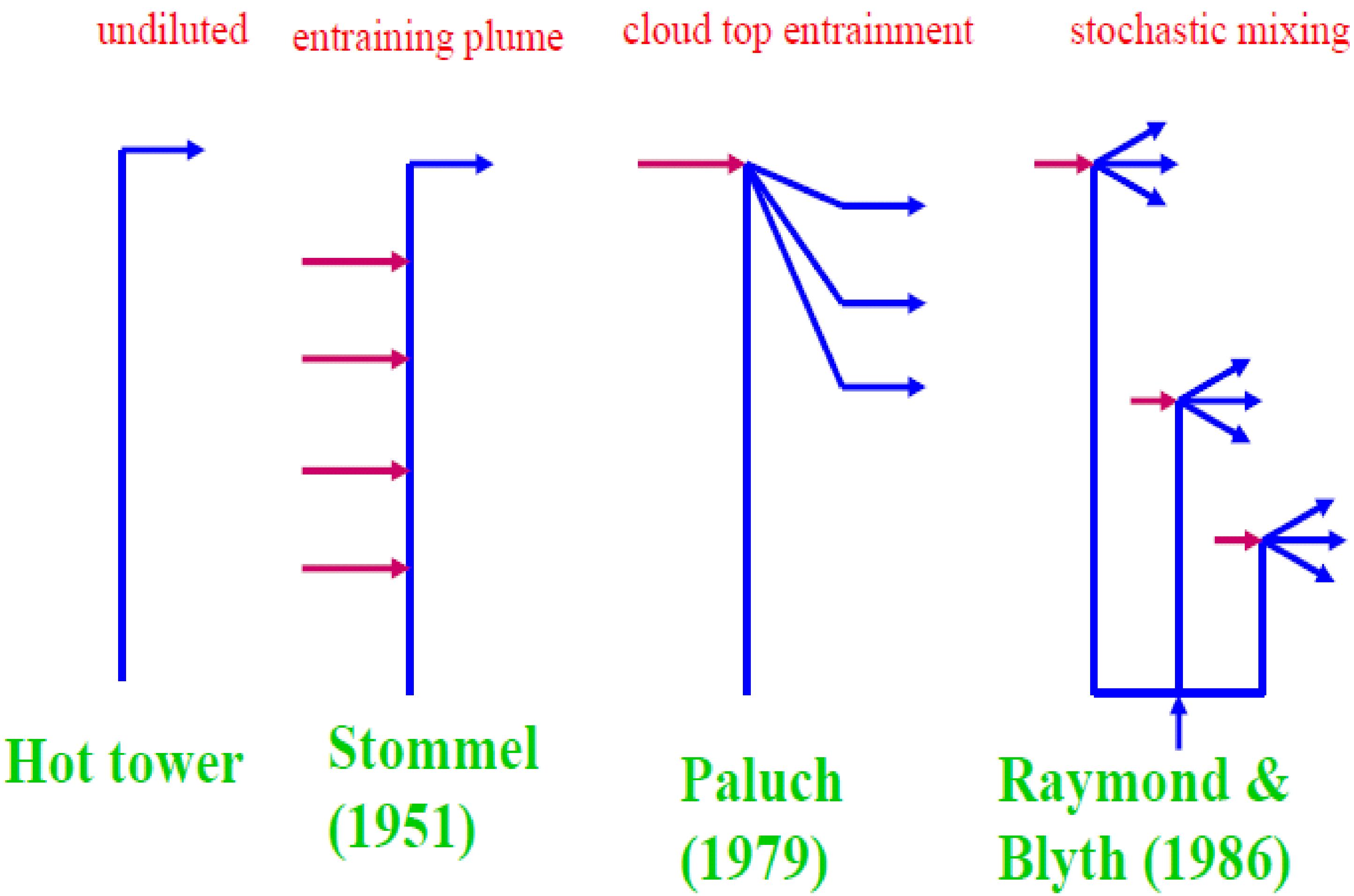
Entrainment and detrainment

Outline

- Making estimations of entrainment
- Ouline of some key methods and issues
  - Source of entraining air
  - Buoyancy sorting
  - Relative humidity dependence
  - Stochastic mixing

Vertical structure of convection



Direct estimates

- Mass continuity over a homogeneous area gives

$$\frac{\partial\sigma}{\partial t} + \frac{1}{A}\oint \hat{\mathbf{n}}\cdot(\mathbf{u}-\mathbf{u}_{\mathrm{int}})dl + \frac{\partial\sigma w_u}{\partial z} = 0$$

- Hard to evaluate, particularly to get reliable estimates of interface velocity $\mathbf{u}_{\mathrm{int}}$
- Need to make careful subgrid interpolation
  (e.g., Romps 2010, Dawe and Austin 2011)
- Typically gives larger values than we use in practice because
  - detraining air near cloud edge is typically less "cloud-like" than χ_{bulk}
  - entraining air near cloud edge is typically less "environment-like" than χ

LES diagnoses

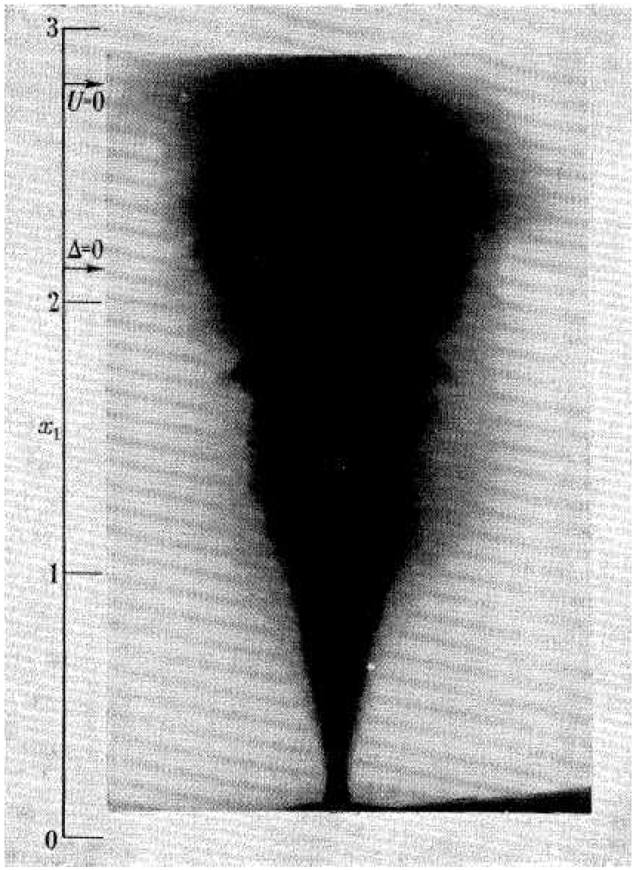
- Can make bulk estimate directly from parameterization formulae

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon - \delta$$

$$\frac{\partial M\chi}{\partial z} = M(\varepsilon_\chi \chi - \delta_\chi \chi_{\mathrm{bulk}})$$

- where $\varepsilon = E/M$ and $\delta = D/M$
- Sampling is a key issue to define “cloud” and “environment”
  - “cloud core” $q_l > 0$, $\theta_v > \overline{\theta_v}$ often chosen

Morton tank experiments



- water tank experiments (Morton et al 1956)
- growth described by fractional entrainment rate,

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon \simeq \frac{0.2}{R}$$

- The form is essentially a dimensional argument
- Used for cloud models from the 1960s on

Key Issues

- lateral or cloud-top entrainment?

  i.e., diffusion-type mixing at cloud edge or a more organized flow structure dominates

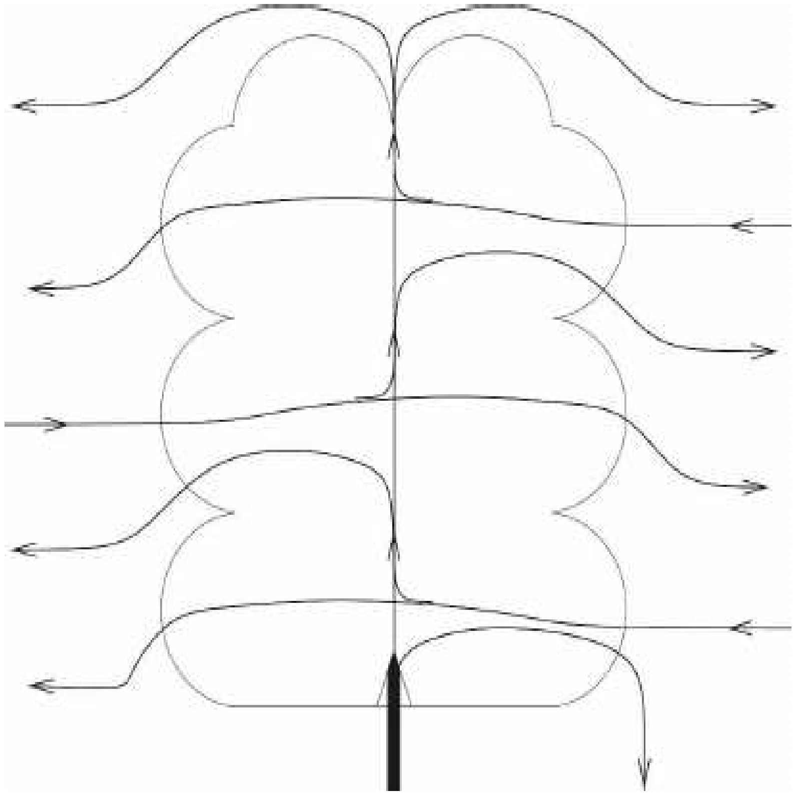
- importance of detrainment?

  unlike the lab:

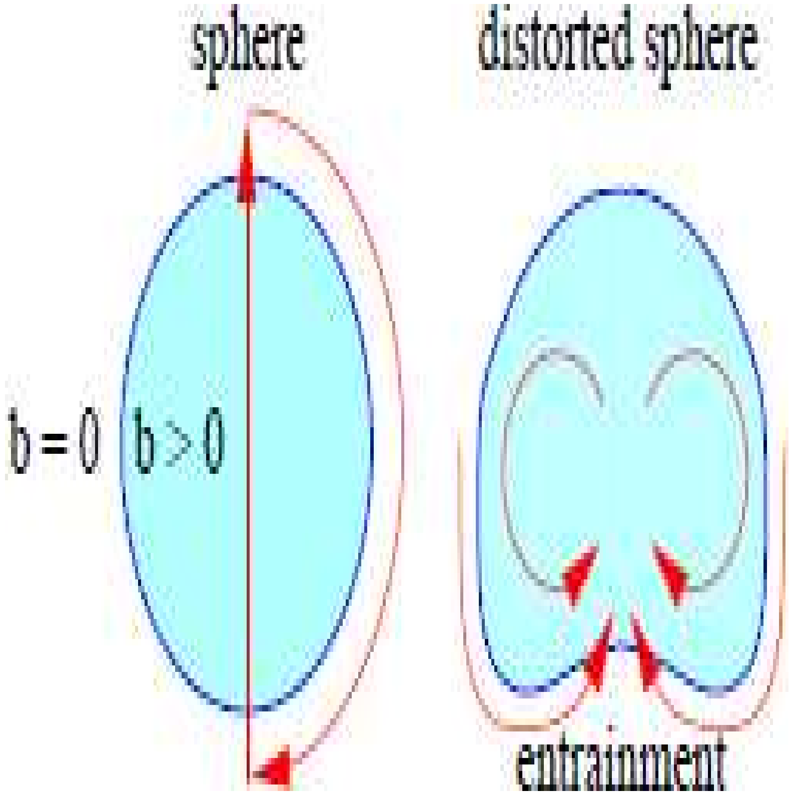
  1. turbulent mixing and evaporative cooling can cause negative buoyancy
  2. stratification means that cloud itself becomes negatively buoyant

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon_{dyn} + \varepsilon_{turb} - \delta_{dyn} - \delta_{turb}.$$

Source of Entraining Air

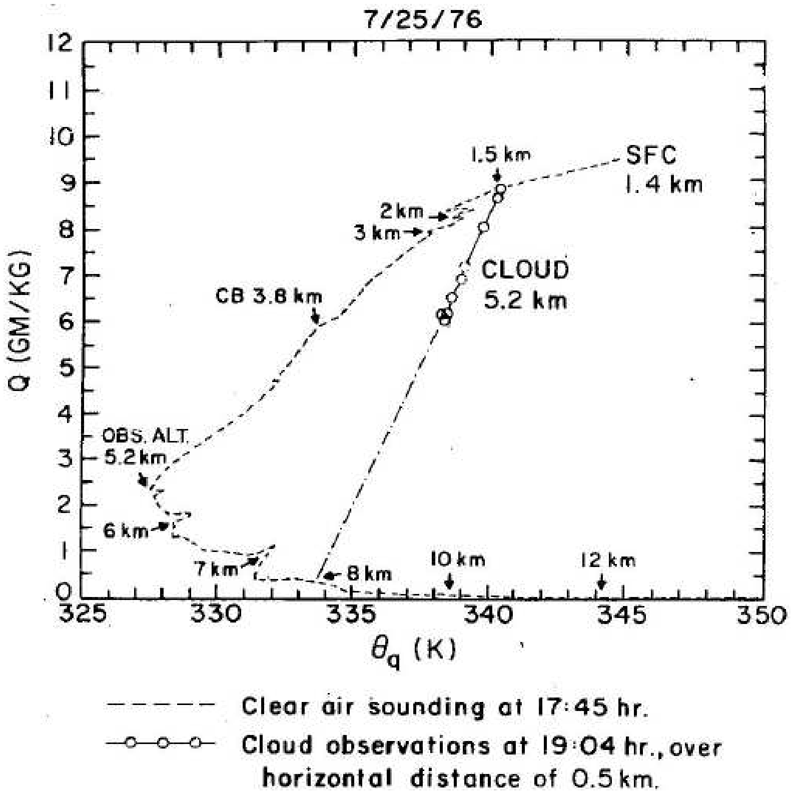


lateral entrainment
usual parameterization
assumption



cloud-top entrainment

Paluch diagrams



(Paluch 1979)

- plot conservative variables (eg, θ_e and q_T)
- in-cloud values fall along mixing line
- extrapolate to source levels: cloud-base and cloud-top
- health warning: in-cloud T is not a trivial measurement

Cloud-top entrainment

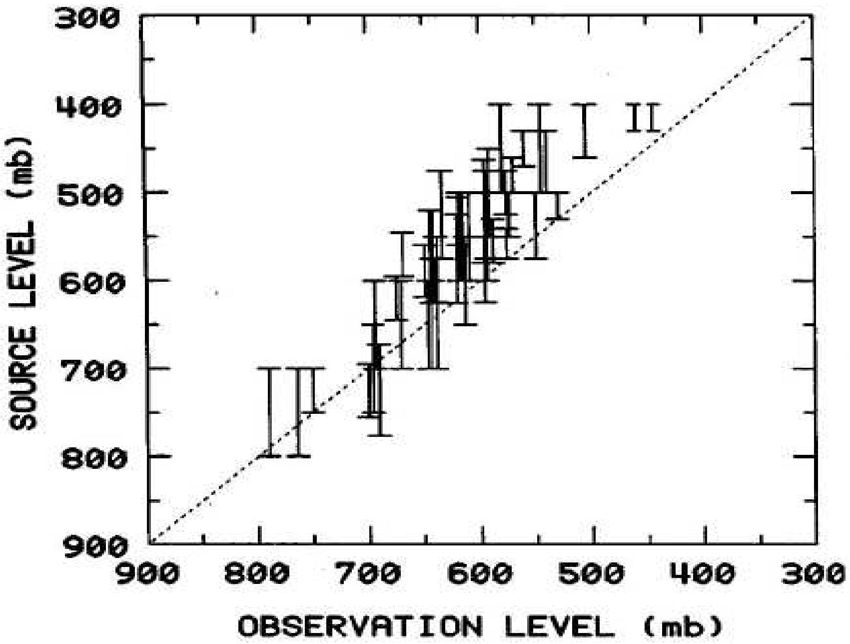


FIG. 10. The source level from which air was entrained into the cloud, as a function of the observation level in the cloud, for 44 cases taken from 44 different regions for which source levels could be determined. The error bars indicate the approximate ranges that are consistent with the observations.

(Blyth et al 1988)

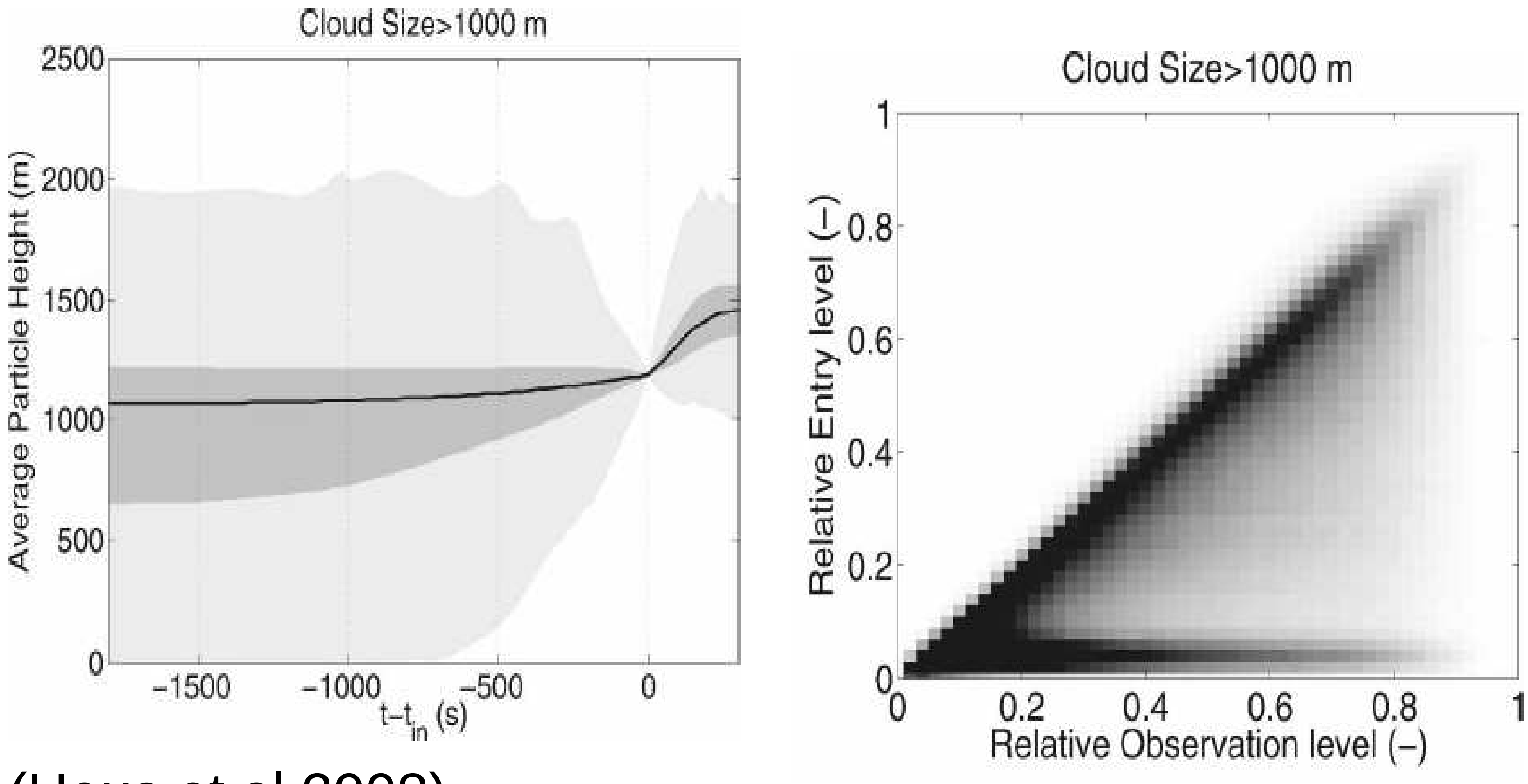
implied source level well above measurement level

Interpretations of Paluch

- Criticized because data points can line up without implying two-point mixing

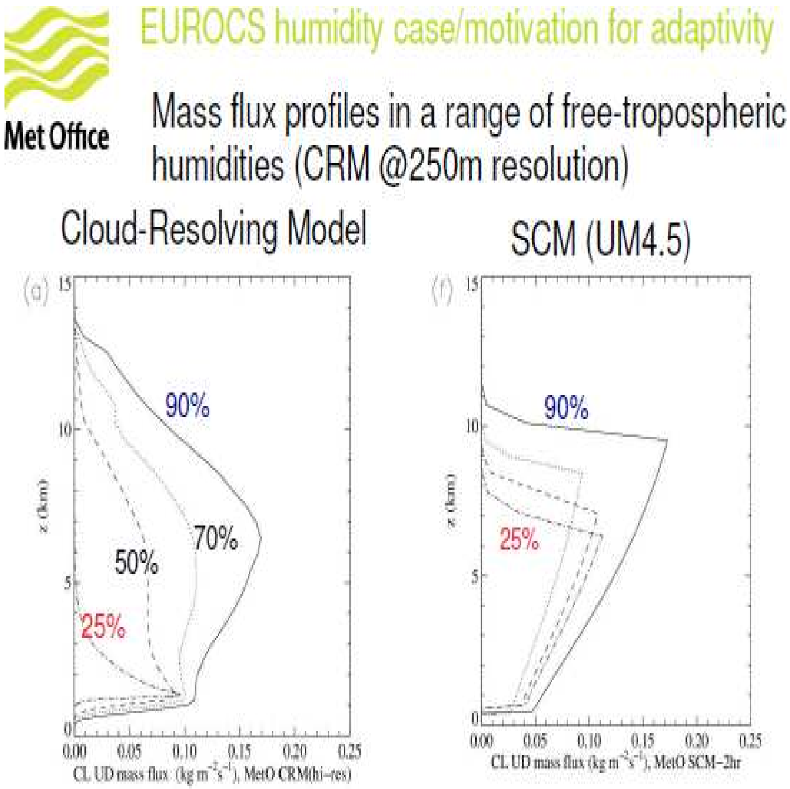
  eg, Taylor and Baker 1991; Siebesma 1998
- Boing et al 2014, "On the deceiving aspects of mixing diagrams of deep cumulus convection"
- correlations implied because parcels from below likely to be positively buoyant and those from below negatively bouyant

LES Analysis



(Heus et al 2008)

Actual Formulations

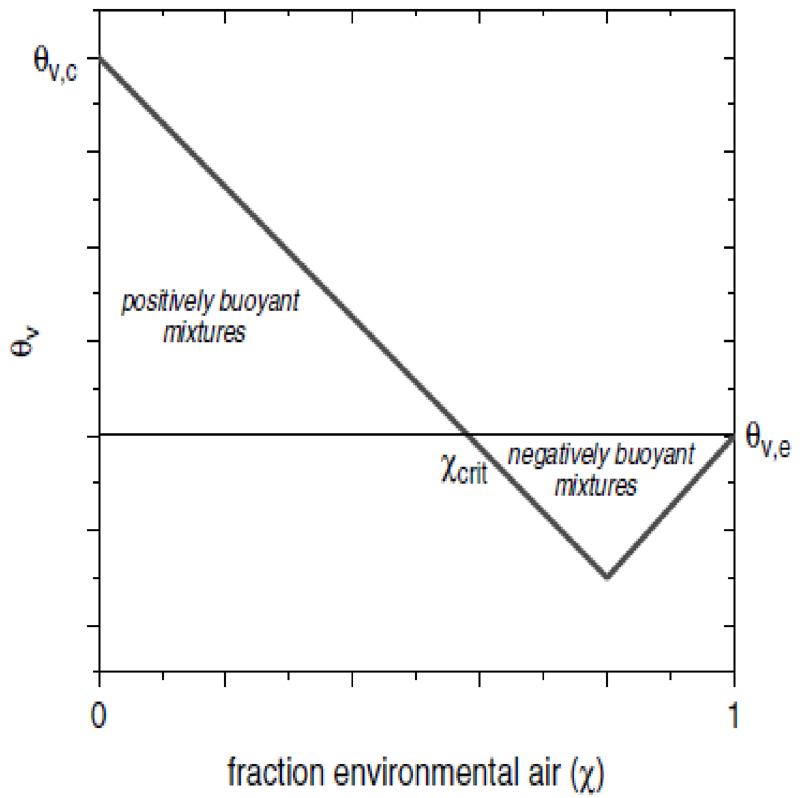


- Much can be done by formulating E and D as better functions of the environment
- e.g. Bechtold et al 2008 revised ECMWF scheme to have entrainment with explicit RH dependence

Stochastic mixing model

- Introduced by Raymond and Blyth (1986) and implemented into Emanuel (1991) scheme
- consider separate parcels from cloud base each of which mixes with air at each level up to cloud top
- mixed parcels spawn further parcels each of which can mix again with air at each level from the current one up to cloud top
- can incorporate lateral and cloud-top mechanisms
- how to proportion the air into different parcels?
- Suselj et al (2013) have explicitly stochastic treatment with Poisson process: unit chance of mixing 20% of the mass per distance travelled

Buoyancy Sorting and Kain-Fritsch



- Ensemble of cloud/environment mixtures: retain buoyant mixtures in-plume and detrain negatively buoyant
- evaporative cooling can make mixture $\theta_v <$ environmental θ_v

pdf of mixtures

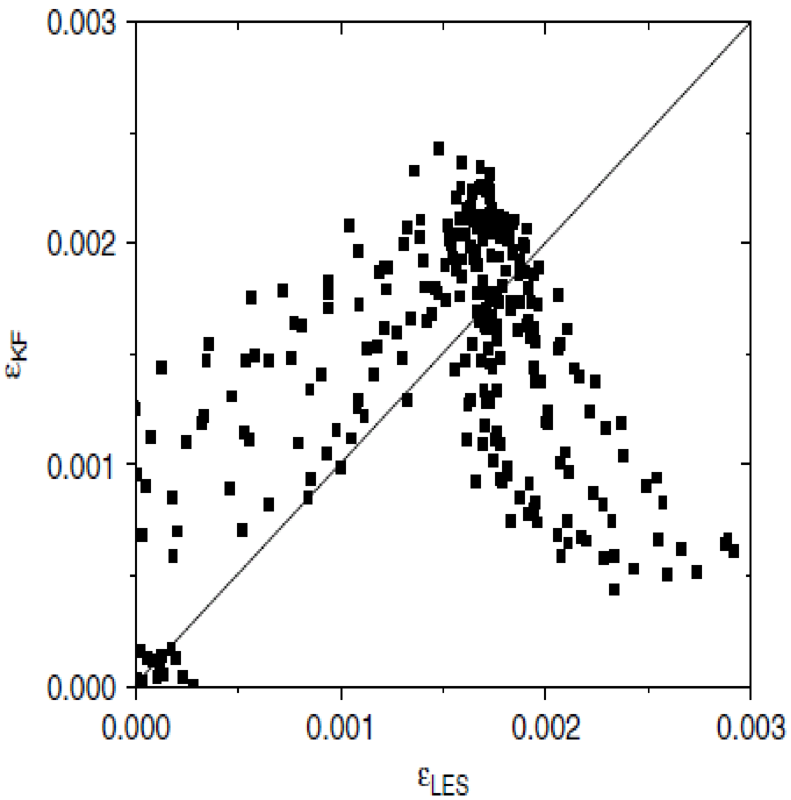
- To complete calculations, also need PDF for occurrence of the various mixtures
- This has to be guessed
- Uniform pdf gives

$$\varepsilon_{KF} = \varepsilon_0 \chi^2_{crit}$$

$$\delta_{KF} = \varepsilon_0 (1 - \chi_{crit})^2$$

where ε_0 is the fraction of the cloud that undergoes some mixing

BOMEX LES estimates



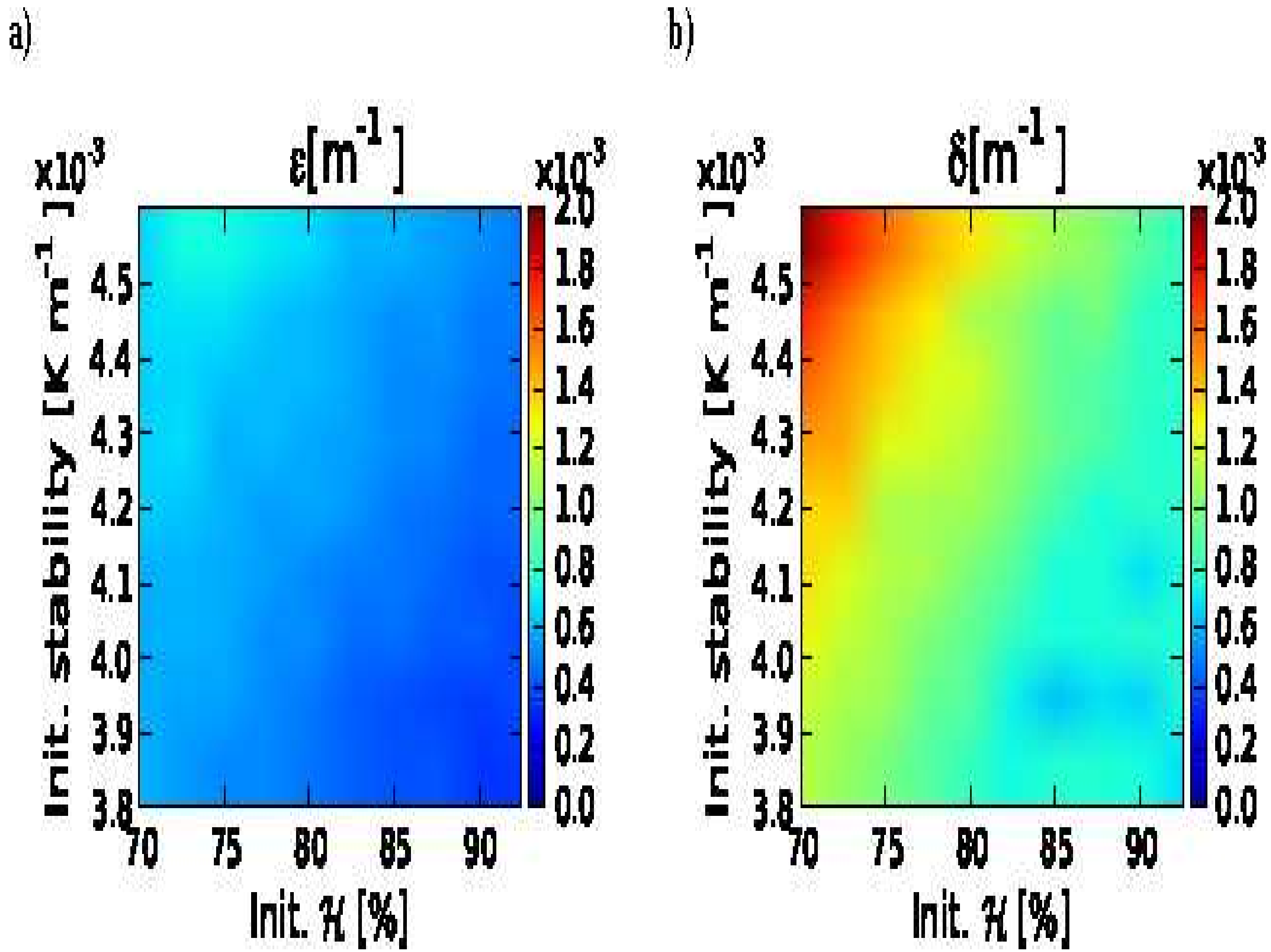
From BOMEX case

- dry conditions $\longrightarrow$ small χ_{crit} $\longrightarrow$ weak dilution

$$\varepsilon_{KF} = \varepsilon_0 \chi_{crit}^2$$

- various fixes possible (Kain 2004, Bretherton and McCaa 2004)

Detrainment variations



Boing et al 2012

Detrainment variations

- Variations of LES estimates dominated by δ not ε
- Variations dominated by cloud-area not by in-cloud w
  (e.g. Derbyshire et al 2011)

Conclusions

- Small clouds are shallower: larger fractional entrainment due to mixing on dimensional grounds
- Some progress on process-level analysis of entrainment and detrainment, but difficult to translate into reliable E and D for use in bulk scheme

  main issue is *how much* of the cloudy material mixes in each way
- Distribution of cloud tops affected by environment
- This controls the organized detrainment contribution
- which seems to be an important control on the overall bulk profile

Closure

Outline

- Objective of closure
- Quasi-equilibrium, Arakawa and Schubert formulation
- CAPE and its variants
- Moisture closure
- Boundary-layer based closures

Objective

We need to calculate the total mass flux profile,

$$M = \sum_i M_i = \eta(z) M_B(z_B)$$

- $\eta(z)$ comes entrainment/detrainment formulation
- $M_B = M(z_B)$ remains, the overall amplitude of convection

Practical Issue

- A practical convection scheme needs to keep the parent model stable

  Settings may err on the defensive side to remove potential instability

- not all diagnostic relationships for M_B are appropriate

$$M_B = k\frac{C_p\overline{w'T'}_0 + L\overline{w'q'}_0}{\mathrm{CAPE}}$$

  Shutts and Gray 1999

- scaling works well for a set of equilibrium simulations, but not as closure to determine M_B

Convective Quasi-Equilibrium

- Generation rate of convective kinetic energy defined per unit area

$$\int_{z_B}^{z_T} \sigma \rho w_c b dz \equiv M_B A$$

- where the "cloud work function" is

$$A = \int_{z_B}^{z_T} \eta b dz.$$

- For each plume type

$$A(\lambda) = \int_{z_B}^{z_T(\lambda)} \eta(\lambda, z) b(\lambda, z) dz.$$

Convective Quasi-Equilibrium

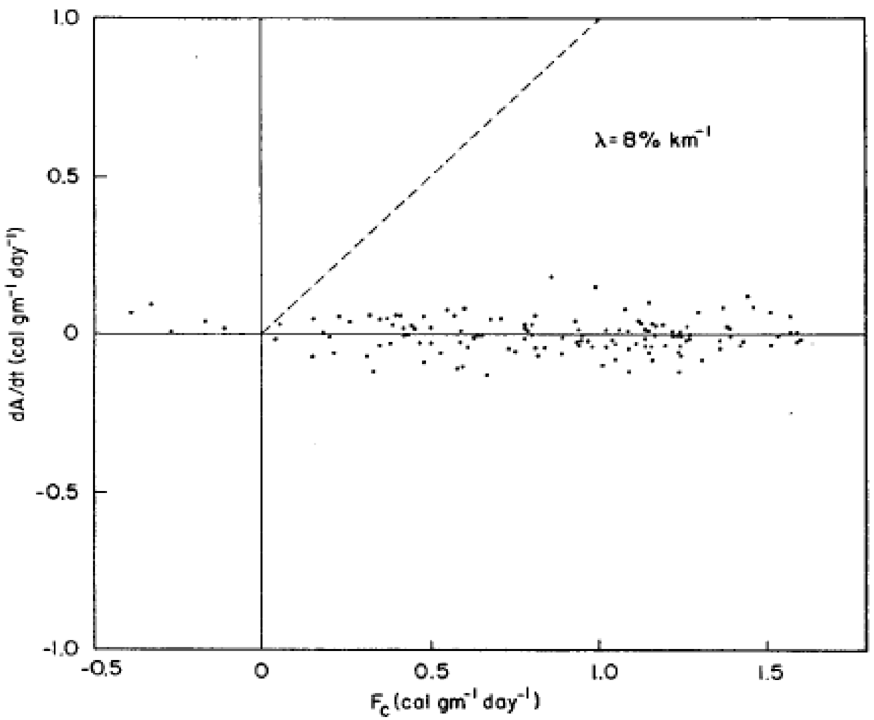
- Taking a derivative of the definition

$$\frac{\partial}{\partial t} A_\lambda = F_{L,\lambda} - D_{c,\lambda}$$

  where

- $F_{L,\lambda}$ is “large-scale” generation: terms independent of M_B
- $D_{c,\lambda}$ is consumption by convective processes: terms dependent on M_B, proportional for entraining plumes with simplified microphysics in AS74
- “scale” not immediately relevant to this derivation which follows by definition
- all of the cloud types consume the CWF for all other types

Convective Quasi-Equilibrium



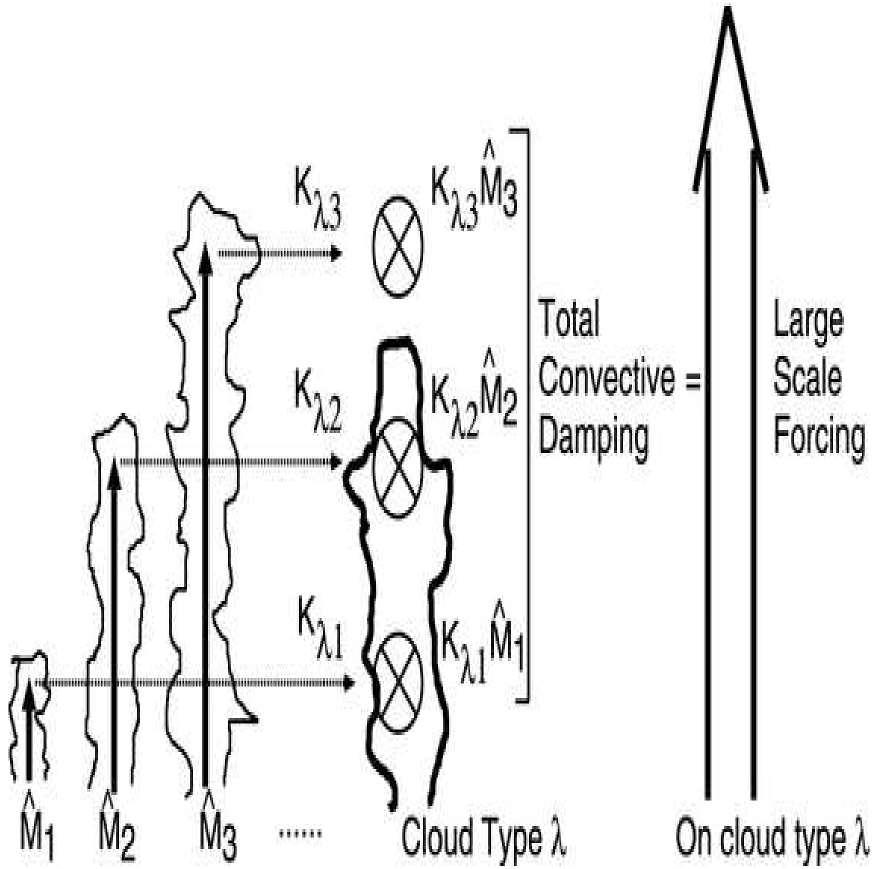
A stationary solution to the CWF tendency equation

$$F_{L,\lambda} - D_{c,\lambda} = 0$$

$$\sum_{\lambda'} \mathcal{K}_{\lambda\lambda'} M_{B,\lambda'} = F_{L,\lambda}$$

Assumes $\tau_{\mathrm{LS}} \gg \tau_{\mathrm{adj}}$

Using CQE



$$\sum_{\lambda'} \mathcal{K}_{\lambda\lambda'} M_{B,\lambda'} = F_{L,\lambda}$$

- $F_{L,\lambda}$ is known from parent model
- $\mathcal{K}_{\lambda\lambda'}$ is known from the plume model
- invert matrix $\mathcal{K}$ to get $M_{B,\lambda}$

Issues with CQE calculation

1. The resulting $M_{B,\lambda}$ is not guaranteed positive
   various fixes possible, eg Lord 1982; Moorthi and Suarez 1992
2. the equilibrium state is not necessarily stable
3. $\eta(z,\lambda)$ and $b(z,\lambda)$ depend on $T(z)$ and $q(z)$. If the $A(\lambda)$ form a near-complete basis set for T and q, then stationarity of all A would imply highly- (over-?) constrained evolution of T and q

Some CWF variants

$$A(\lambda) = \int_{z_B}^{z_T(\lambda)} \eta(\lambda, z) b(\lambda, z) dz$$

1. CAPE$= A(\lambda = 0)$, ascent without entrainment
2. CIN: negative part of integated non-entraining parcel buoyancy
3. Diluted CAPE: ascent with entrainment, but differs from CWF by taking $\eta = 1$ in integrand
4. PEC (potential energy convertibility): bulk A estimate by choosing a different normalization
5. Other quantities investigated based on varying the limits of the integral

   (e.g. “parcel-environment” CAPE of Zhang et al 2002, 2003)

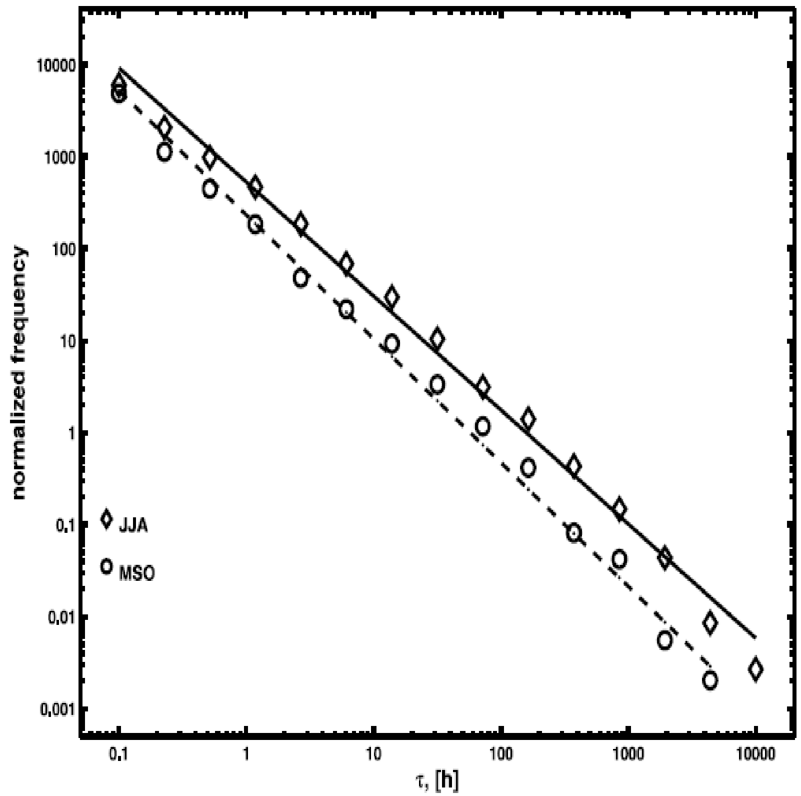
CQE Validity

- Zimmer et al (2010) timescale for CAPE consumption rate

$$\tau \sim \mathrm{CAPE}/P$$

  assuming precipitation rate $P \sim (d\mathrm{CAPE}/dt)_{\mathrm{conv}}$

- P is average within 50 km radius and 3 hr window



- 2/3 of events have less than 12 hours

Operational CAPE closure

- In many operational models assumed that convection consumes CAPE at a rate that is determined by a characteristic closure time–scale τ_c.

$$M_B \propto \left.\frac{dCAPE}{dt}\right|_{\mathrm{conv}} = -\frac{CAPE}{\tau_c}$$

(Fritsch and Chappell 1980)

- Conceptually, maintains idea of timescale separation, but recognizes finite convective-consumption timescale
- Many variations on this basic theme:
- As well as variations of the CAPE-like quantity, some experiments with a functional form for τ_c

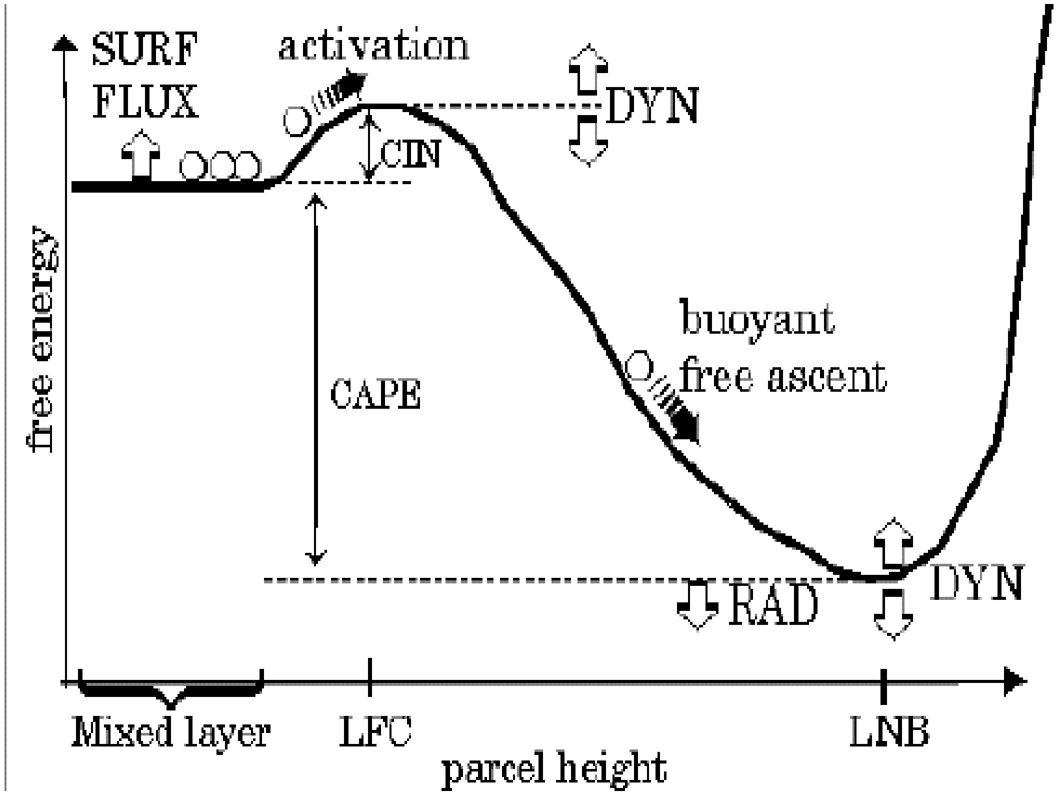
Moisture-based closure

- large–scale supply of moisture balanced against consumption by convective processes
- some methods consider only large–scale convergence, but others add surface fluxes
- remains a popular approach since original proposal by Kuo 1974
- especially for applications to models of tropical deep convection
- Emanuel 1994, causality problem assuming convection is driven by moisture rather than by buoyancy
- tendency for grid–point storms

PBL-based closures

- Mapes 1997 deep convection may be controlled by:
  - equilibrium response to increases in instability
  - the ability to overcome CIN (activation control)
- On large-scales, CIN will always be overcome somewhere and equilibrium applies
- On smaller scales, PBL dynamics producing eddies that overcome CIN may be important
- Mapes 2000 proposed $M_B \sim \sqrt{\mathrm{TKE}}\exp(-k\mathrm{CIN}/\mathrm{TKE})$

Control of deep convection



Which is right?

- Buoyancy-based, moisture-convergence-based and PBL-based methods all have some intuitive appeal
- Analyses are bedevilled by “chicken-and-egg” questions
- Convection “consumes” moisture and CAPE on the average, but not always, and the exceptions matter
- e.g., shallow convection
- Various analyses attempt to correlate rainfall (note not M_B!) with various factors
  - results, while interesting, are typically not conclusive
  - and correlations typically modest (or even anti!)
  - and different for different regions

(Sherwood and Warlich 1999, Donner and Phillips 2003, Zhang et al 2002, 2003, 2009, 2010, Glinton 2014)

Conclusions

- Cloud work function is a measure of efficiency of energy generation rate
- CAPE is a special case, as are various other measures
- Quasi-equilibrium if build-up of instability by large-scale is slow and release at small scales is fast
- Similar QE ideas can be formulated for the variants, and for moisture
- QE is a often a good basis for a closure calculation, but is not always valid, and may not be a good idea to apply it very strictly

News ideas (if time!):
1. Stochastic Aspects of Convection
2. Prognostic aspects of convection

Stochastic Aspects of Convection

Stochastic Effects

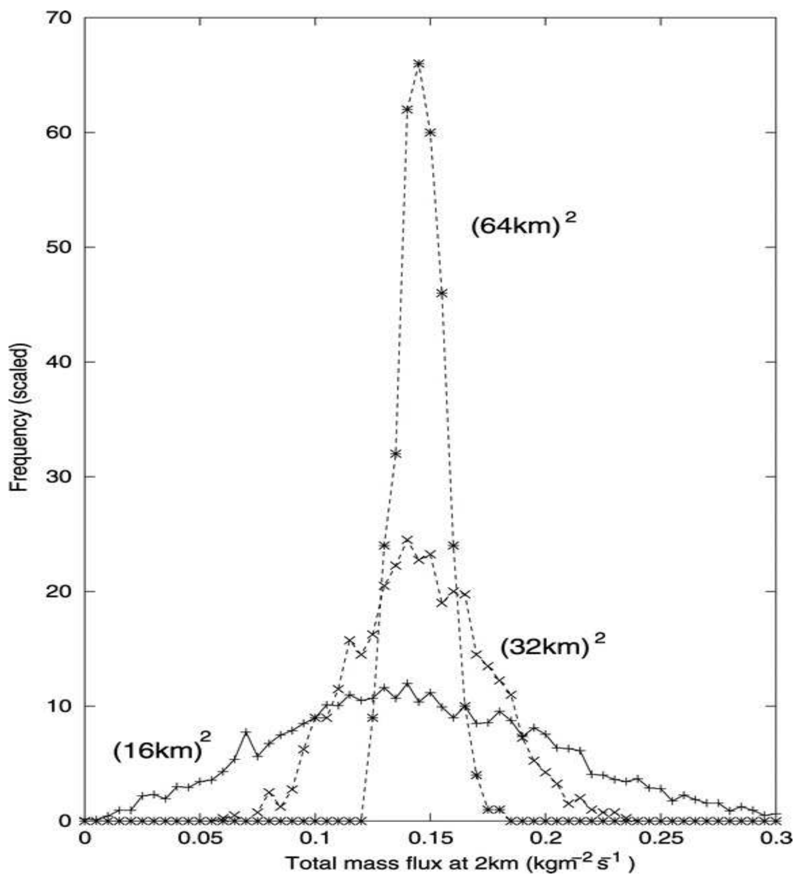
Standard assumption: enough plumes to treat statistically, as found within

> a region of space-time large enough to contain an ensemble of cumulus clouds but small enough to cover only a fraction of a large-scale disturbance
> (Arakawa and Schubert 1974)

But:

- Convective instability is released in discrete events
- The number of clouds in a GCM grid-box is not large enough to produce a steady response to a steady forcing

Convective variability



- Convection on the grid-scale is unpredictable, but randomly sampled from a pdf dictated by the large scale
- To describe the variability arising from fluctuations about equilibrium, we must consider the partitioning of the total mass flux M into individual clouds, m_i

pdf for m

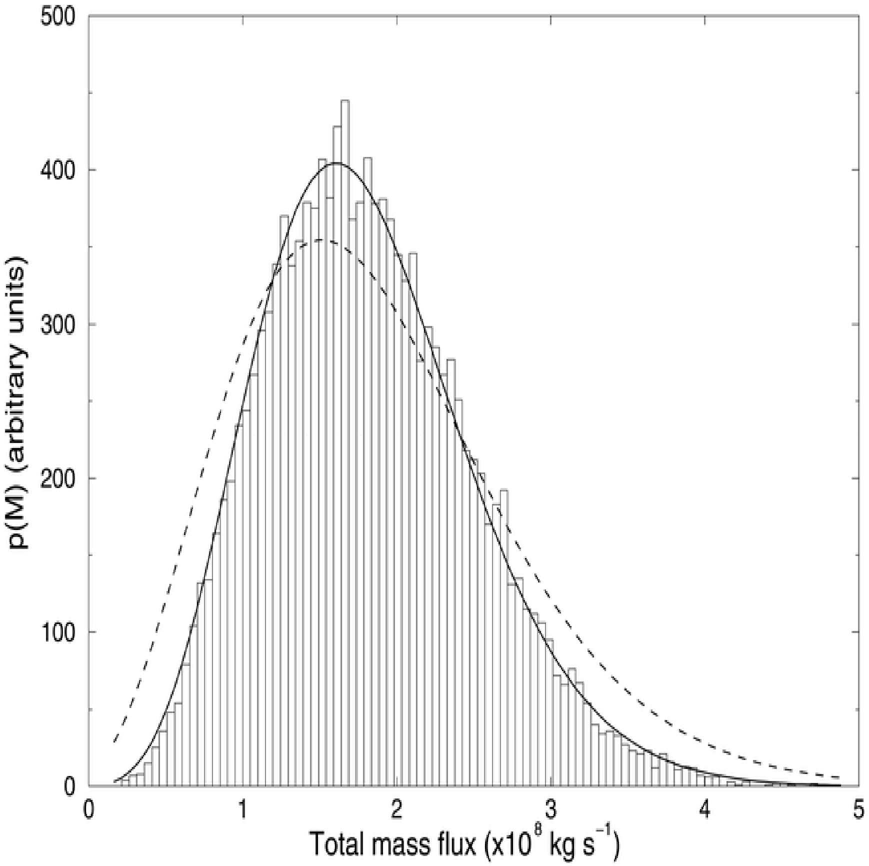
- Our assumptions about clouds as discrete, independent objects in a statistical equilibrium with a large-scale, macroscopic state are directly equivalent to those for an ideal gas
- So the pdf of m is a Boltzmann distribution

$$p(m)dm = \frac{1}{\langle m \rangle} \exp\left(\frac{-m}{\langle m \rangle}\right) dm$$

- Remarkably good and robust in CRM data

  Cohen and Craig 2006; Shutts and Palmer 2007; Plant and Craig 2008; Davies 2008; Davoudi et al 2010
- which also reveals $\langle m \rangle$ to be nearly independent of forcing

pdf for M



- Number of clouds is not fixed, unlike number of gas particles
- If they are randomly distributed in space, number in a finite region given by Poisson distribution
- pdf of the total mass flux is a convolution of this with the Boltzmann

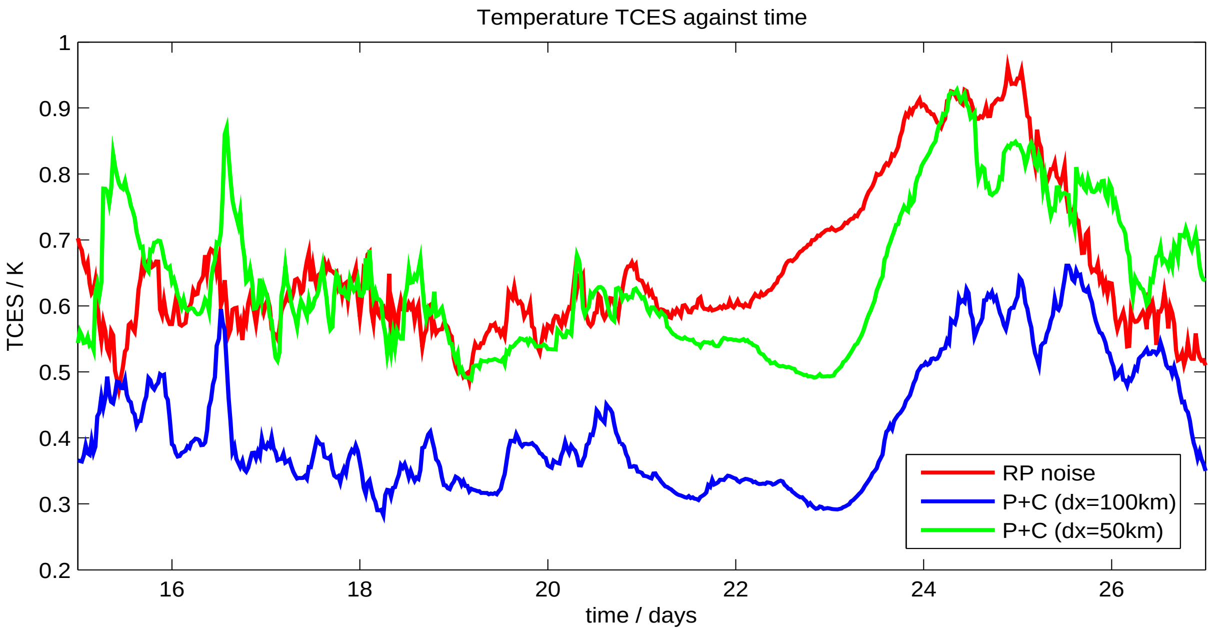
Stochastic parameterization

- Grid-box state $\neq$ large-scale state
  space average over $\Delta x \neq$ ensemble average
- We must parameterize convection on the grid-scale as being unpredictable, but randomly sampled from a known pdf dictated by the large-scale

Important note: **None** of these scales is fixed in a simulation!

Practical implementation

Single-column test with Plant-Craig (2008) parameterization



- Spread similar to random parameters or multiplicative noise for $\Delta x = 50$km
- Stochastic drift similar to changing between deterministic parameterizations

The University of Reading

Prognostic aspects of convection

Why consider time dependence?

- For relatively rapid forcings, we may wish to consider a prognostic equation for cloud-base mass flux
- Even for steady forcing, it is not obvious
  - that a stable equilibrium must be reached
  - which equilibrium might be reached

Systems for time dependence

- From the definitionof cloud work fucntion

$$\frac{dA}{dt} = F - \gamma M$$

where A and γ are calculable with a plume model

- The convective kinetic energy equation is

$$\frac{dK}{dt} = AM - \frac{K}{\tau_D}$$

- Need further assumption to close these energy equations

Closing this system

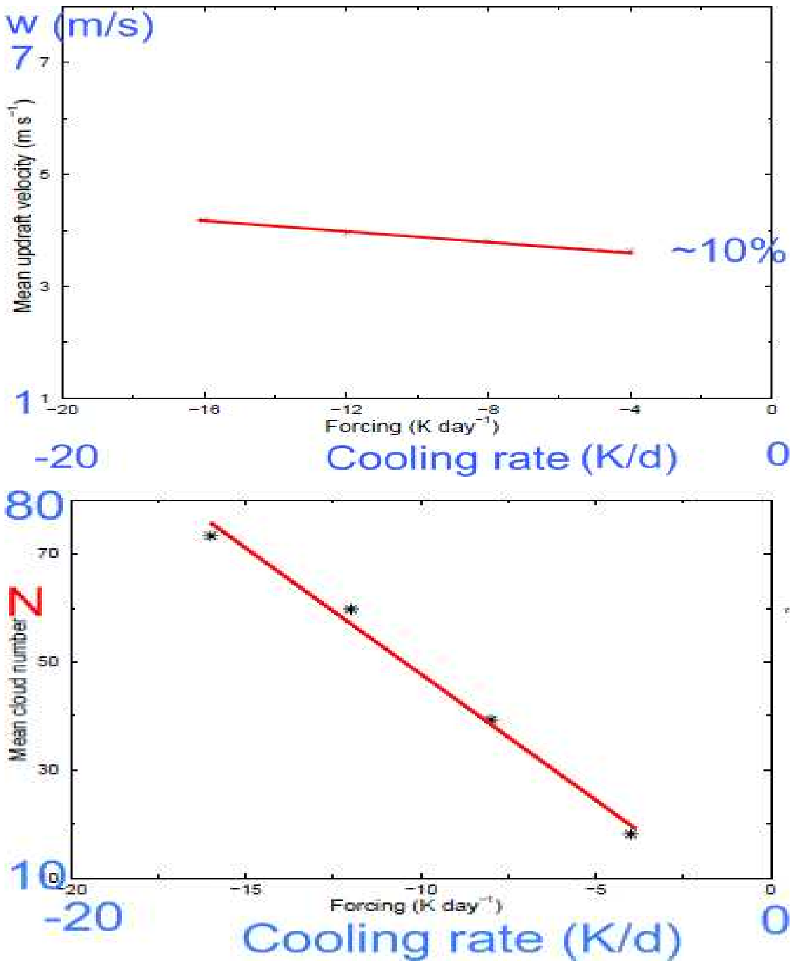
- Pan and Randall (1998) and others postulate

$$K_i \sim M_i^2$$

(Recall $K_i \sim \sigma_i w_i^2$ and $M_i = \rho\sigma_i w_i$ so $p \approx 2$ if variations in w dominate variations in K and M)

- For a bulk system, the time dependence is a damped oscillator that approaches equilibrium after a few τ_D

CRM data for changes in mass flux



Increased forcing linearly increases the mass flux, $\rho\sigma w$

- achieved by increasing cloud number N
- not the in-cloud velocities
- nor the sizes of clouds

(Cohen 2001)

Yano and Plant system

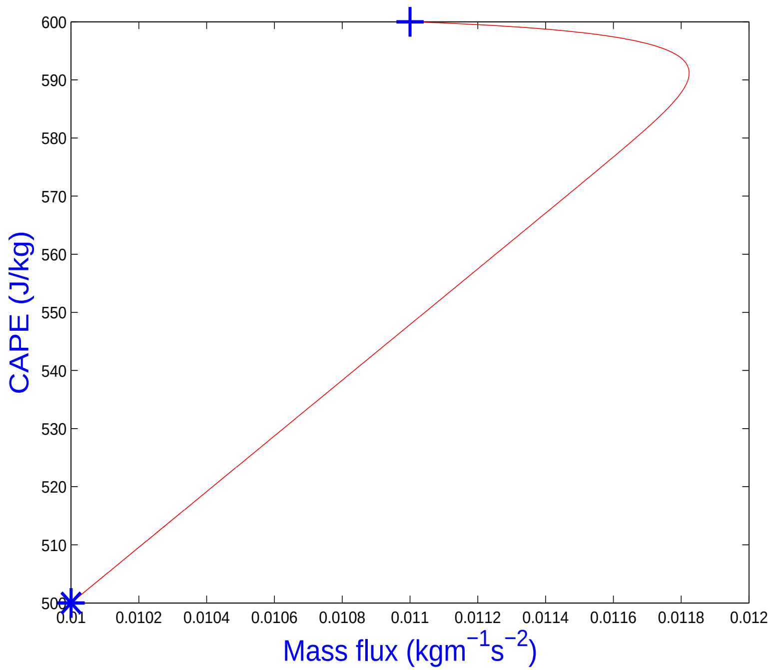
- Yano and Plant (2011) choose $p = 1$. i.e.

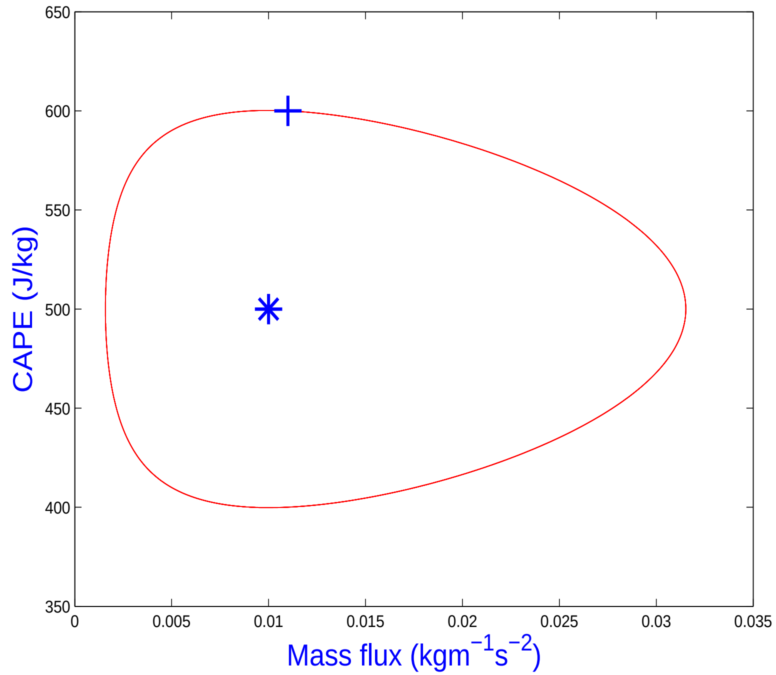
$$K \sim M$$

- Recall $K \sim \sigma w^2$ and $M = \rho\sigma w$ so $p \approx 1$ if variations in σ dominate variations in K and M
- This is consistent with scalings and CRM data for changes in mass flux with forcing strength

  Emanuel and Bister 1996; Robe and Emanuel 1996; Grant and Brown 1999; Cohen 2001; Parodi and Emanuel 2009
- Time dependence is periodic orbit about equilibrium state

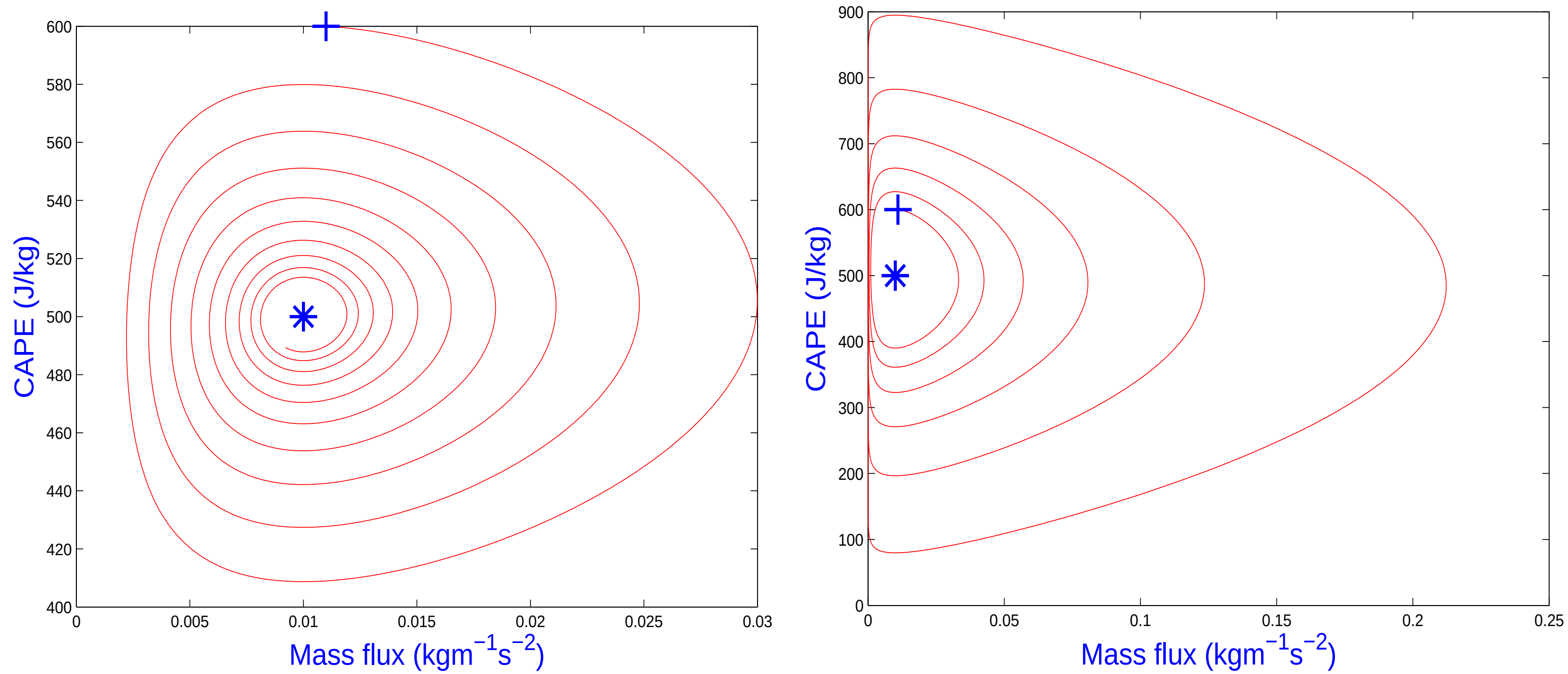
Illustrative results





Pan & Randall (left) and Yano & Plant (right) systems

Illustrative results



$p = 1.01$ (left) and $p = 0.99$ (right)

- The CRM data supports $p \approx 1$ but > 1
- Equilibrium is reached but more slowly as $p \longrightarrow 1$ from above