Independent Research Fellowship

Dynamic Turbulence Modelling in the 'Terra-Incognita' of turbulence

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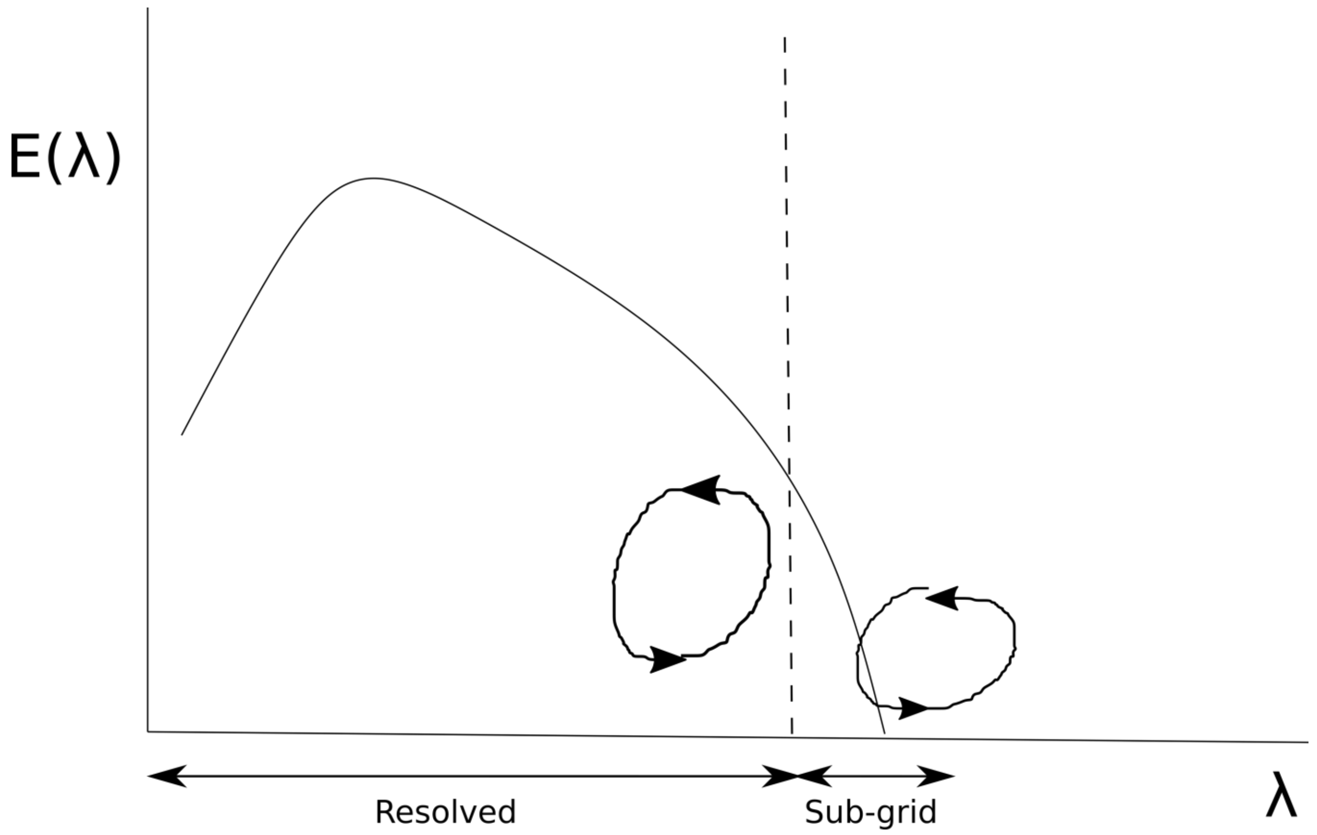
Dynamic Turbulence Modelling

- **Scale similarity** between resolved and subgrid eddies
- **Use smallest resolved fluxes** to diagnose the subgrid scales



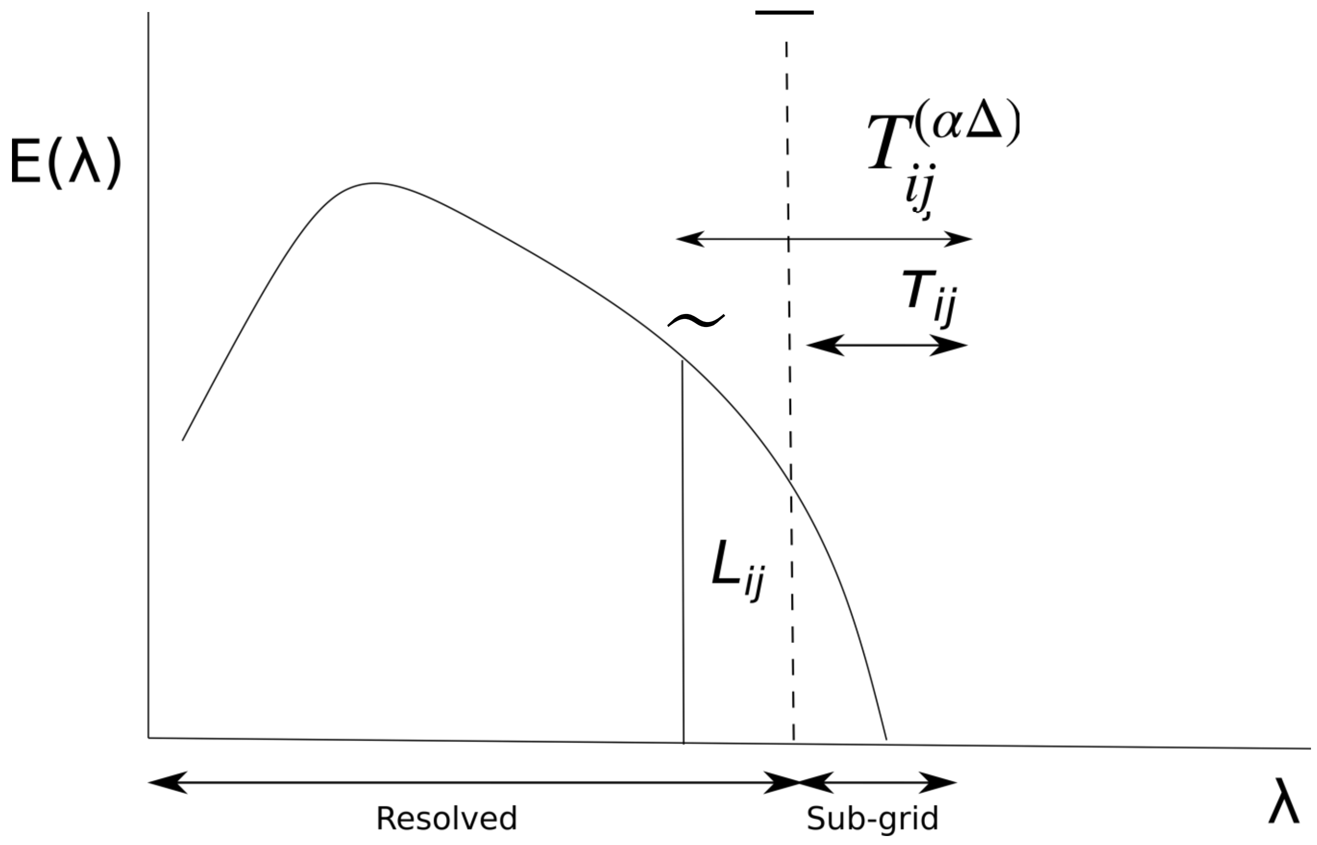
Dynamic Turbulence Modelling

Germano Identity

$$L_{ij} = \widetilde{\bar{u}_i \bar{u}_j} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j = T_{ij}^{(\alpha\Delta)} - \tilde{\tau}_{ij}$$

τ_{ij} : subgrid stress tensor
(Turbulence model)

$$T_{ij}^{(\alpha\Delta)} \sim C_{(\alpha\Delta)}\, \Phi_{ij}(\alpha\Delta)$$



Dynamic Turbulence Modelling (scale dependent)

Germano Identity

$$Q_{ij} = \widehat{\bar{u}_i\bar{u}_j} - \hat{\bar{u}}_i\hat{\bar{u}}_j = T_{ij}^{(\alpha^2\Delta)} - \hat{\tau}_{ij}$$

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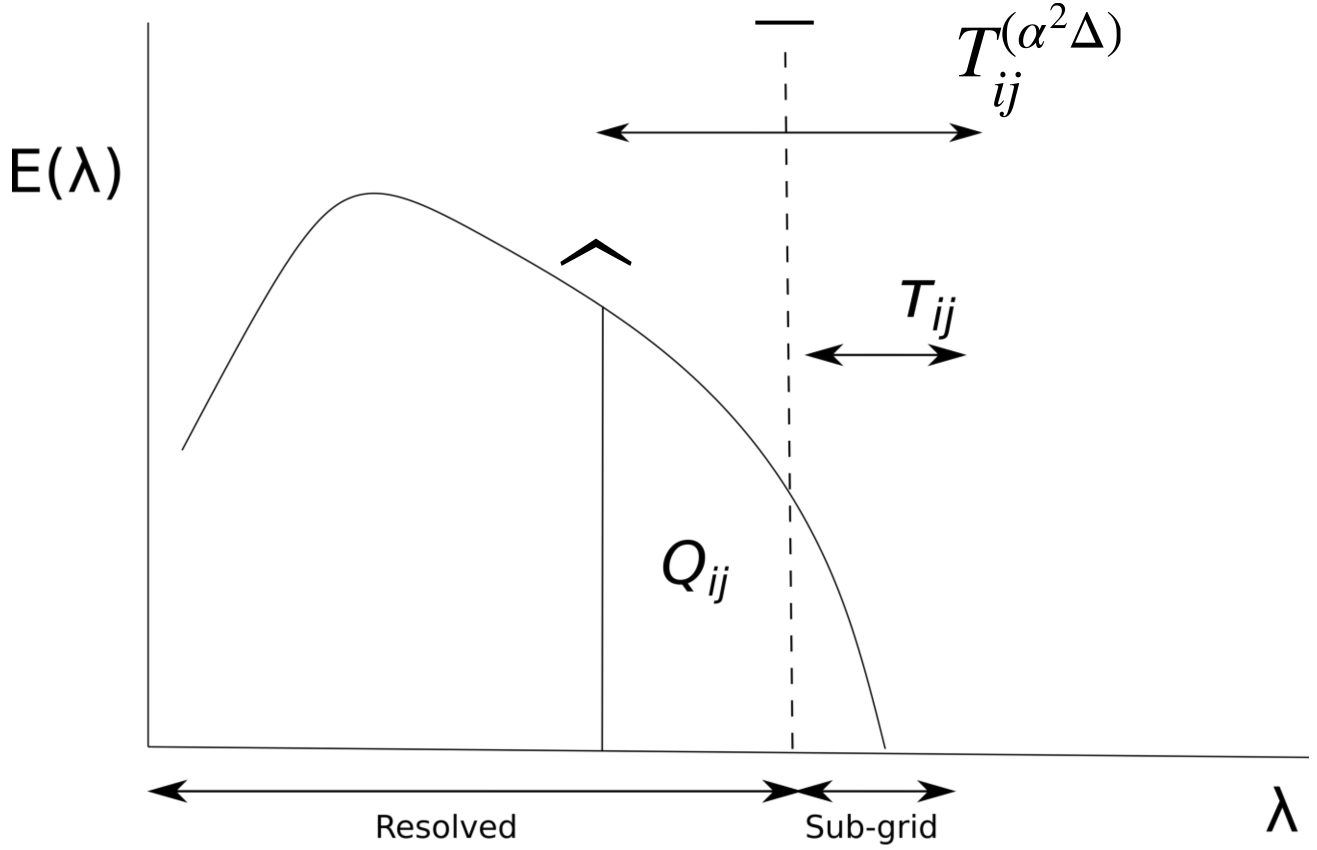

Dynamic Turbulence Modelling (scale dependent)

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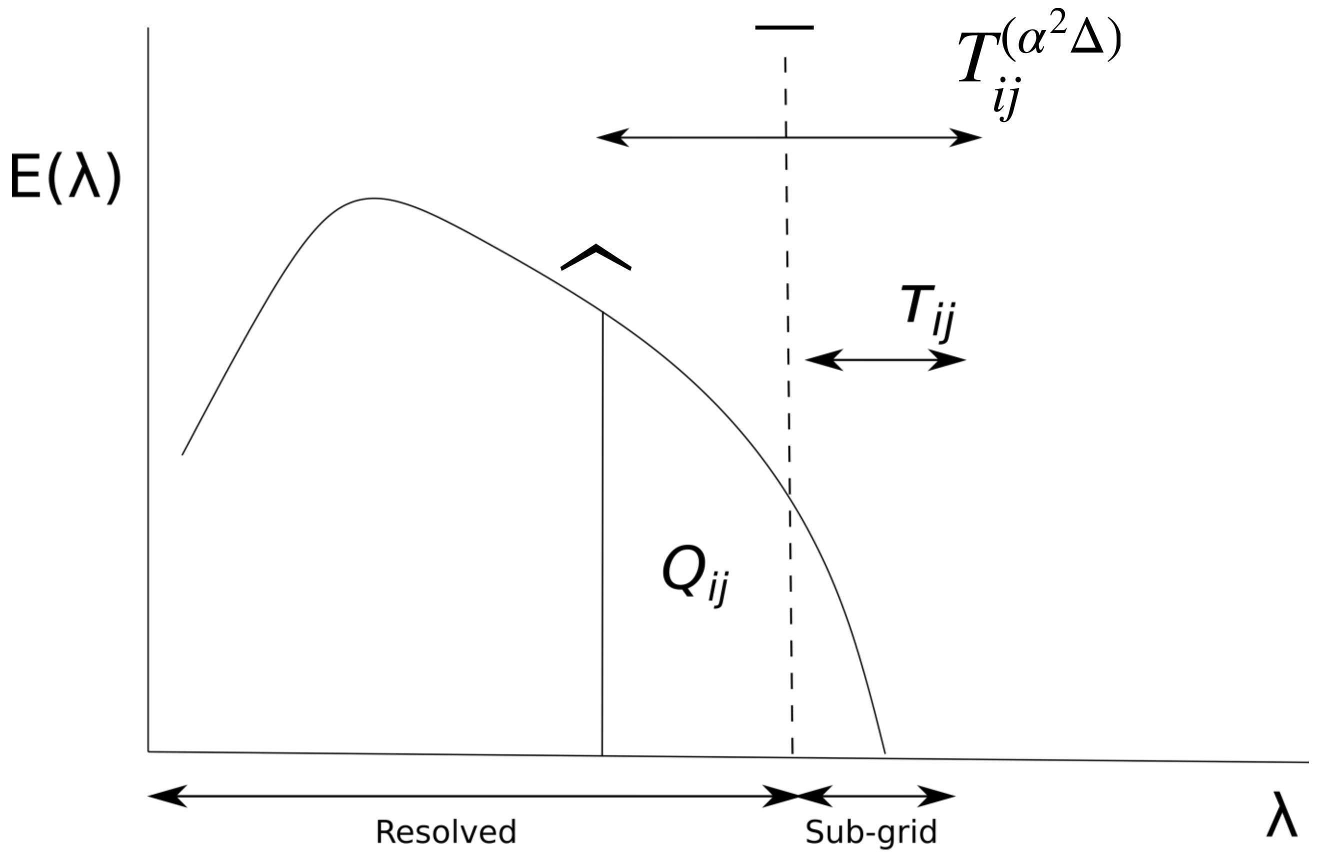


$$\beta = \frac{C_{(\alpha^2\Delta)}}{C_{(\alpha\Delta)}}$$

C ➡ Scale dependent

β ➡ Scale invariant

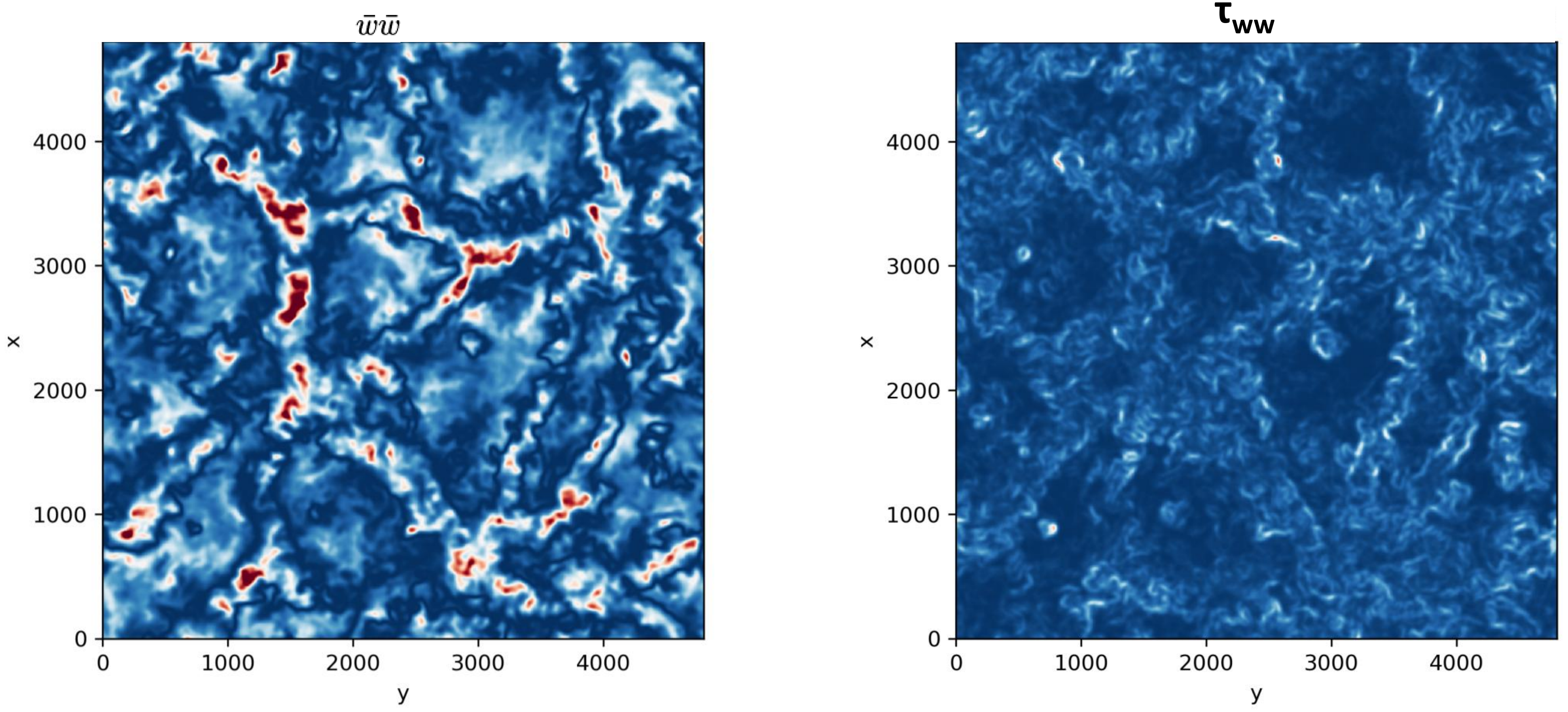
(Bou-Zeid et al. 2005)

Dynamic Turbulence Modelling

Filter a Δx = 5 m MONC CBL LES at a scale of Δ = 40 m

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

z = 250 m



Dynamic Turbulence Modelling

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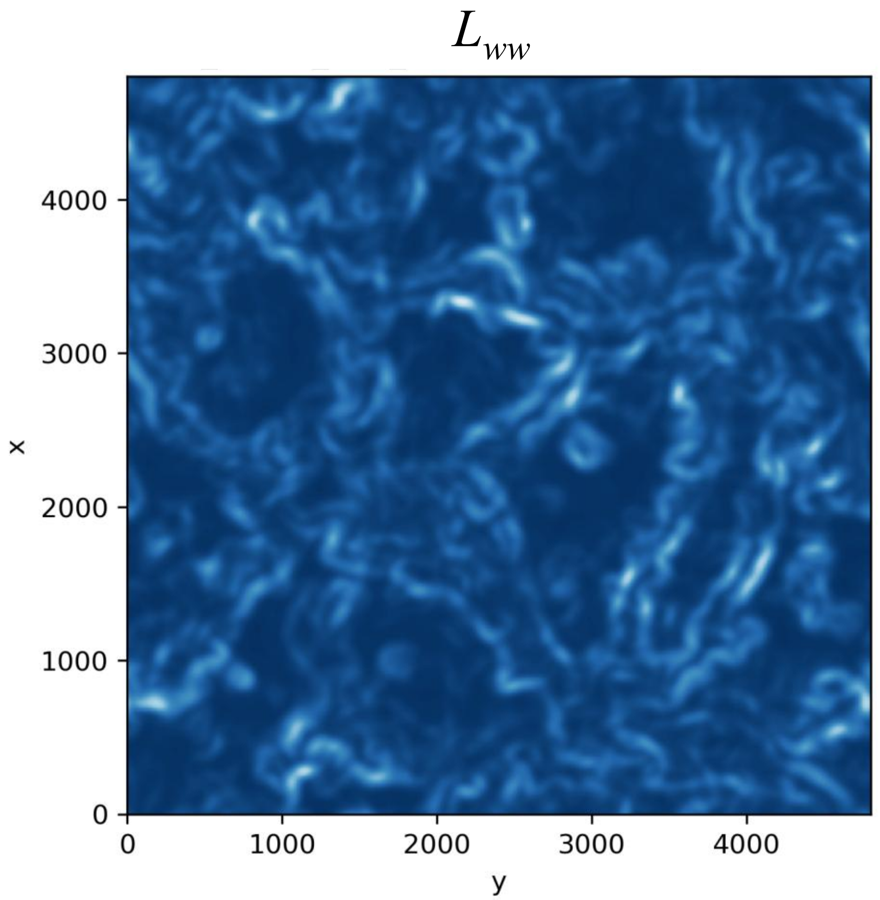
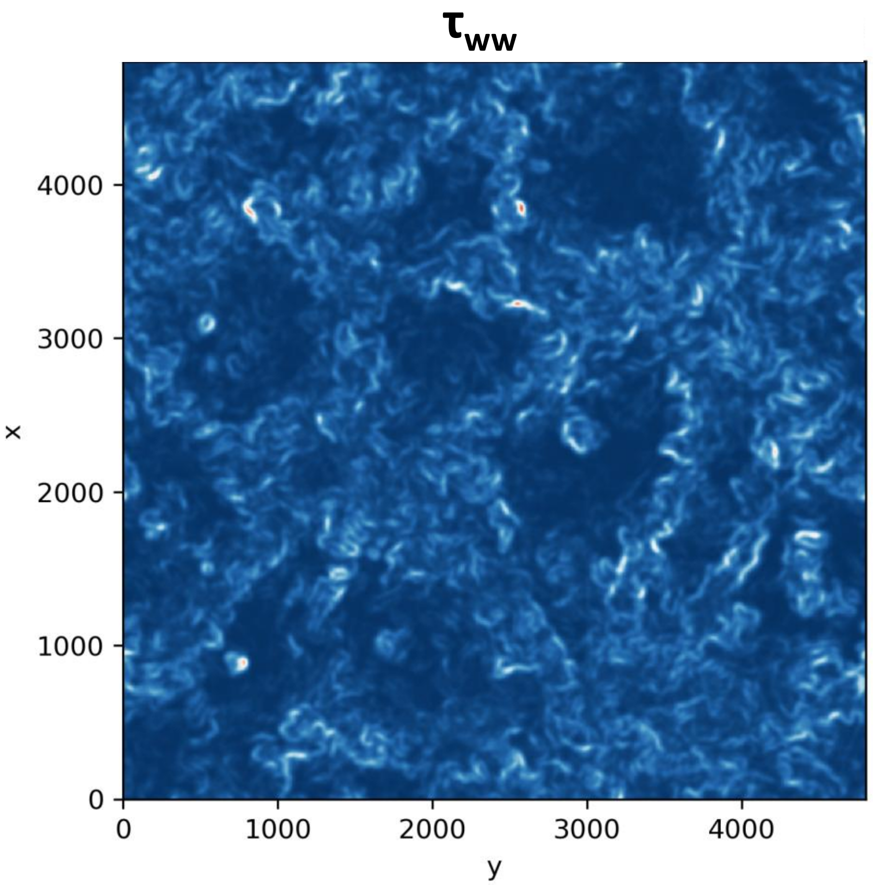


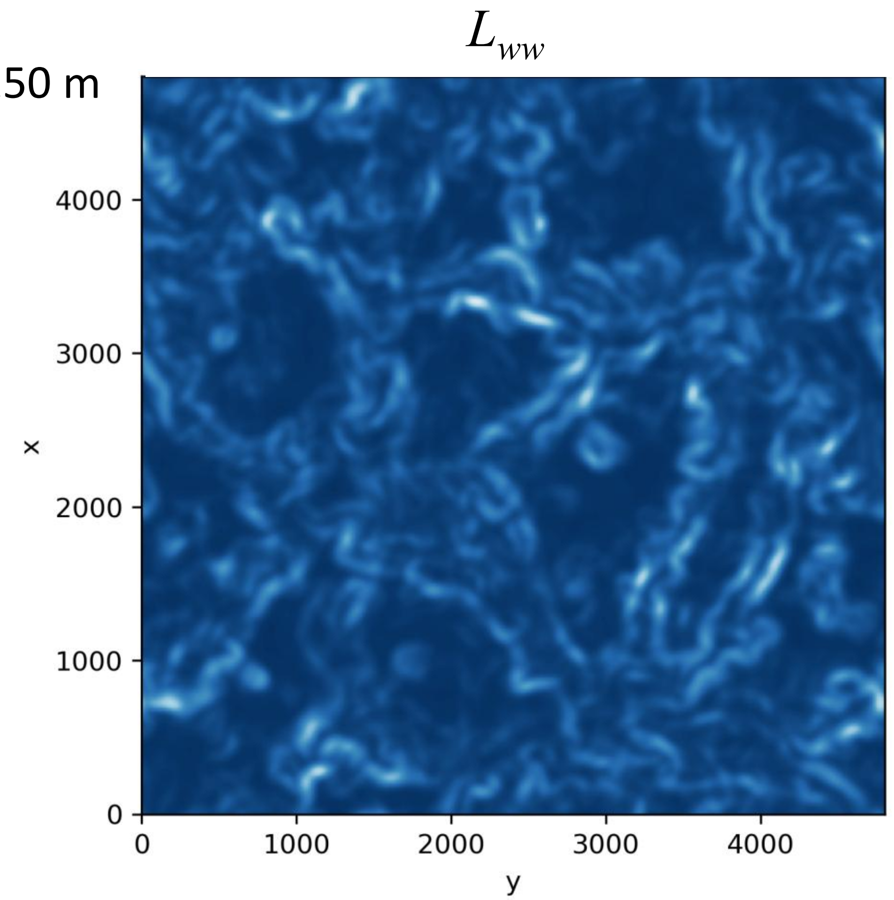


← Scale similarity →

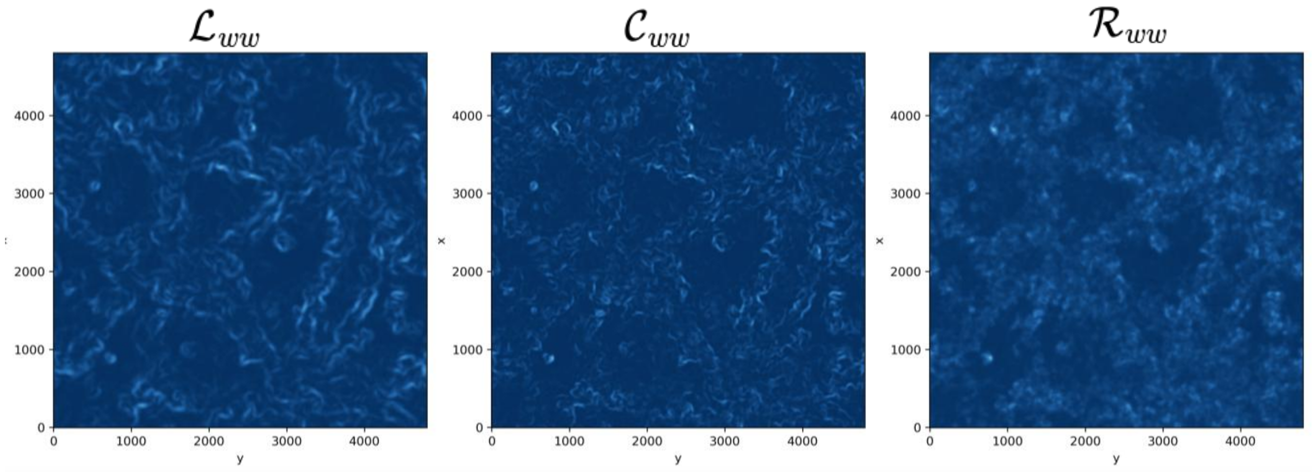
Dynamic Turbulence Modelling

Filter a dx = 5 m MONC LES at a scale of Δ = 40 m

z = 250 m



$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j = \mathcal{L}_{ij} + \mathcal{C}_{ij} + \mathcal{R}_{ij}$$



Dynamic Smagorinsky (eddy-viscocity)

- **Smagorinsky:** local equilibrium between dissipation and turbulent production

$$\tau_{ij} = -2(\boxed{C_s}\Delta)^2|\bar{S}|\bar{S}_{ij}\bar{f}_m$$

$$h_i = -\frac{(C_s\Delta)^2}{\boxed{\mathrm{Pr}}}|\bar{S}|\frac{\partial\bar{\theta}}{\partial x_i}\bar{f}_h$$

$$\left(\lambda = C_S\Delta\right)$$

$$\underbrace{\boxed{\frac{1}{\lambda^2} = \frac{1}{(kz)^2} + \frac{1}{(C_S\Delta)^2}}}_{\text{SMAG}}$$

Dynamic Mixed Model (SMAG + Leonard stress)

Spatially filtered equations

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j = \mathcal{L}_{ij} + \mathcal{C}_{ij} + \mathcal{R}_{ij}$$

$$\tau_{ij} = \overbrace{-2(C_s\Delta)^2 |\bar{S}| \bar{S}_{ij} \bar{f}_m}^{\mathcal{R}_{ij}} + \overbrace{C_n(\widetilde{\bar{u}_i \bar{u}_j} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j)}^{\mathcal{L}_{ij} + \mathcal{C}_{ij}}$$

$$h_i = -\frac{(C_s\Delta)^2}{\mathrm{Pr}} |\bar{S}| \frac{\partial \bar{\theta}}{\partial x_i} \bar{f}_h + C_{n\theta}(\widetilde{\bar{u}_i \bar{\theta}} - \tilde{\bar{u}}_i \tilde{\bar{\theta}})$$

$$(C_n = C_{n\theta} = 1)$$

Simulations

- Met Office/NERC Cloud Model (**MONC**)
- Met Office Unified Model (**UM**)

LES: Δx = 50 m, Δz = 20 m

Grey – zone runs : Δx = 200 m – 800 m

MONC vertical resolution ARM: Δz = 40 m
LBA: Δz = 50 m – 200 m

UM vertical resolution LBA: L80 (stretched)

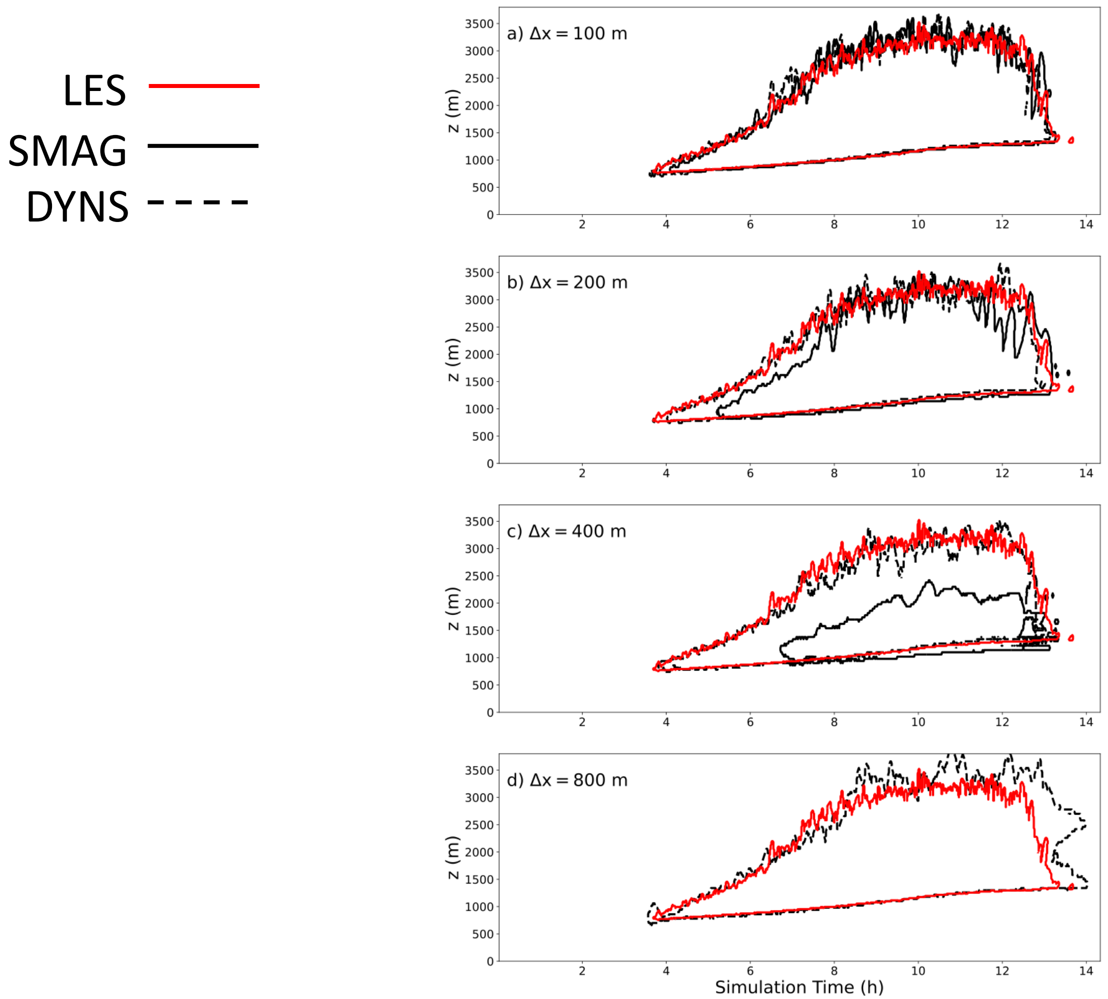
ARM

Diurnal cycle of Shallow Cu over land

LBA

Shallow to deep convection transition over the Amazon

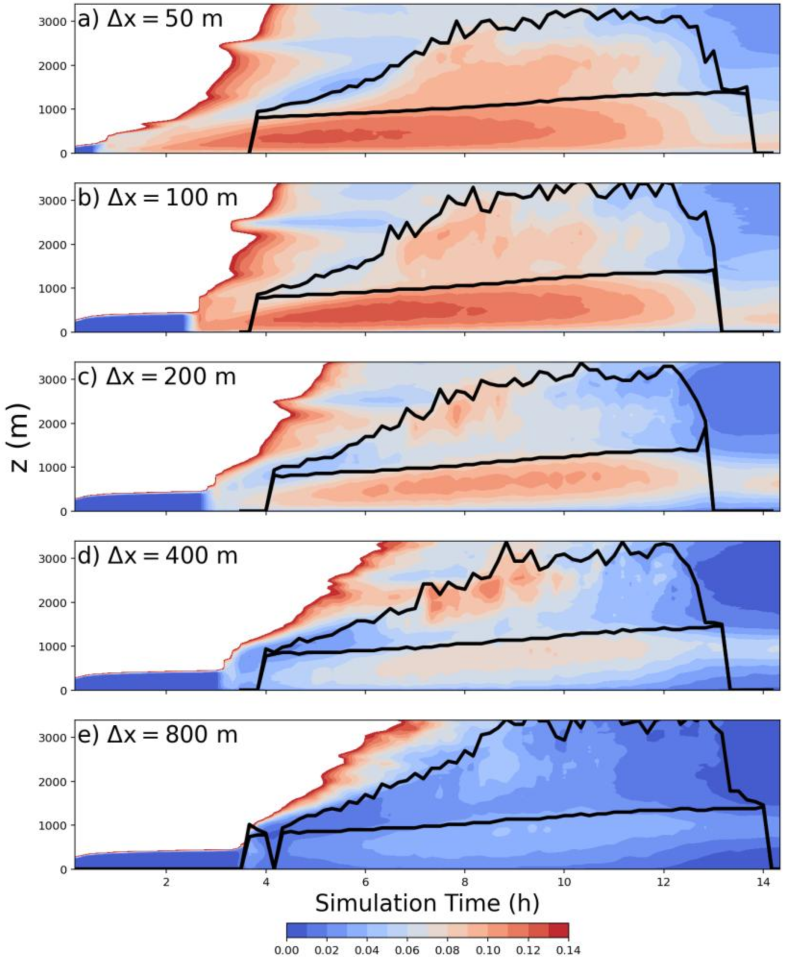
Diurnal cycle of shallow convection over land (ARM) – MONC simulations



(Efstathiou, 2023)

Diurnal cycle of shallow convection over land (ARM) – MONC simulations

Smagorinsky Coefficient

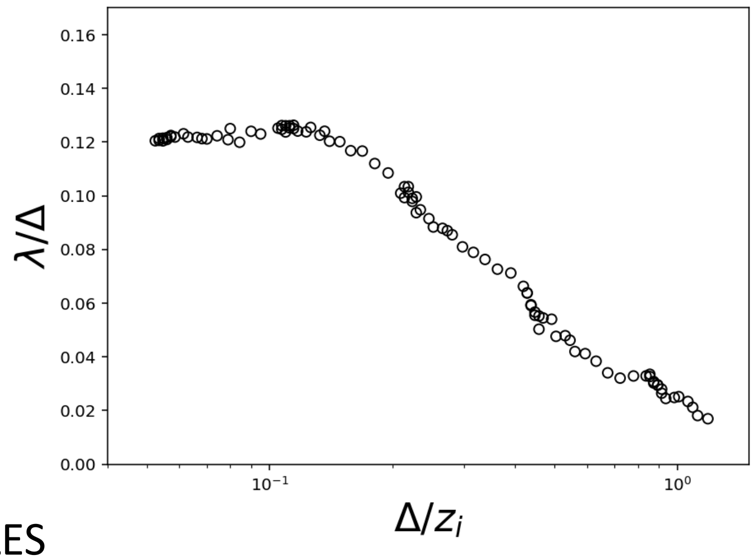


λ / Δ

Diurnal cycle of shallow convection over land (ARM) – MONC simulations

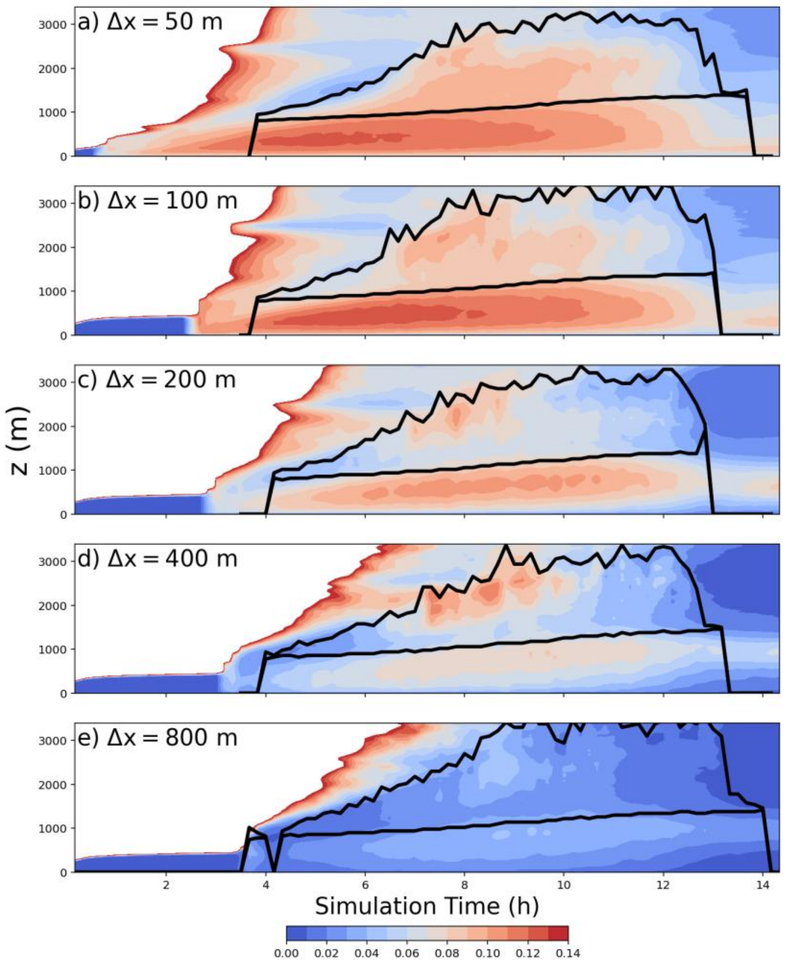
Smagorinsky Coefficient

$z = 0.5\ z_i$



LES

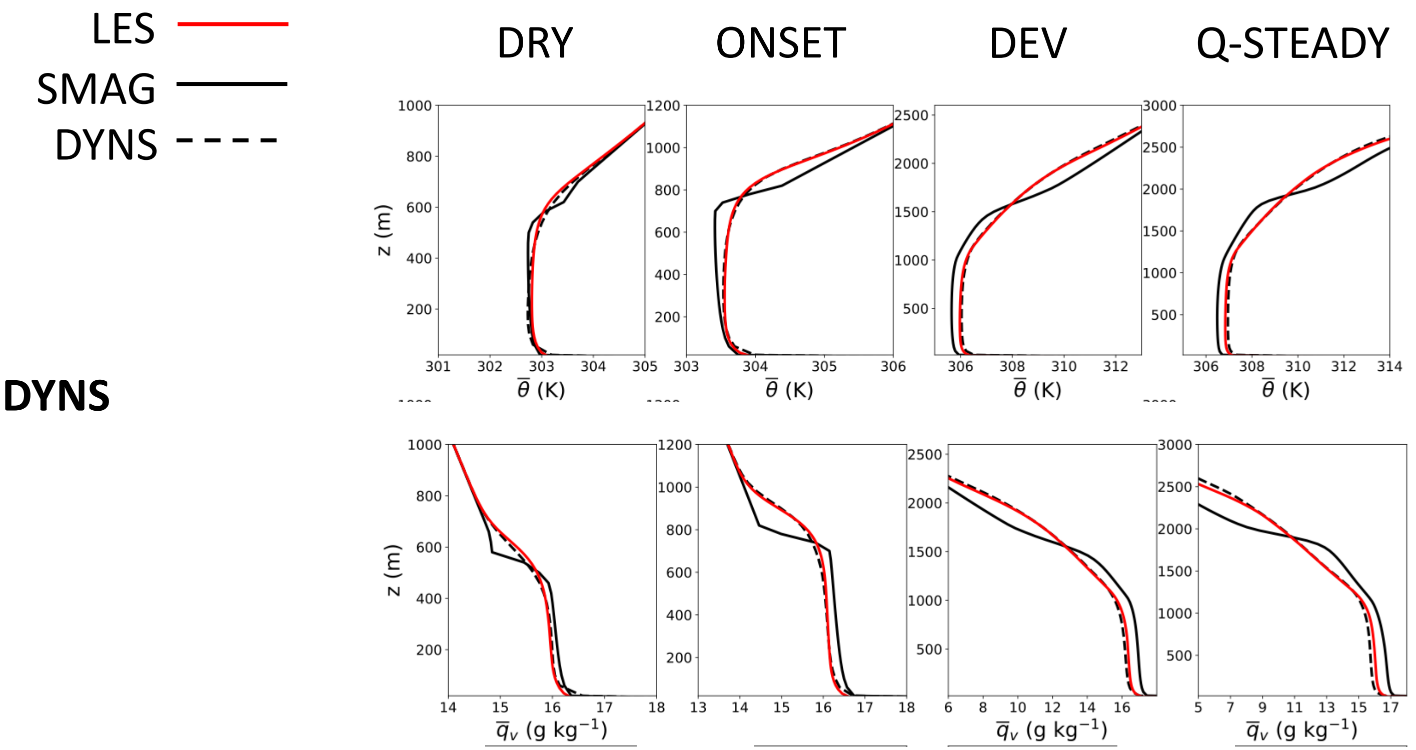
RANS



λ/Δ

Diurnal cycle of shallow convection over land (ARM)

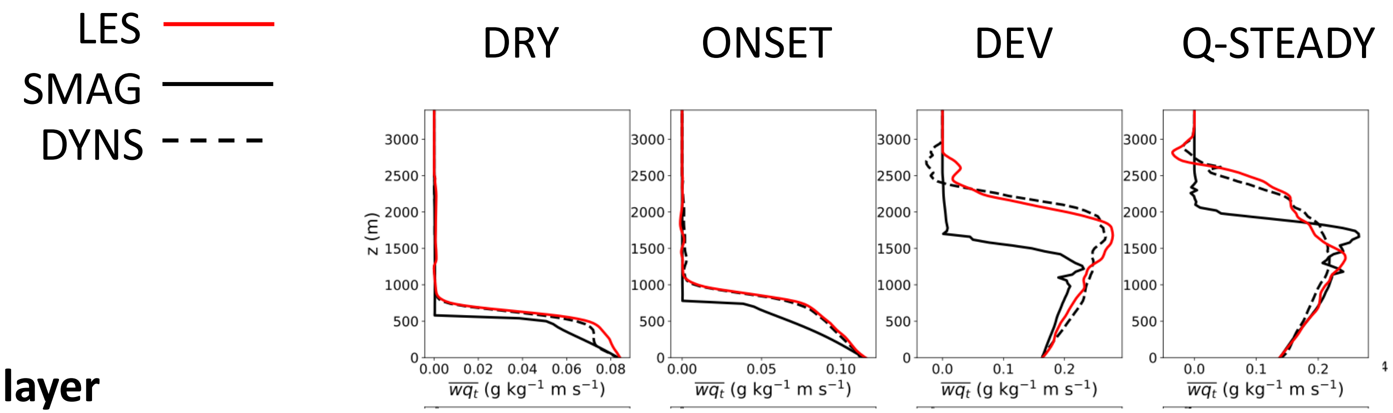
Δx = 400 m



Improved CBL Representation for DYNS

Diurnal cycle of shallow convection over land (ARM)

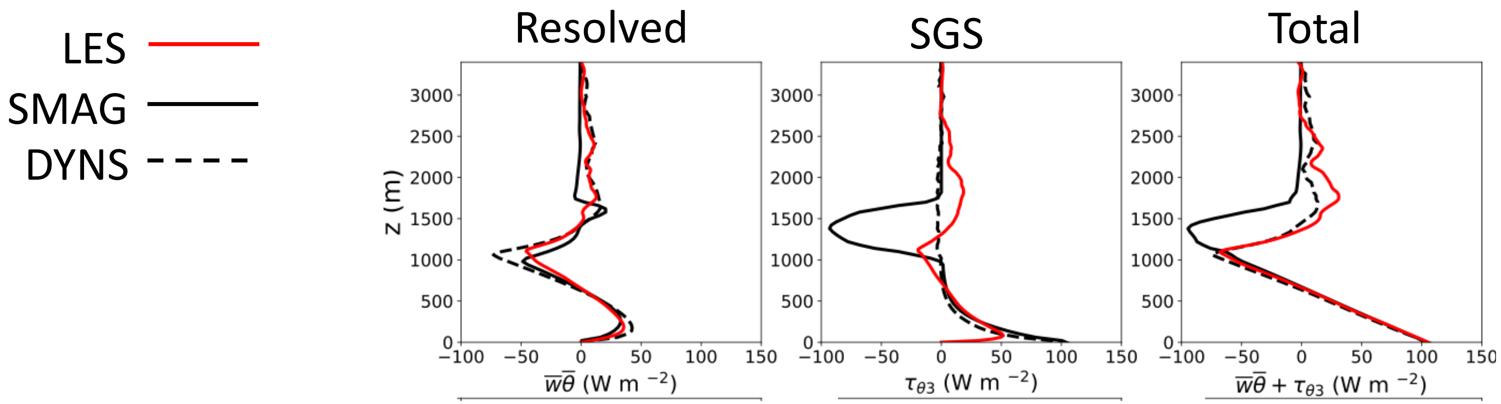
Δx = 400 m



Improved water transport at cloud layer

Diurnal cycle of shallow convection over land (ARM)

Δx = 400 m



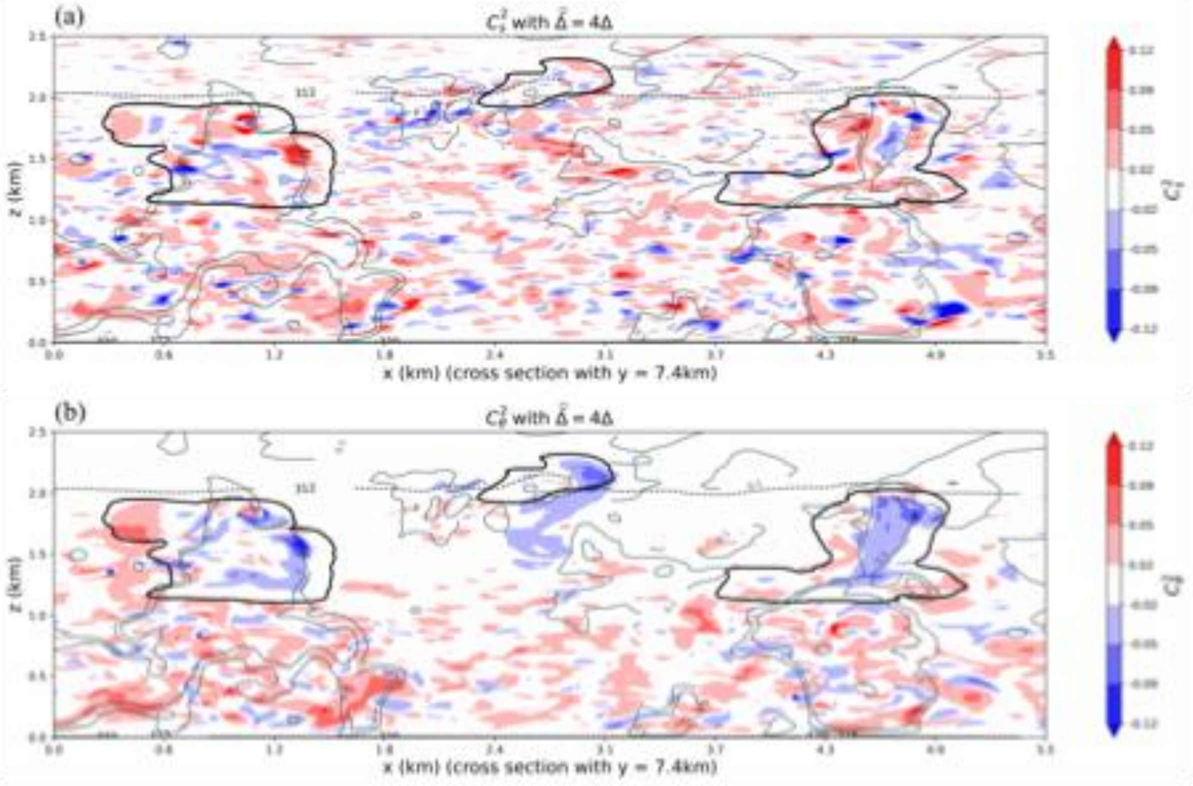
Dynamic Smagorinsky solely depends on resolved flow

Diurnal Convection over land (ARM)

(Alanna Power, PhD thesis 2025)

Momentum

Heat



Diagnostic dynamic derivation of coefficients

(Efstathiou et al. 2024)

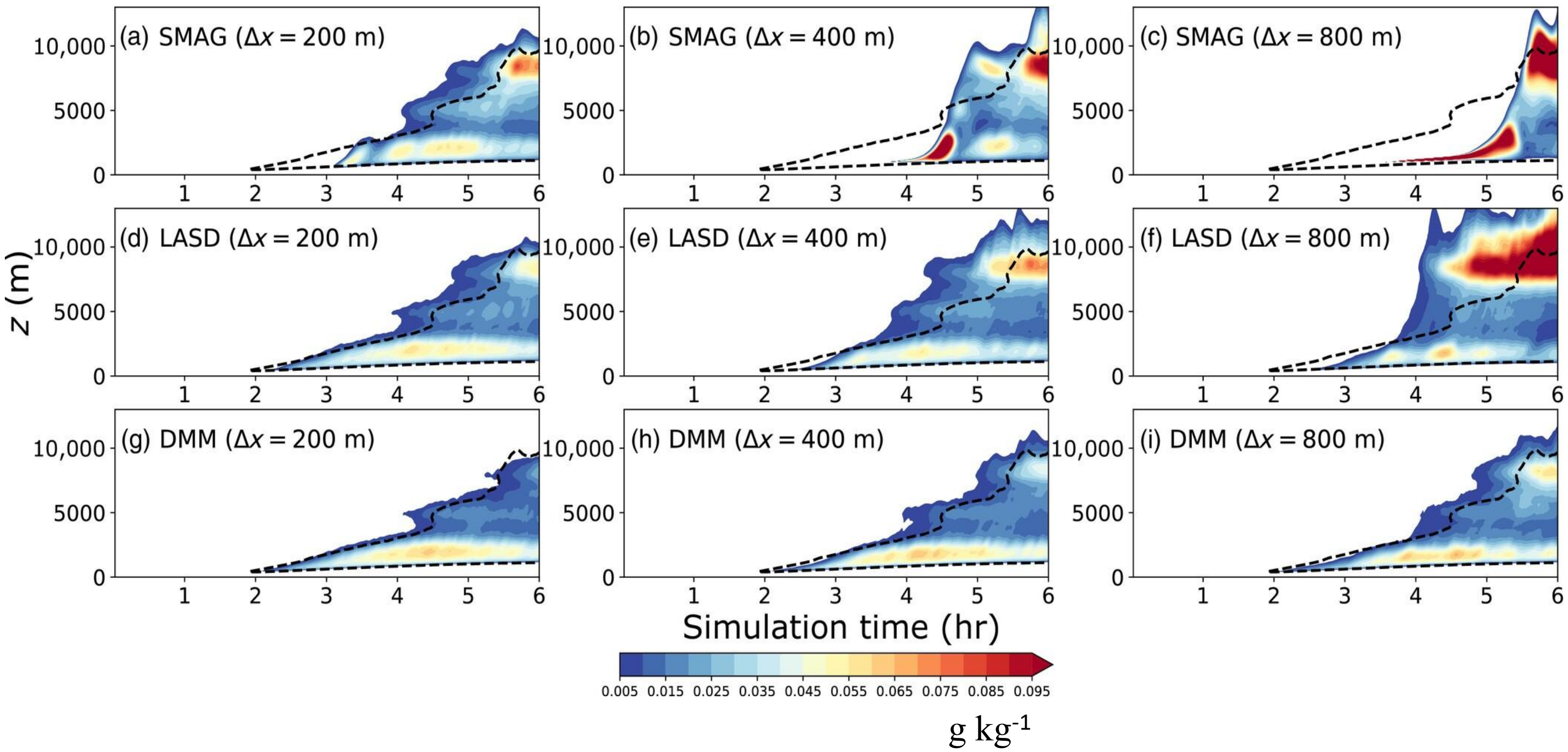
LBA Case study

Hydrometeor evolution $(q_l + q_s + q_i + q_g)$

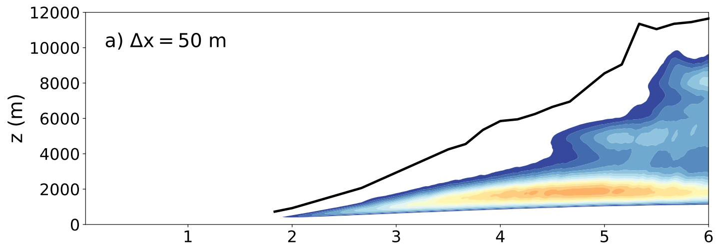
SMAG

DYNS

DMM



MONC LES

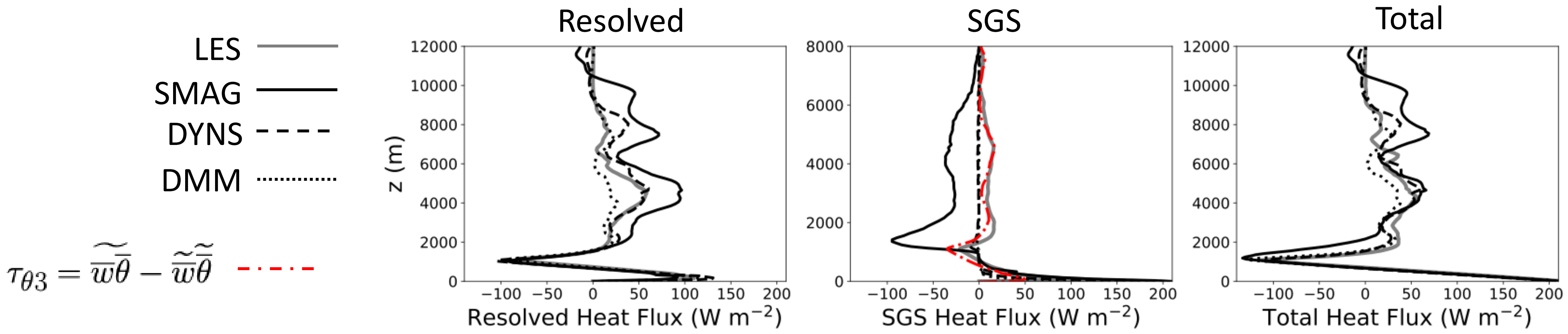


Transition from Shallow to Deep Convection (LBA)

(Efstathiou et al. 2024)

Δx = 400 m

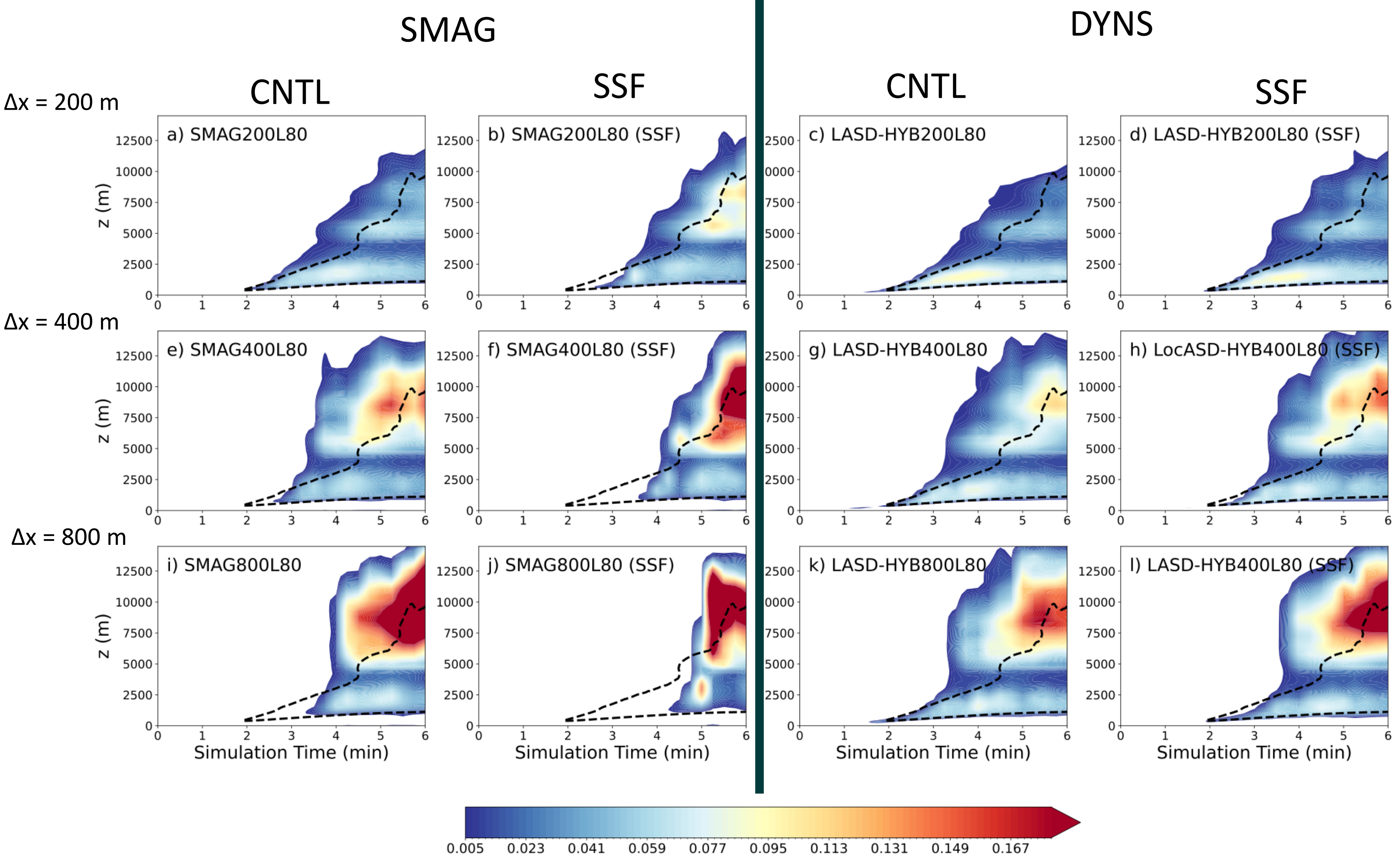
Deep stage (Cb) (t = 6 h)



Transition from Shallow to Deep Convection (LBA) – UM

Sensitivity on stability functions

CNTL: Less diffusive
SSF : More diffusive



Hydrometeor evolution

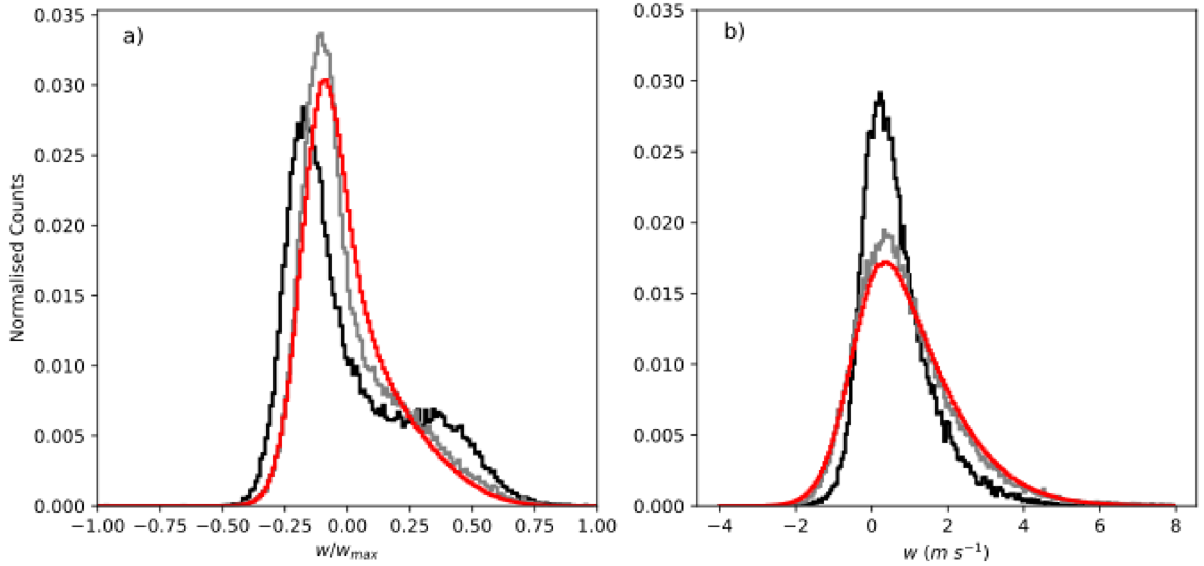
Transition from Shallow to Deep Convection (LBA) – UM

Shallow convection stage (t = 3.5 h) Δx = 400 m

CNTL RUN

LES
SMAG
DYNS

$z = 0.5\ z_i$ In-cloud

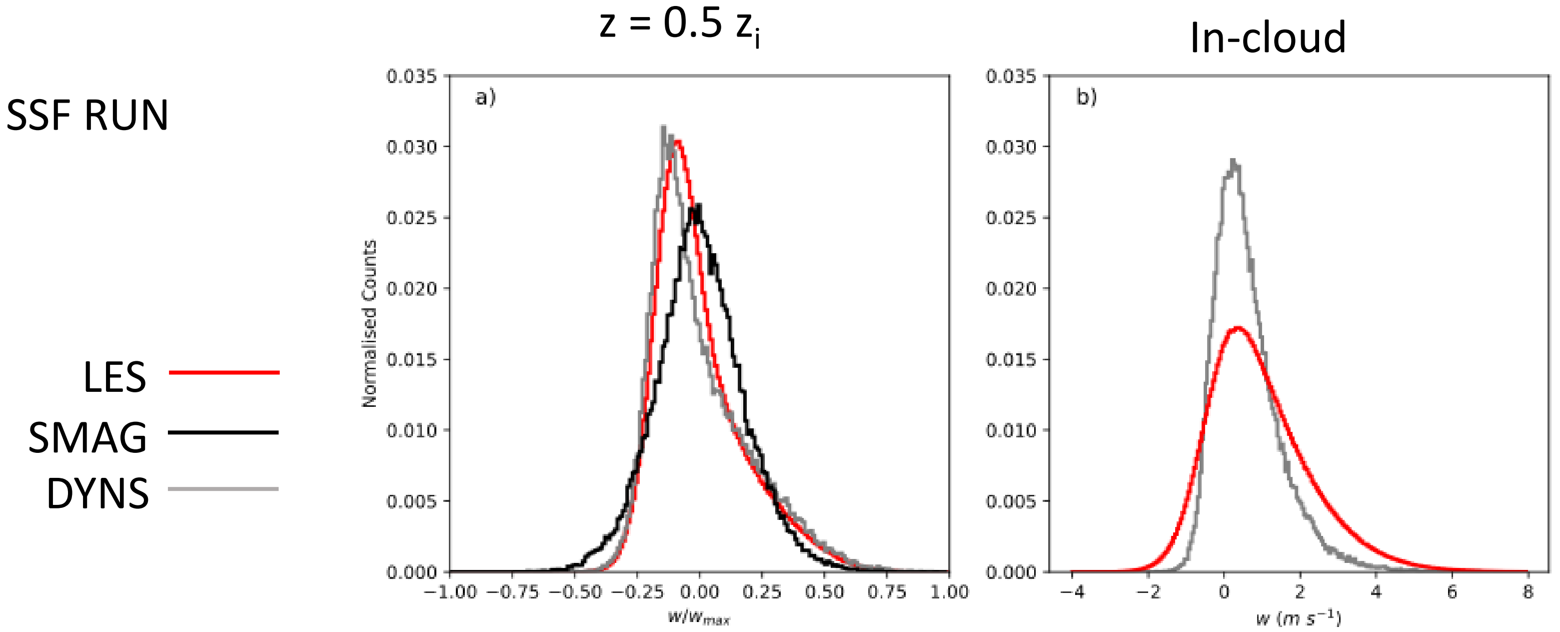


Distribution of vertical velocities

Transition from Shallow to Deep Convection (LBA) – UM

Shallow convection stage (t = 3.5 h)   Δx = 400 m

SSF RUN



Distribution of vertical velocities

Transition from Shallow to Deep Convection (LBA)

Impact of less diffusive stability functions

Δx = 400 m

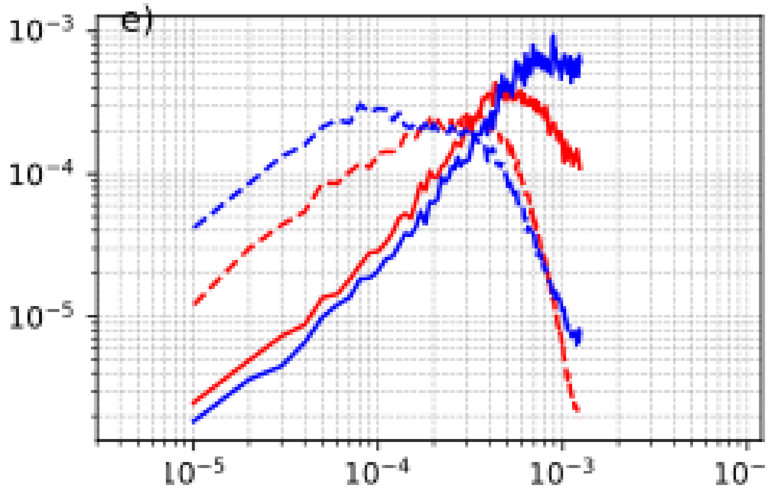
SMAG (x)

DYNS (x)

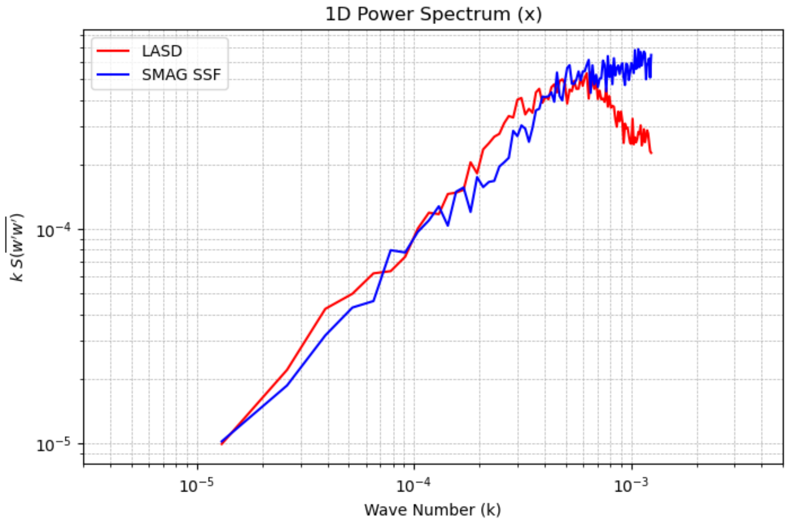
SMAG (y)

DYNS (y)

UM



MONC

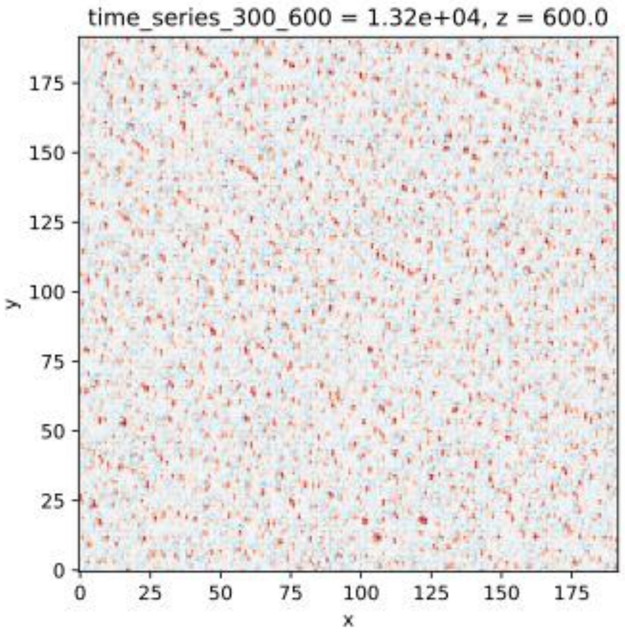


Shallow convection stage (t = 3.5 h)

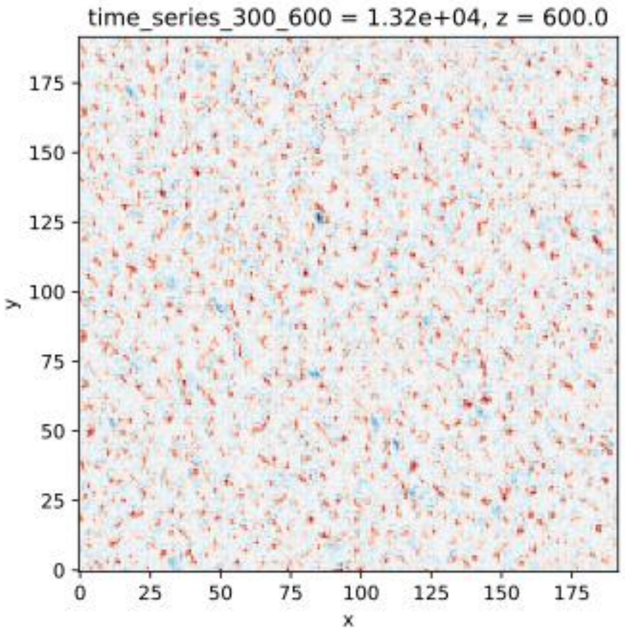
Transition from Shallow to Deep Convection (LBA) – UM

Impact of more diffusive stability functions

SMAG SSF



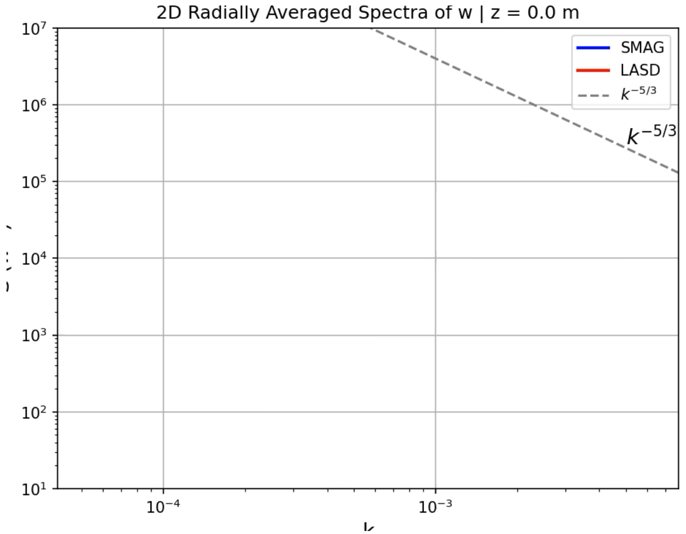
DYNS



Transition from Shallow to Deep Convection (LBA) – UM

2D vertical velocity spectra vs height (z)

Δx = 200 m



Thoughts

- Dynamic approaches
  - Locally adjust dissipation, modulating the resolved field
  - Faster spin-up of convective overturning (handle transitions)
  - Optimised Less sensitivity to adjustable parameters
  - Dependence on the resolved flow field – Stability/scale
- Eddy viscosity limitations
- Backscatter from Leonard terms (counter-gradient fluxes)
- Higher-order terms to capture the transition
- Usability limit