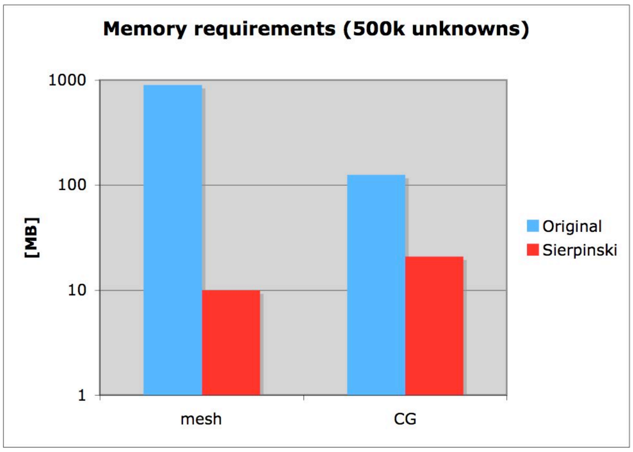
Preliminary result – reentrant corner



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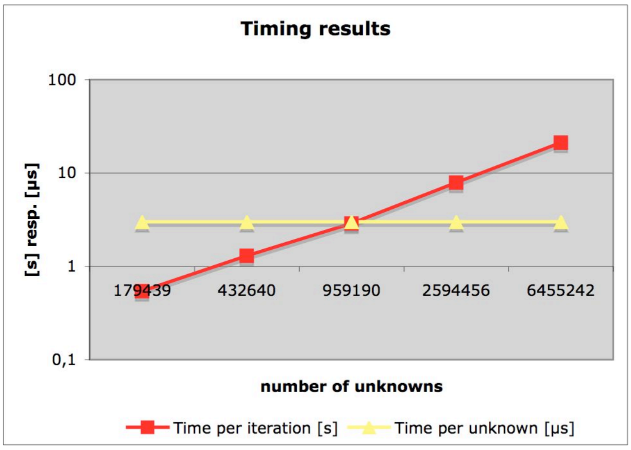
Preliminary result – reentrant corner



- 99% cache hits

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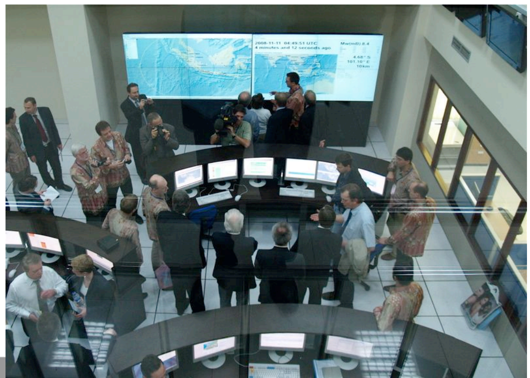
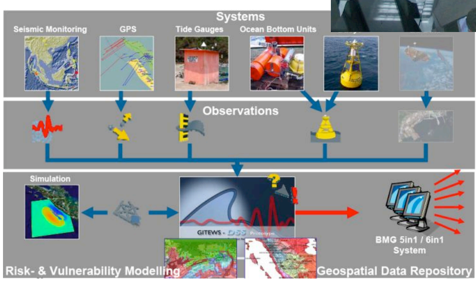
GITEWS Tsunami Early Warning System





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GITEWS Simulation System (SIM)

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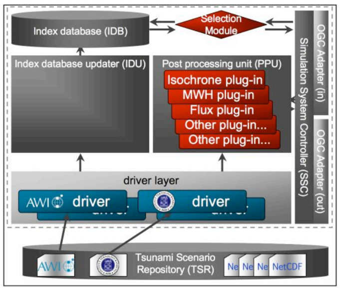
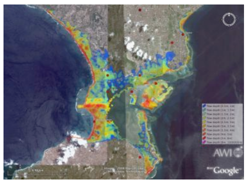
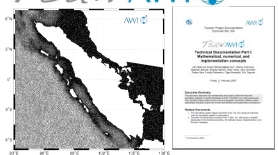
Robust Multi-Sensor Selection



Advanced Simulation Products



TSUNAWI



- Unstructured mesh
- Finite elements
- Non-linear shallow water eq.
- With run-up/inundation

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TSUNAWI Validation I

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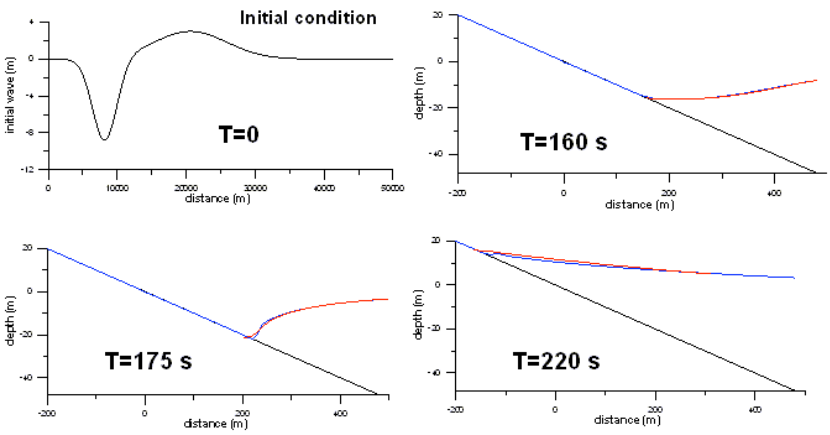
Run-up Benchmark



Analitycal solution — Modelled SSH

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Inundation (Results)

Zur Anzeige wird der QuickTime™
Dekompressor „MPEG-4 Video"
benötigt.

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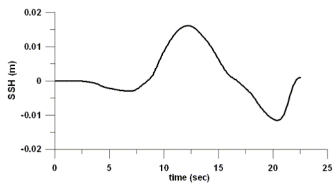
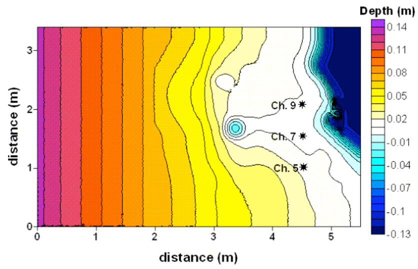
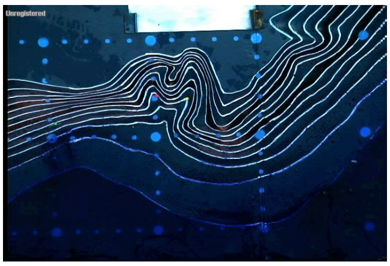
TsunAWI Validation II

Okushiri Island
(1993)



Bathymetry





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TSUNAWI Validation III

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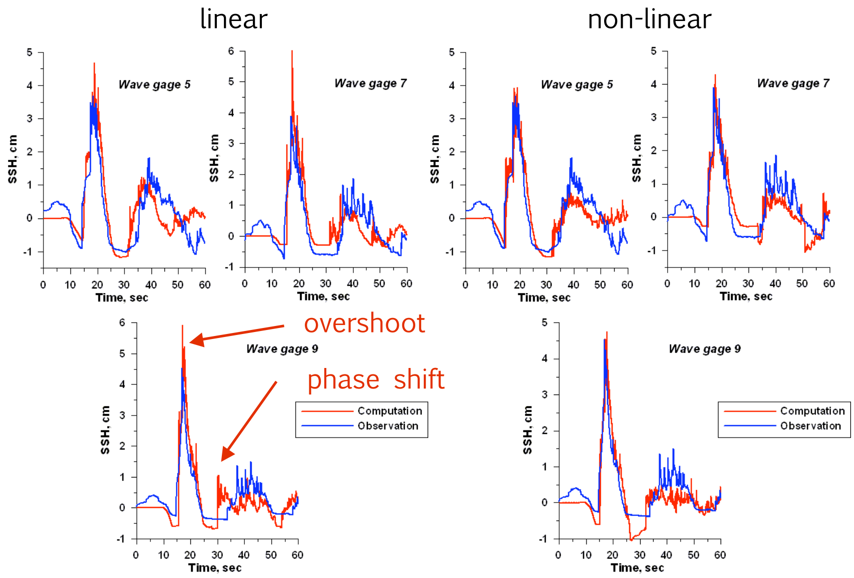

TSUNAWI Validation IV

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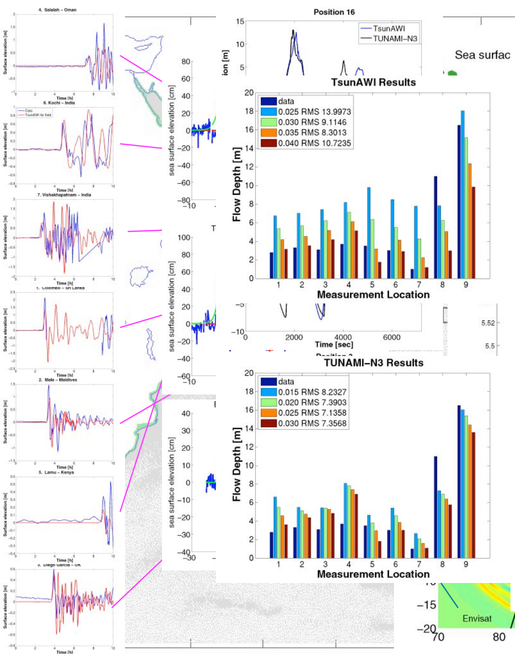

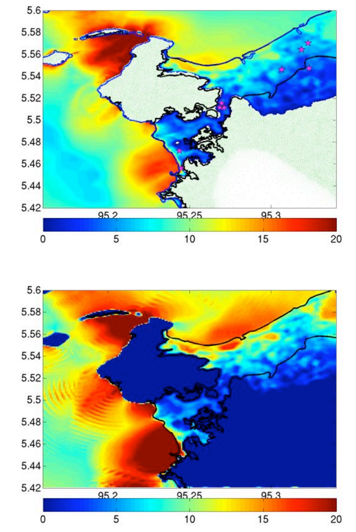

TSUNAWI Animation

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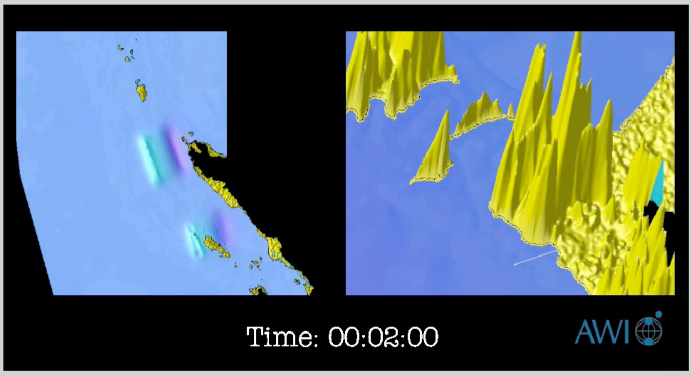

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Preliminary Sumatra Scenario

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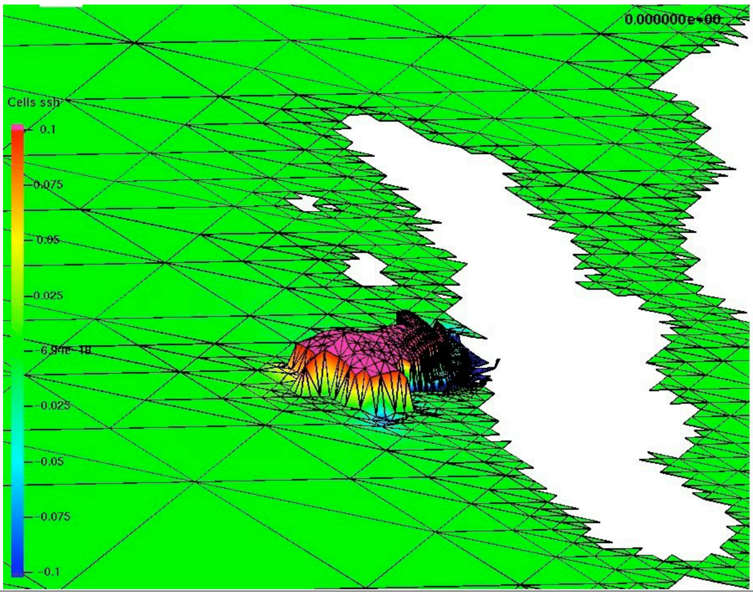

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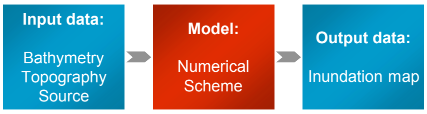
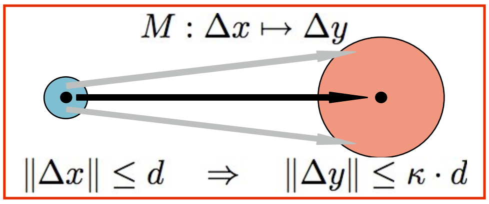
A Word on Reliability and Verification

- Code revisioning
- Validation
- Verification
- Uncertainty quantification



$$M : x \longrightarrow y = M(x)$$



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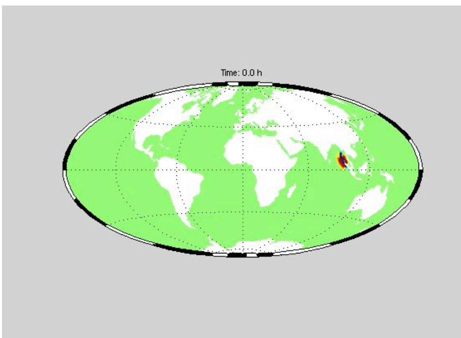
Conclusions

- Simplified application
- Multiple Scales
- Appropriate Numerics
- Refinement criteria
- Efficiency
- Reliability, Uncertainty



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