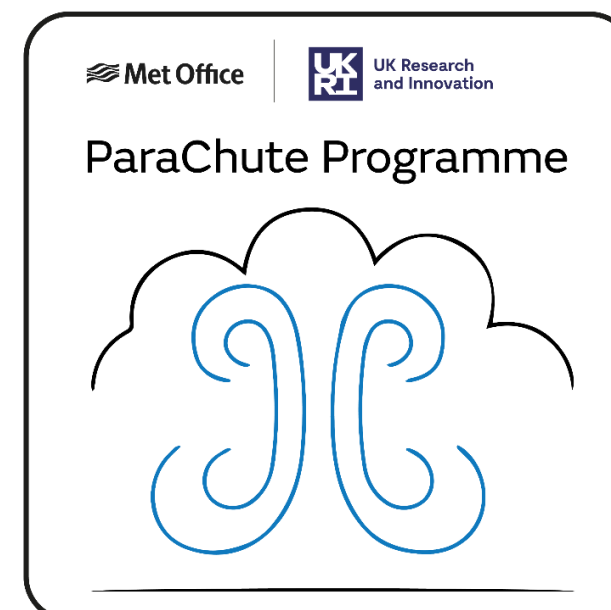


SGS Anisotropy for Turbulent Grey Zone

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+

G Efstathiou, R Beare, P Clark,
R Plant, Y Bao, P Burns



Turbulence closure & the Gray Zone

$$\partial_t \bar{u}_i + \partial_j (\bar{u}_i \bar{u}_j) = -\partial_j \tau_{ij} + \dots ;$$

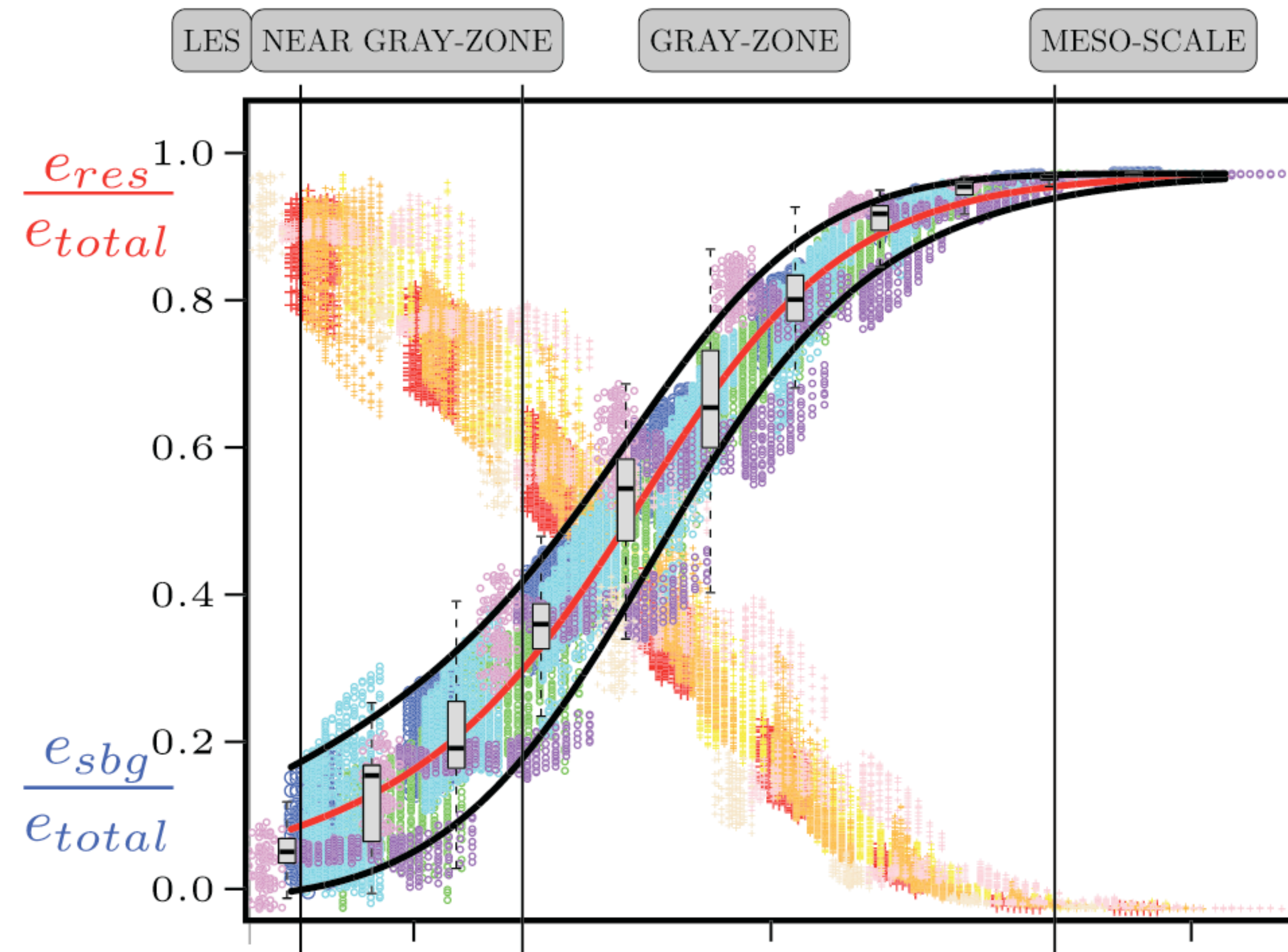
$$\partial_t \bar{\theta} + \partial_j (\bar{u}_j \bar{\theta}) = -\nabla \cdot \tau_\theta + \dots$$

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \quad \tau_{\theta i} = \overline{u_i \theta} - \bar{u}_i \bar{\theta}$$

- Smagorinsky-Lilly eddy-diffusivity

$$\tau_{ij} = -\nu \bar{S}_{ij} \quad \tau_\theta = -\nu_\theta \nabla \bar{\theta}$$

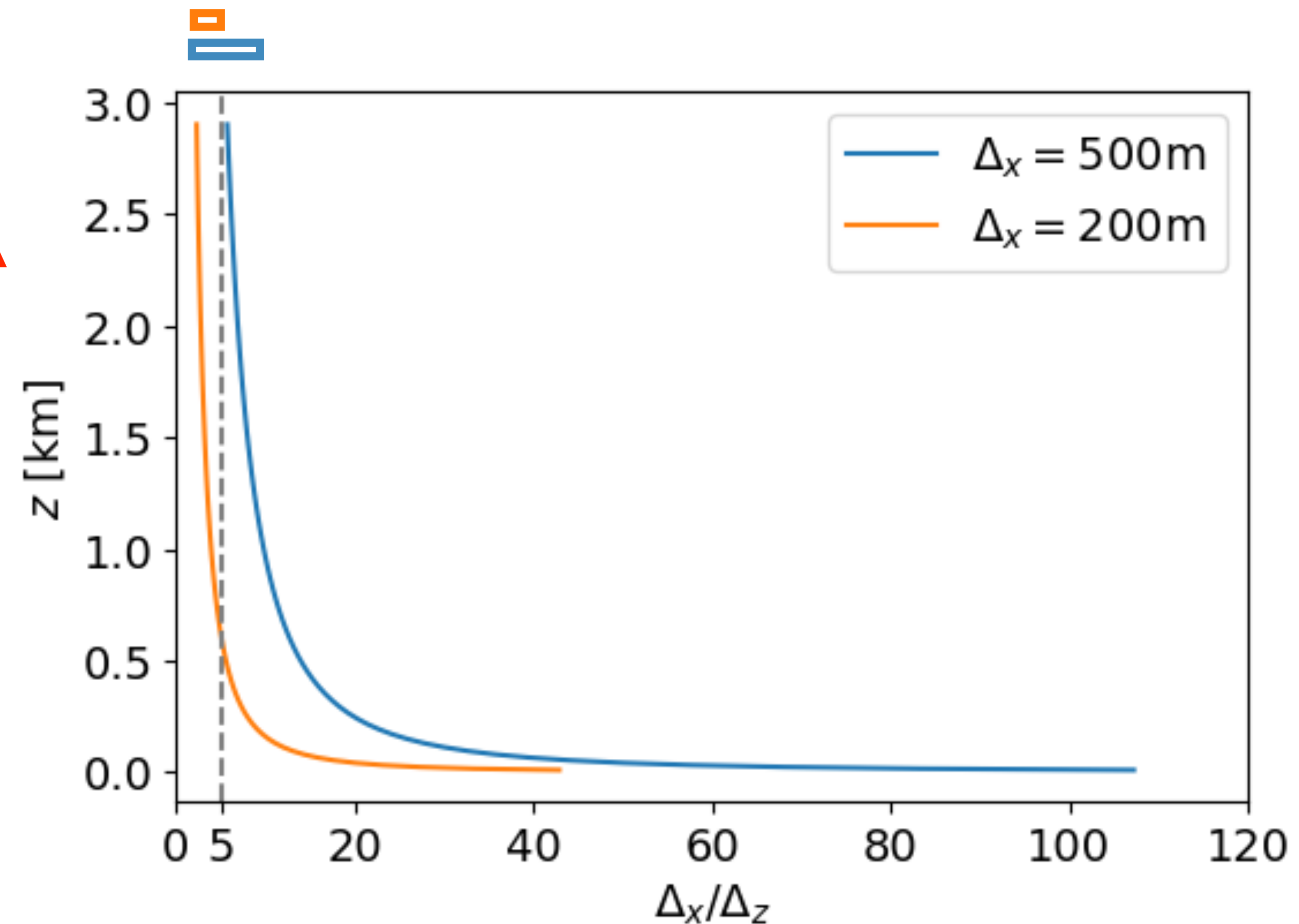
$$\nu = -l^2 |\bar{S}| \quad \nu_\theta = -l_\theta^2 |\bar{S}|$$



Honnert+2002 doi:10.1029/2019JD030317

SGS modelling for Anisotropic grids

- Smagorinsky: $\nu_{\star} = -l_{\star}^2 |\tilde{S}|$; $l = C_s \Delta$
- Numerical error $\rightarrow$ grid scales $O(\Delta^n)$



Grid cells (to scale)

Dynamic model: $l_s(\Delta), l_\theta(\Delta)$

$$\tau(\tilde{u}) = \widetilde{\tau(\bar{u})} + L^{(\sim)}(\bar{u})$$

Germano identity

$$L(\bar{u}) = \widetilde{\bar{u} \bar{u}} - \tilde{u} \tilde{u};$$

$$\bar{l}^2 M(\bar{u}) = L(\bar{u})$$

Scale-similarity: $\bar{l} \leftarrow \tilde{l}$

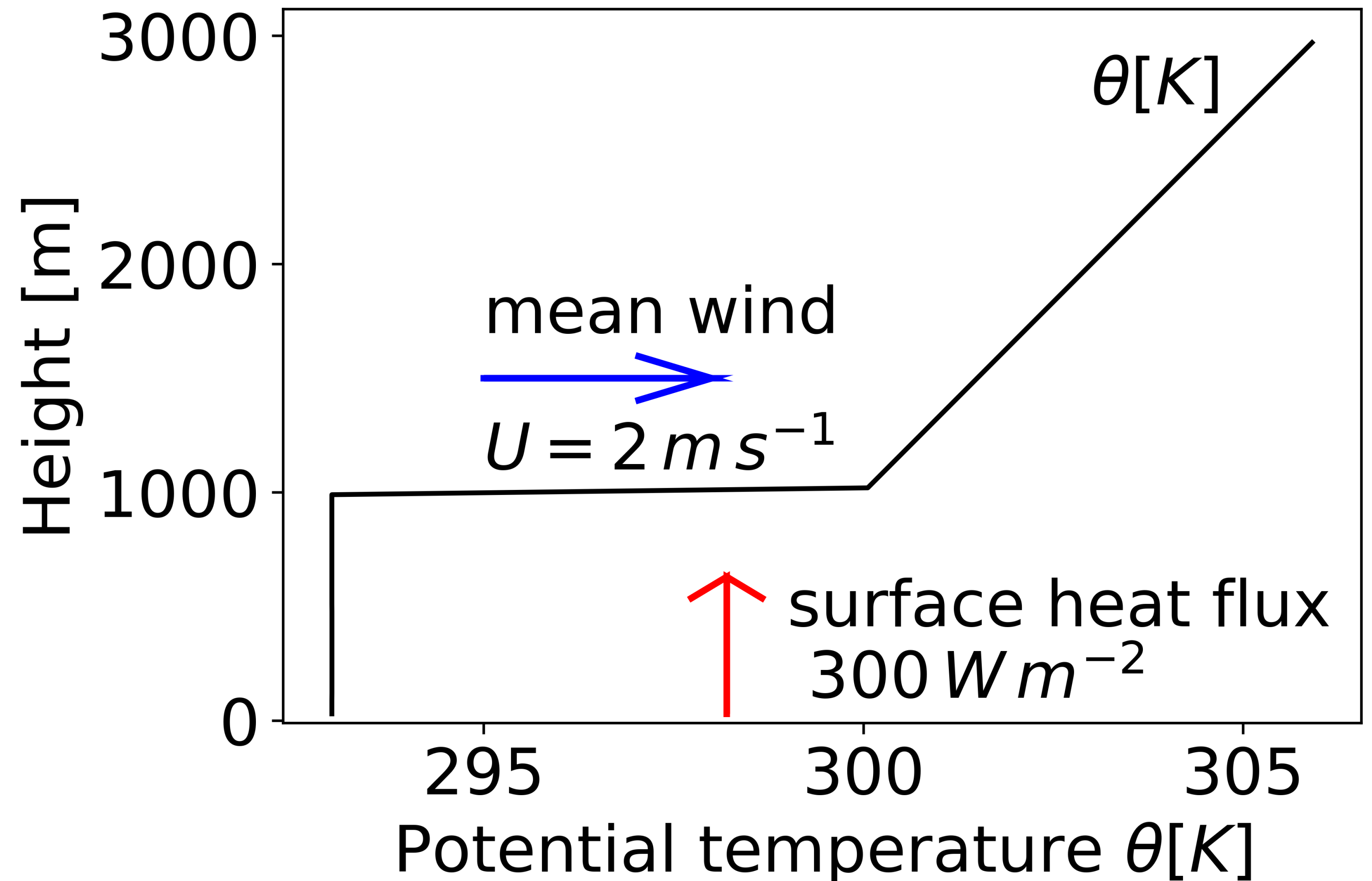
$$\bar{l}^2 M = \widetilde{\tau(\bar{u})} - \tau(\tilde{u})$$

$$\bar{l}^2 = \frac{\langle L(\bar{u}) \cdot M[\tau] \rangle}{\langle M[\tau] \cdot M[\tau] \rangle}$$

Least-square error minimisation (Lilly)

Idealised CBL test

- UK MetOffice Unified Model
- Semi-implicit
- Semi-Lagrangian
- Staggered grid



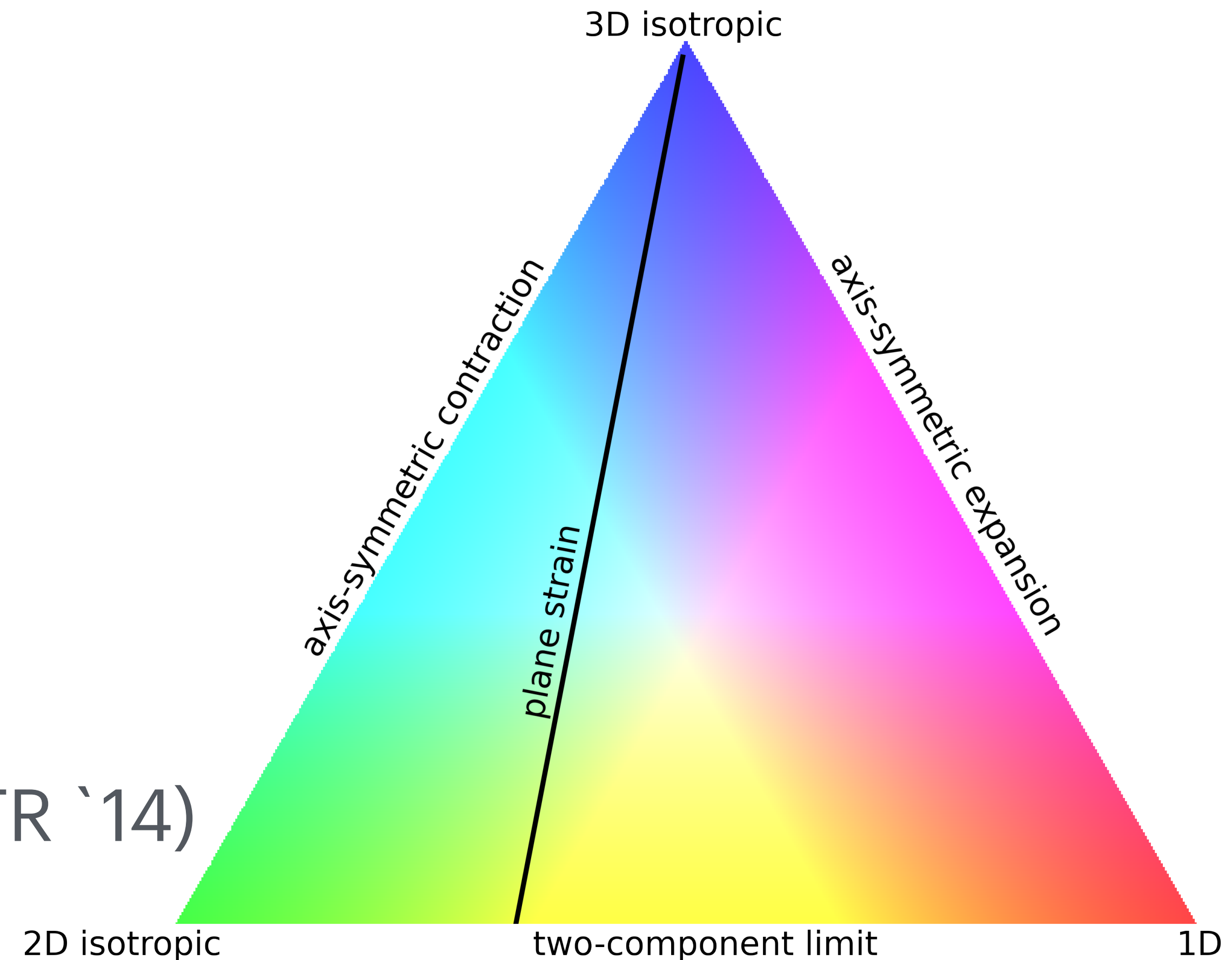
Dynamic anisotropy

$$\text{Anisotropy Tensor: } A[b_{ij}] = \frac{b_{ij}}{b_{kk}} - \frac{1}{3}\delta_{ij}$$

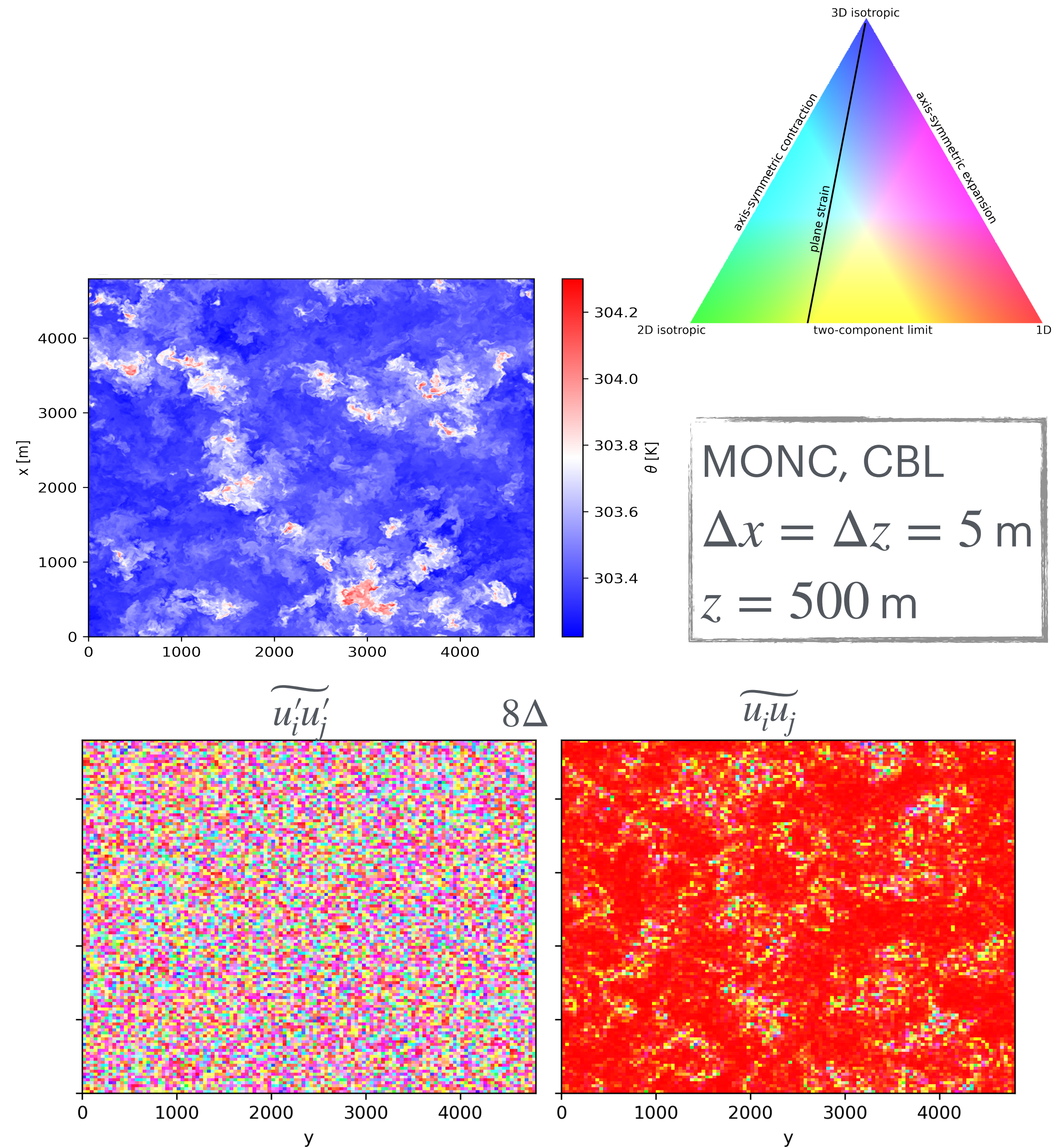
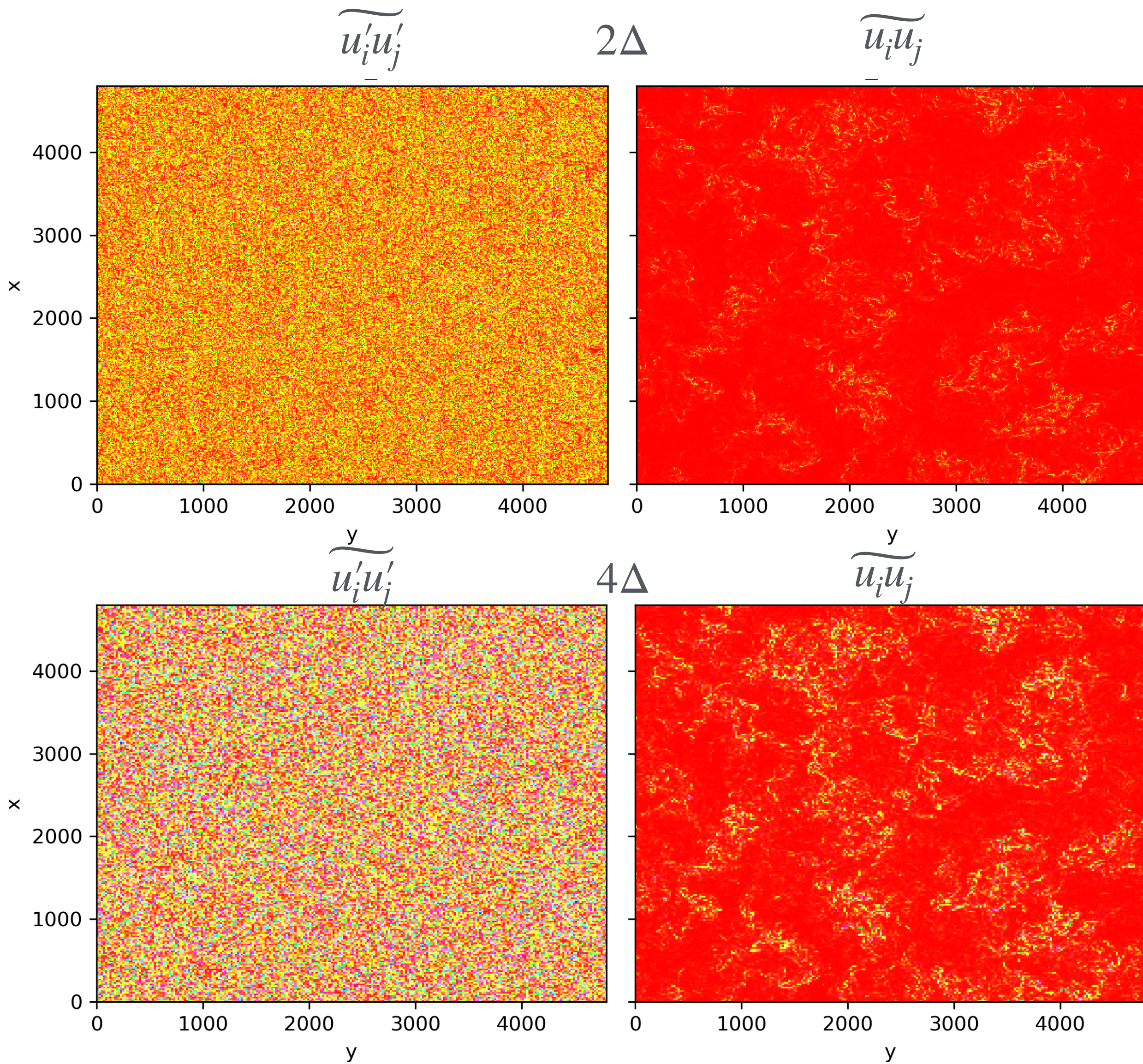
→ e-values: $-1/3 \leq \{\lambda_i\} \leq 2/3$

→ barycentric map

(Banerjee+ J.Turb '07, Emory&laccarino CTR '14)

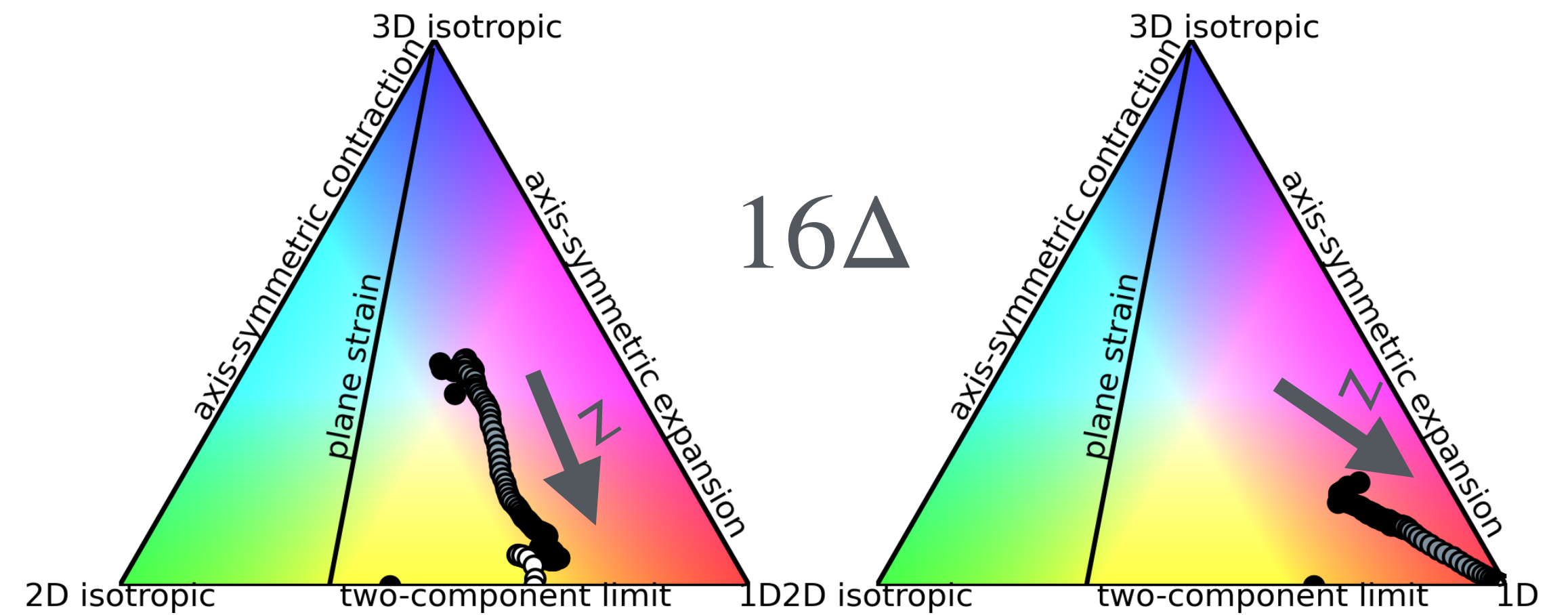
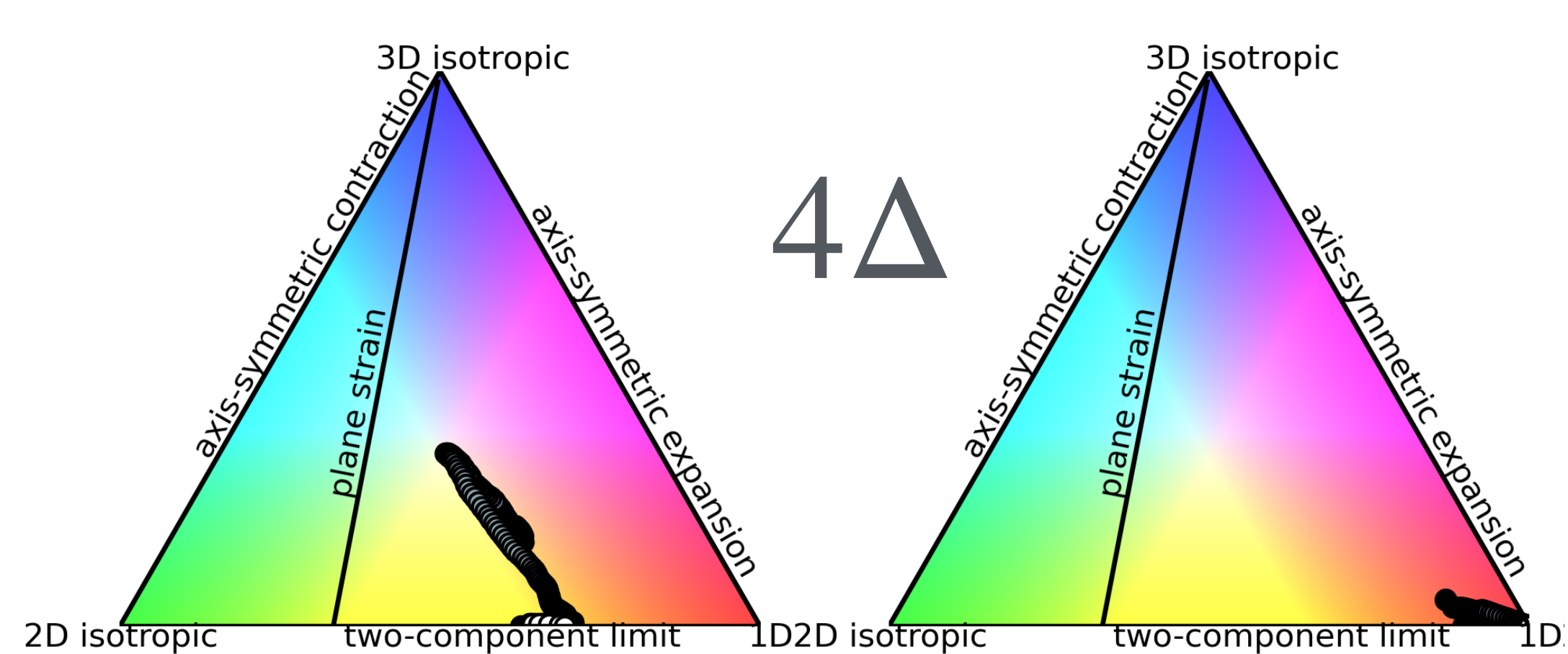
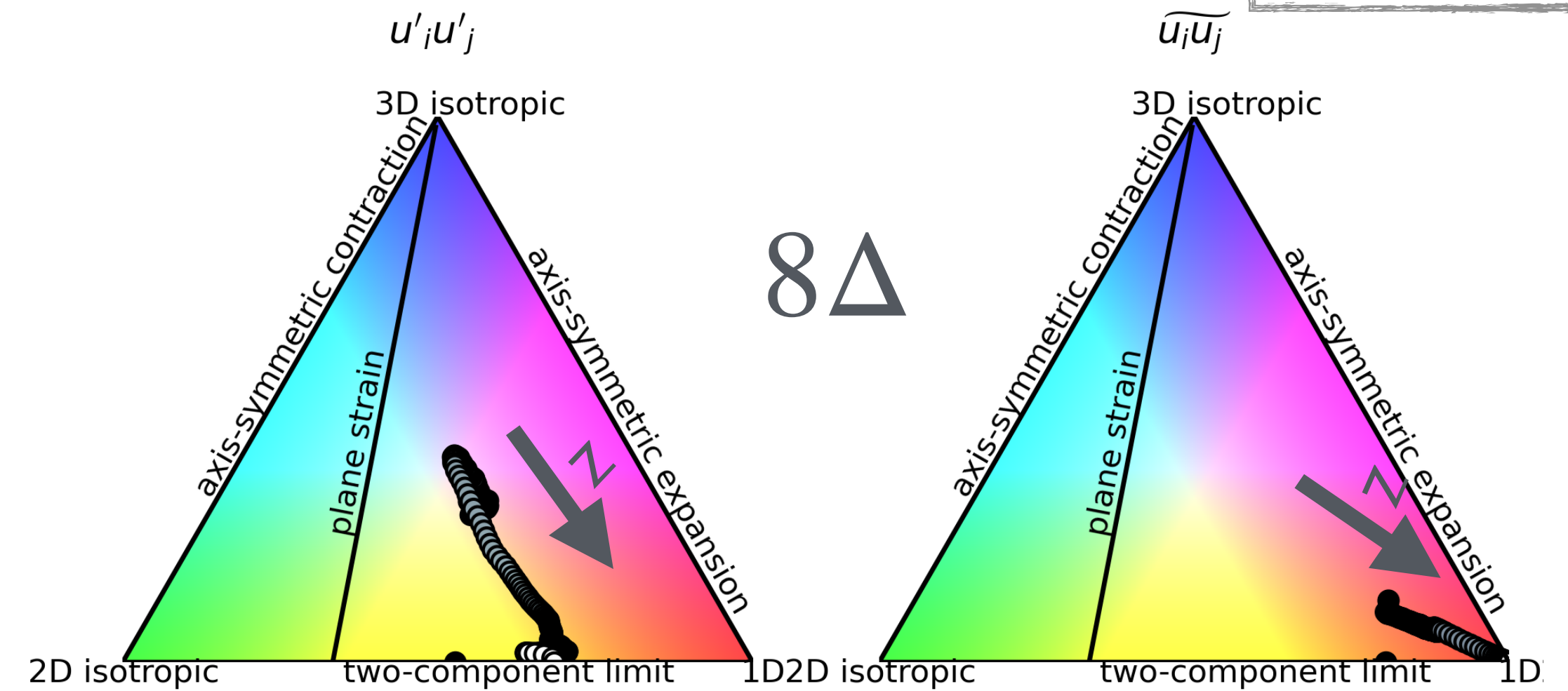
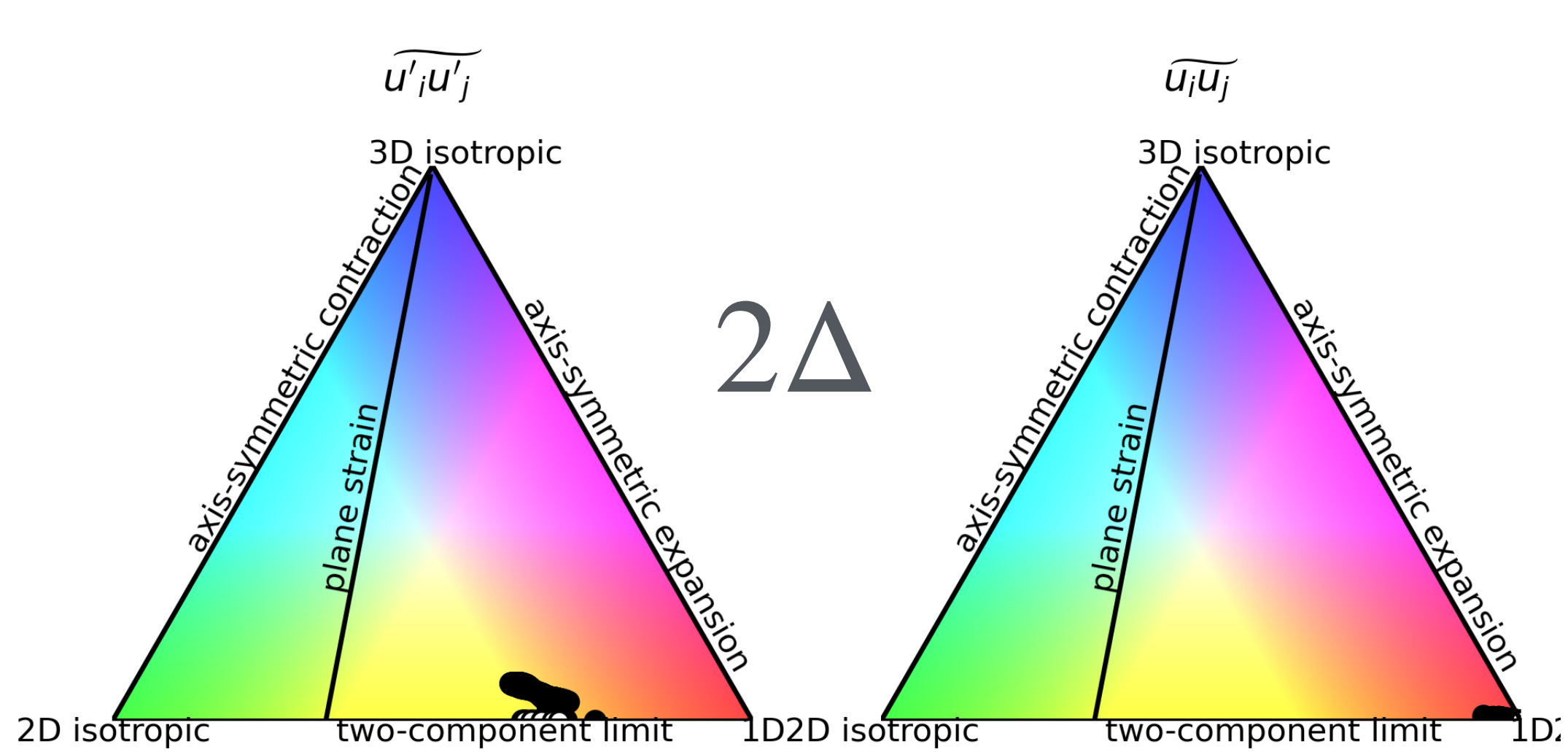


Dynamic anisotropy

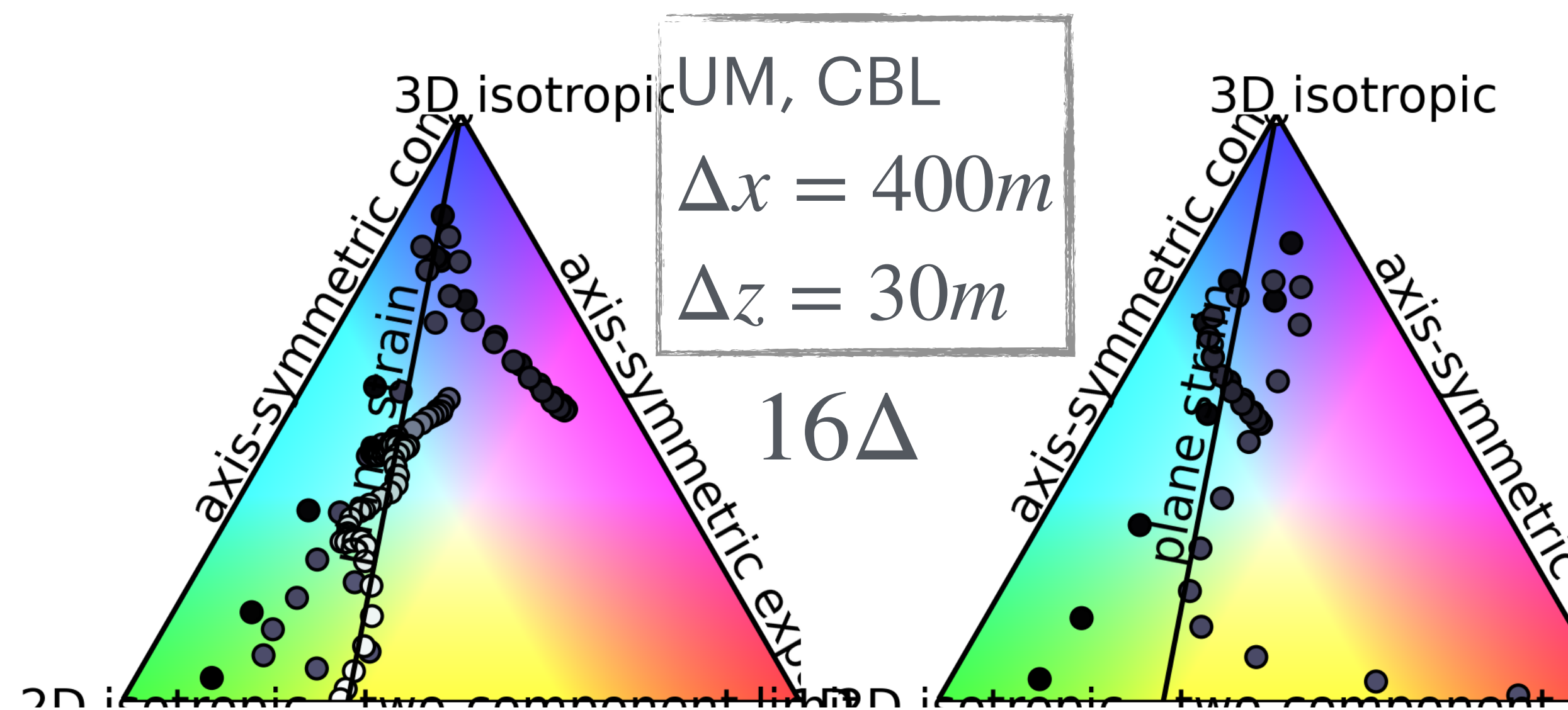
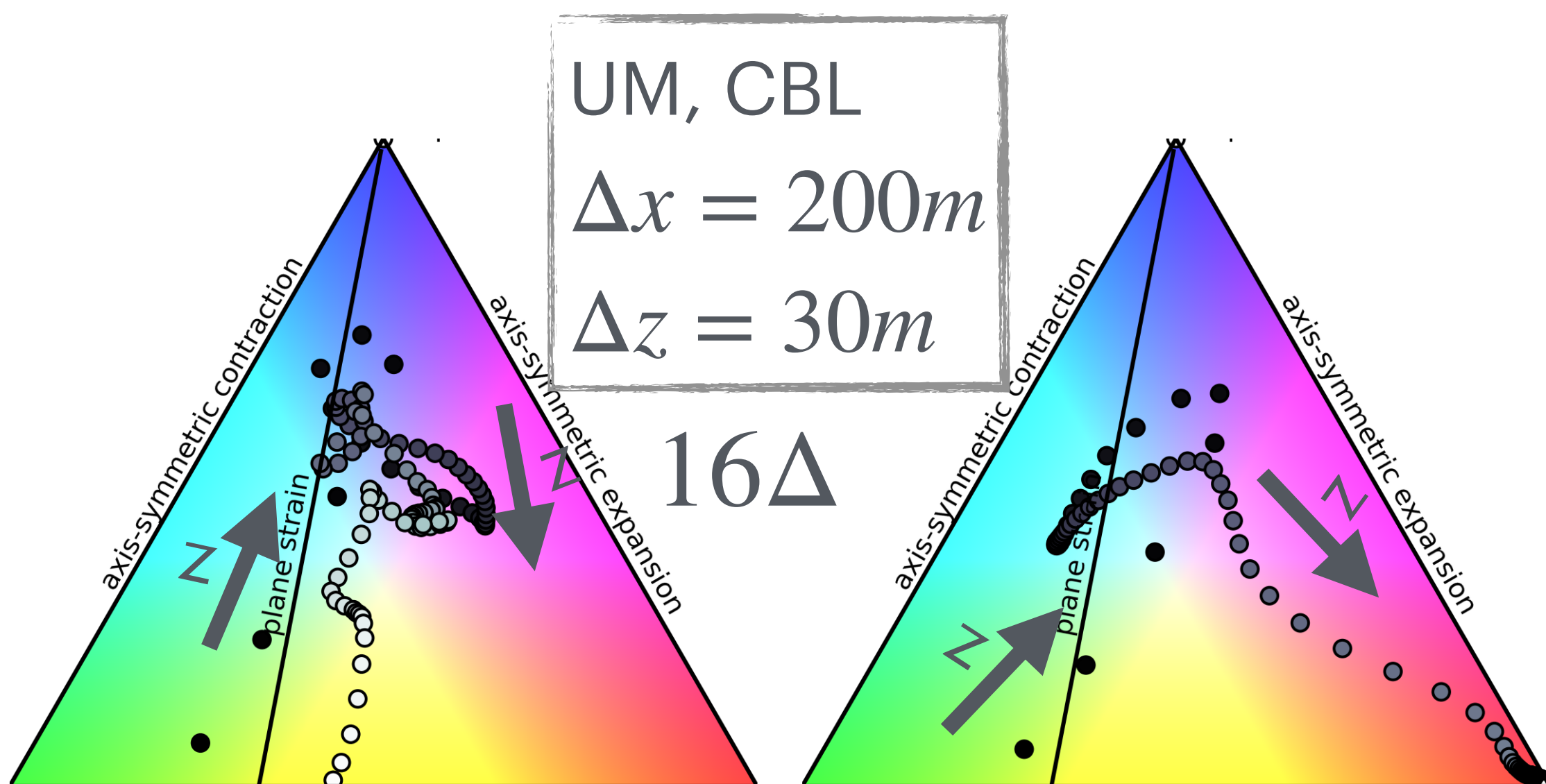
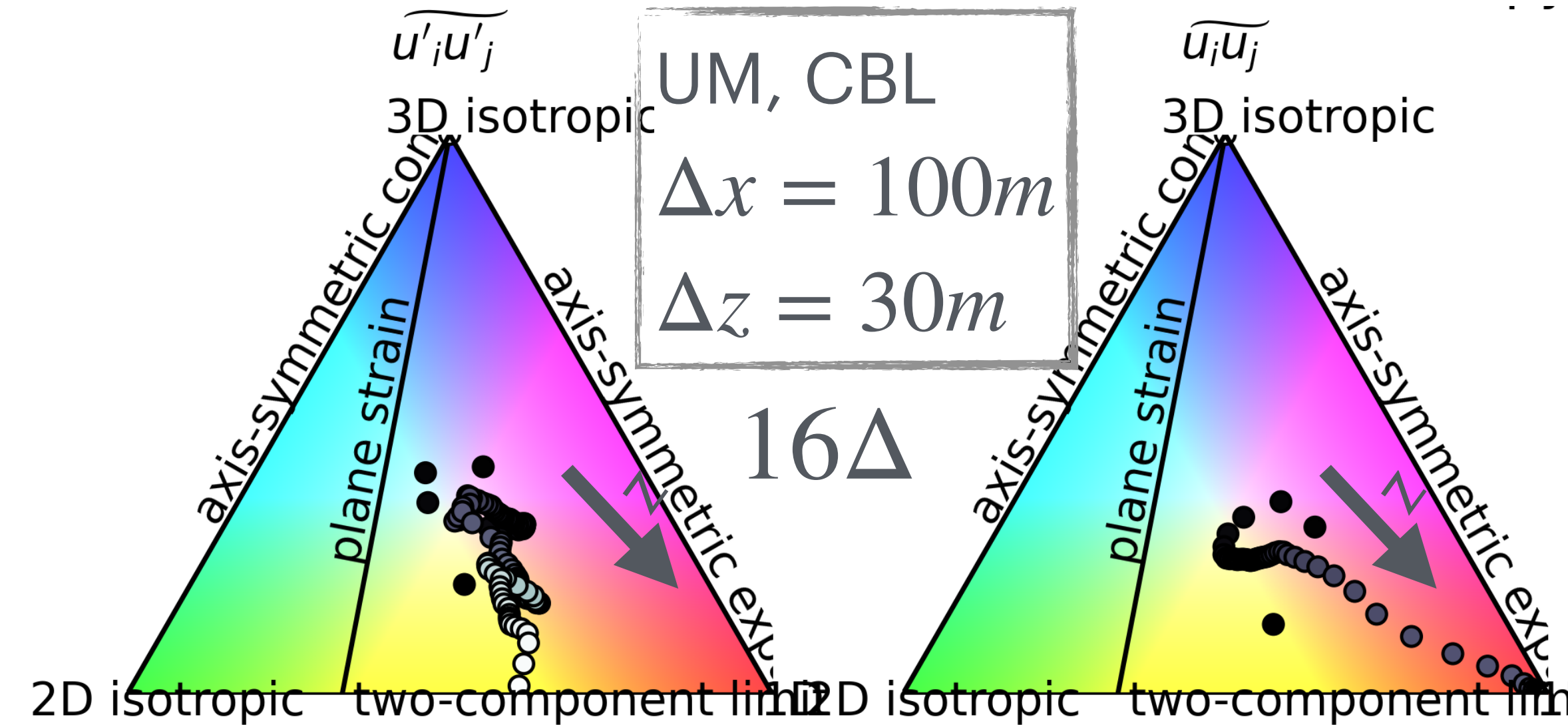
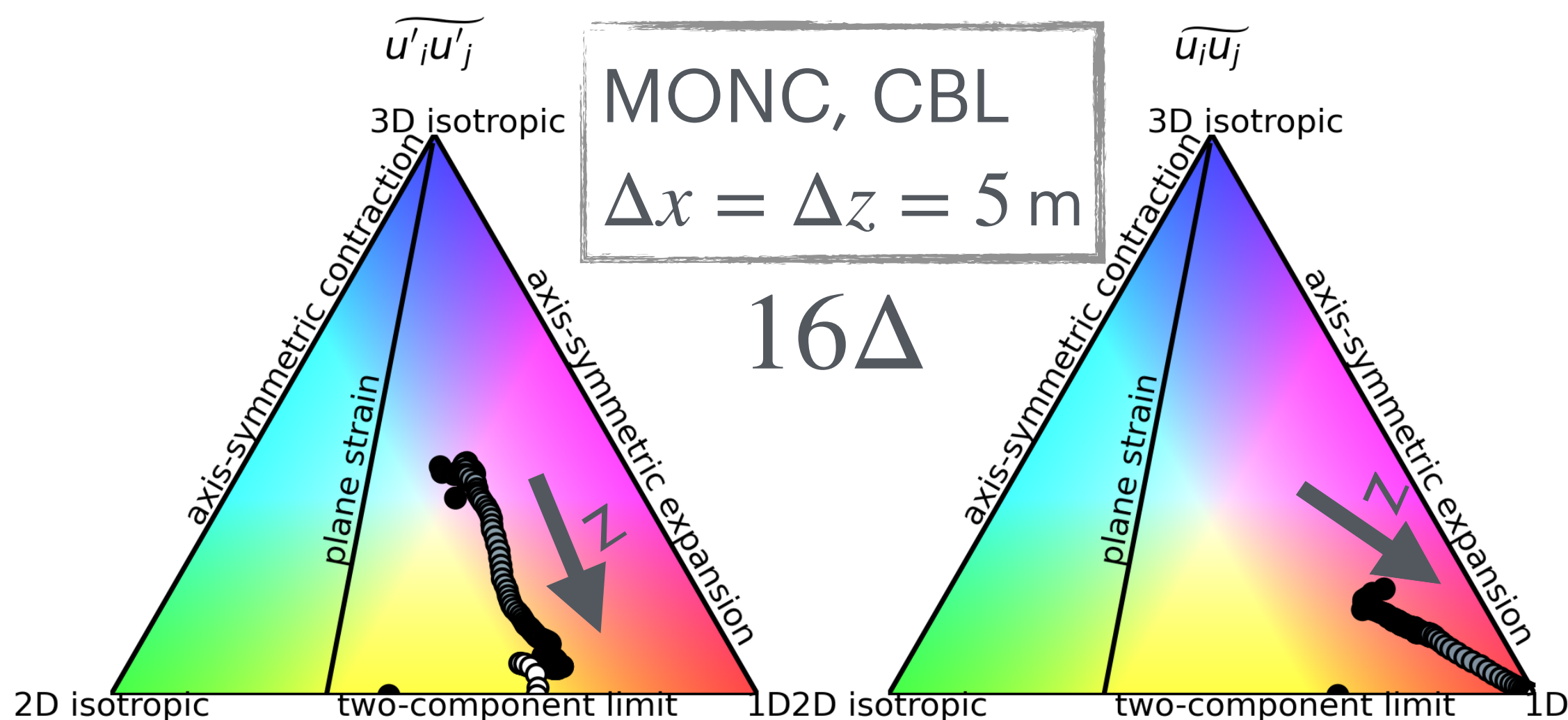


Dynamic anisotropy

MONC, CBL
 $\Delta x = \Delta z = 5 \text{ m}$



Dynamic anisotropy



Anisotropic SGS samples

- $\nu_{\star} = -l_{\star}^2 |\tilde{S}|; l_{\star}^2 = f(\Delta_x, \Delta_z, z/z_i, \kappa)$

- $\nu_h = -l_h^2 S_h; \nu_v = -l_v^2 S_v$

- $\tau_{ij} = \mathbb{P}(S, \Omega, \vec{g}), \text{ e.g.}$
 $l_1^2 |S| S_{ij} + l_2(S_{ik} S_{kj}) + \dots$

Simon & Chow, 2021, BLM

Shi+ 2019, JAMES

Kosovic, 1997, JFM

Carati & Cabot 1996, CTR Proceedings

Scotti+ 1993, Phys. Fluids

Moin 1993, Springer Basel AG

...

Anisotropic Dynamic models

Diagonal model: $\tau_{ij} = - (l_{il}^2 |\tilde{S}| \tilde{S}_{lj} + l_{lj}^2 |\tilde{S}| \tilde{S}_{il})$

(Moin '93) **with** $l_{ij} = \text{diag}(l_{11}, l_{22}, l_{33})$

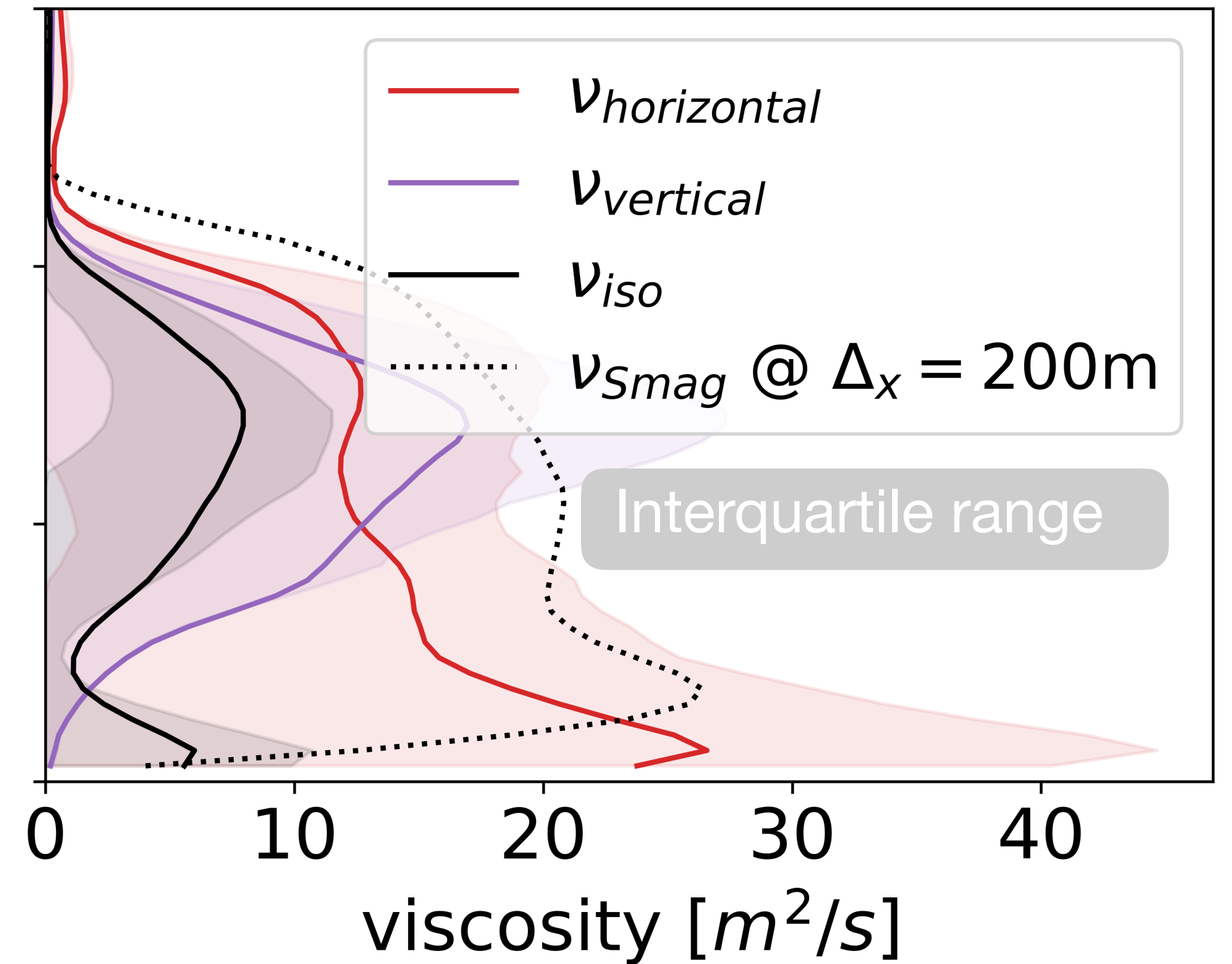
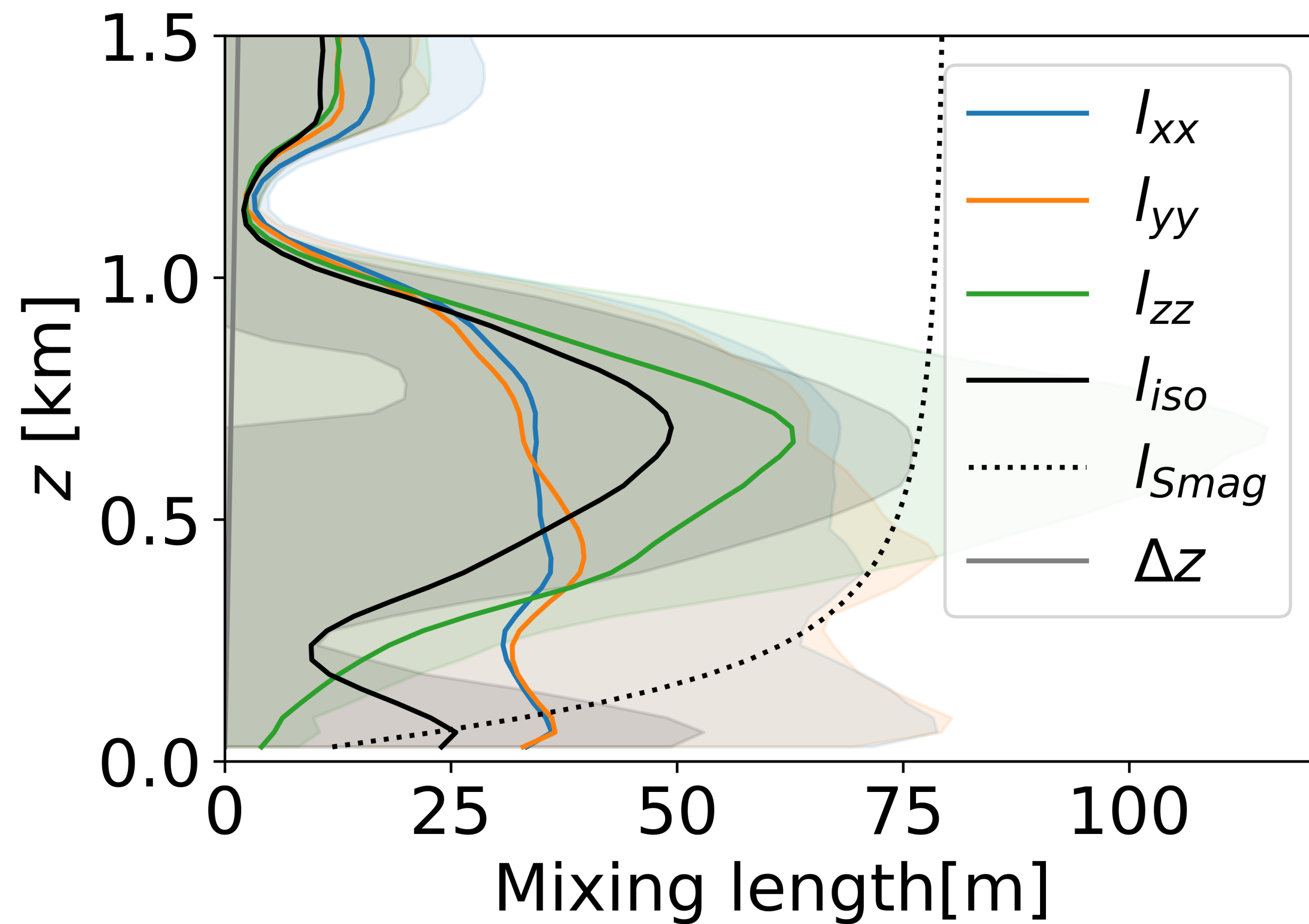
related to full tensor eddy-diffusivity $\tau_{ij} = - \nu_{ijkl} \tilde{S}_{kl} + \text{symmetries}$

(Carati & Cabot '96, CTR)

Regularise by $l_{\alpha\alpha} = \frac{\langle M_{i\alpha} L_{i\alpha} \rangle}{\langle M_{j\alpha} M_{j\alpha} \rangle}$, summing over $\{i, j\}$

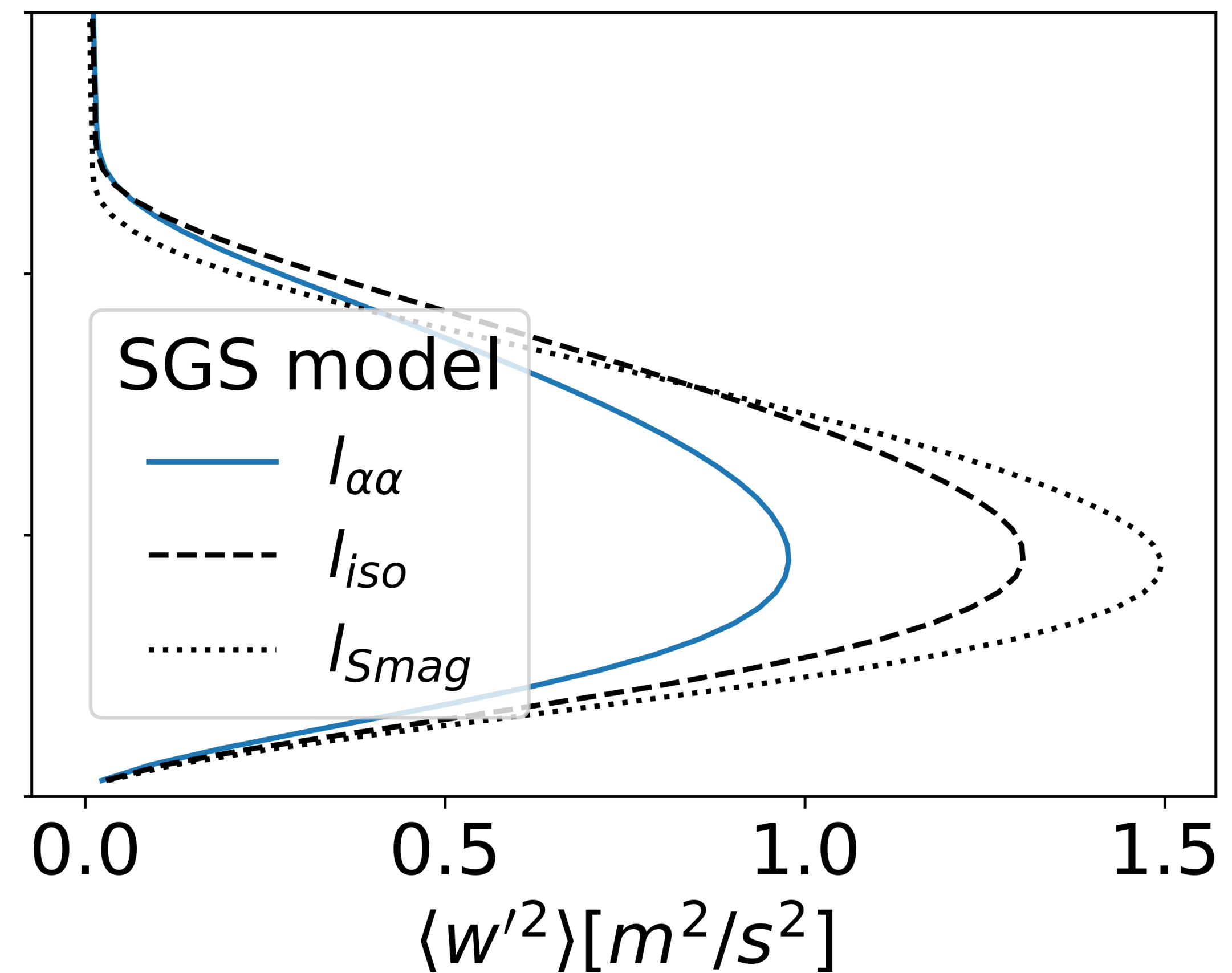
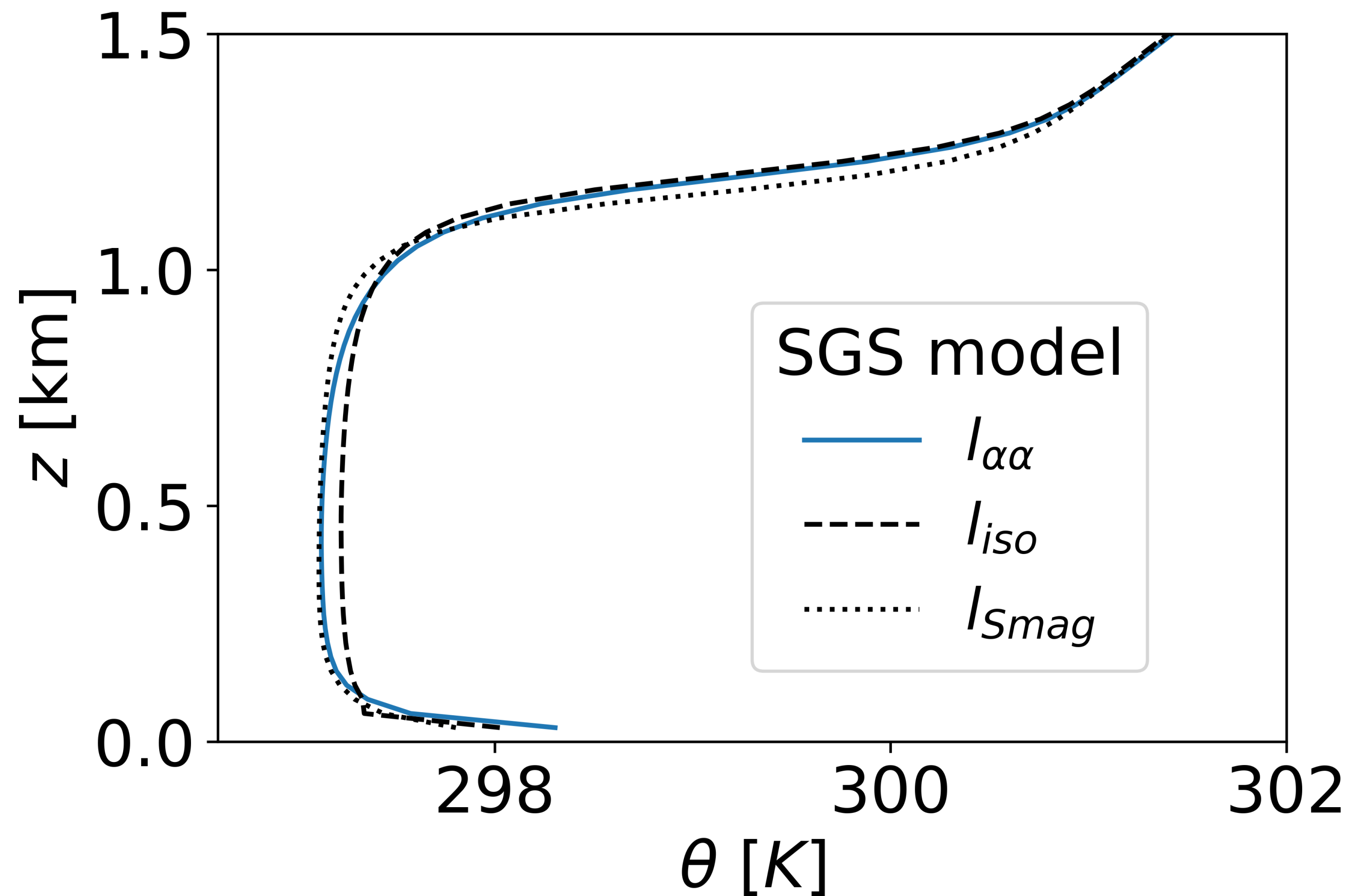
Extended scale similarity: $l_{\alpha\alpha}(\Delta_x) = l_{\alpha\alpha}(2\Delta_x)^2 / l_{\alpha\alpha}(4\Delta_x)$

A priori diagnostics



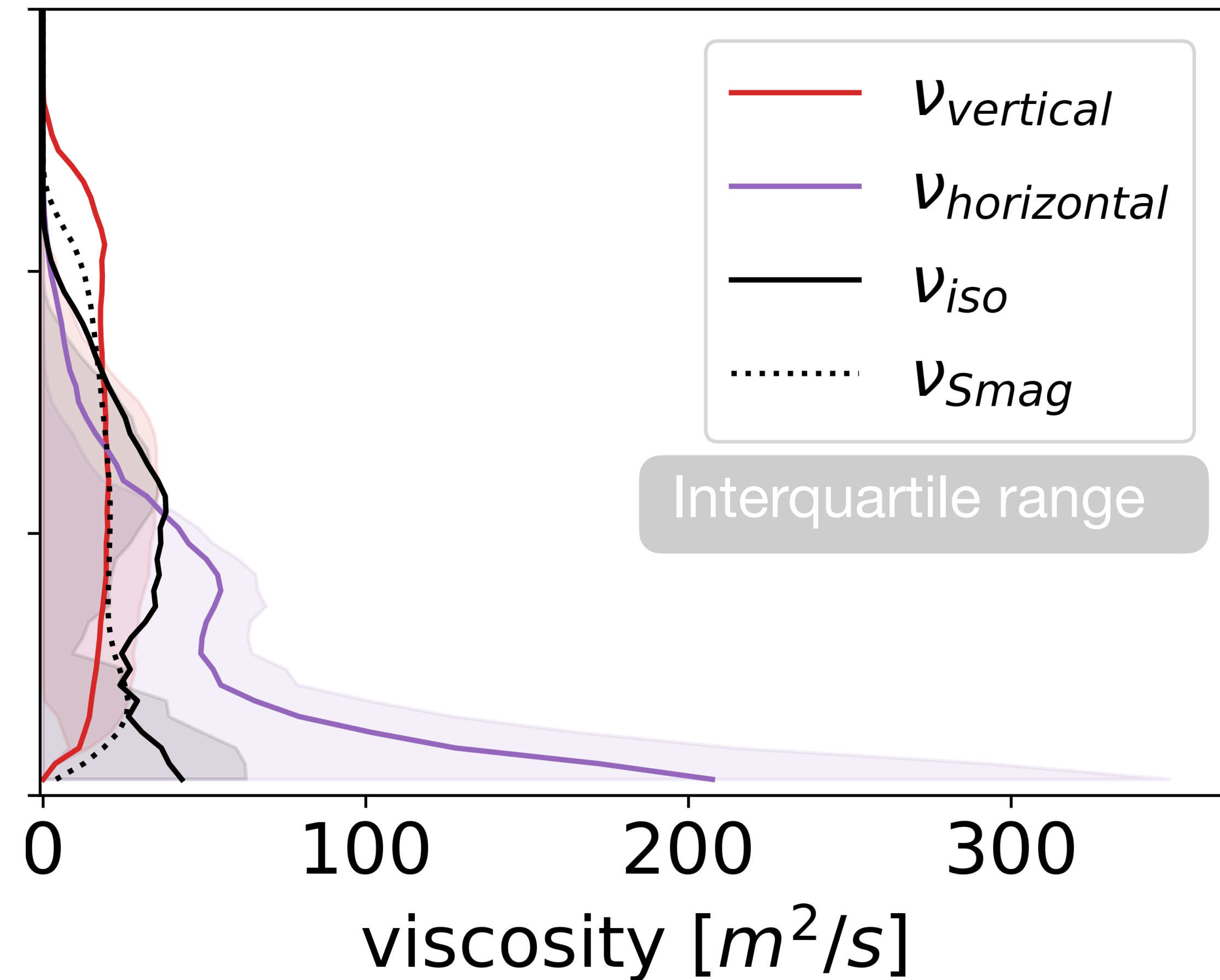
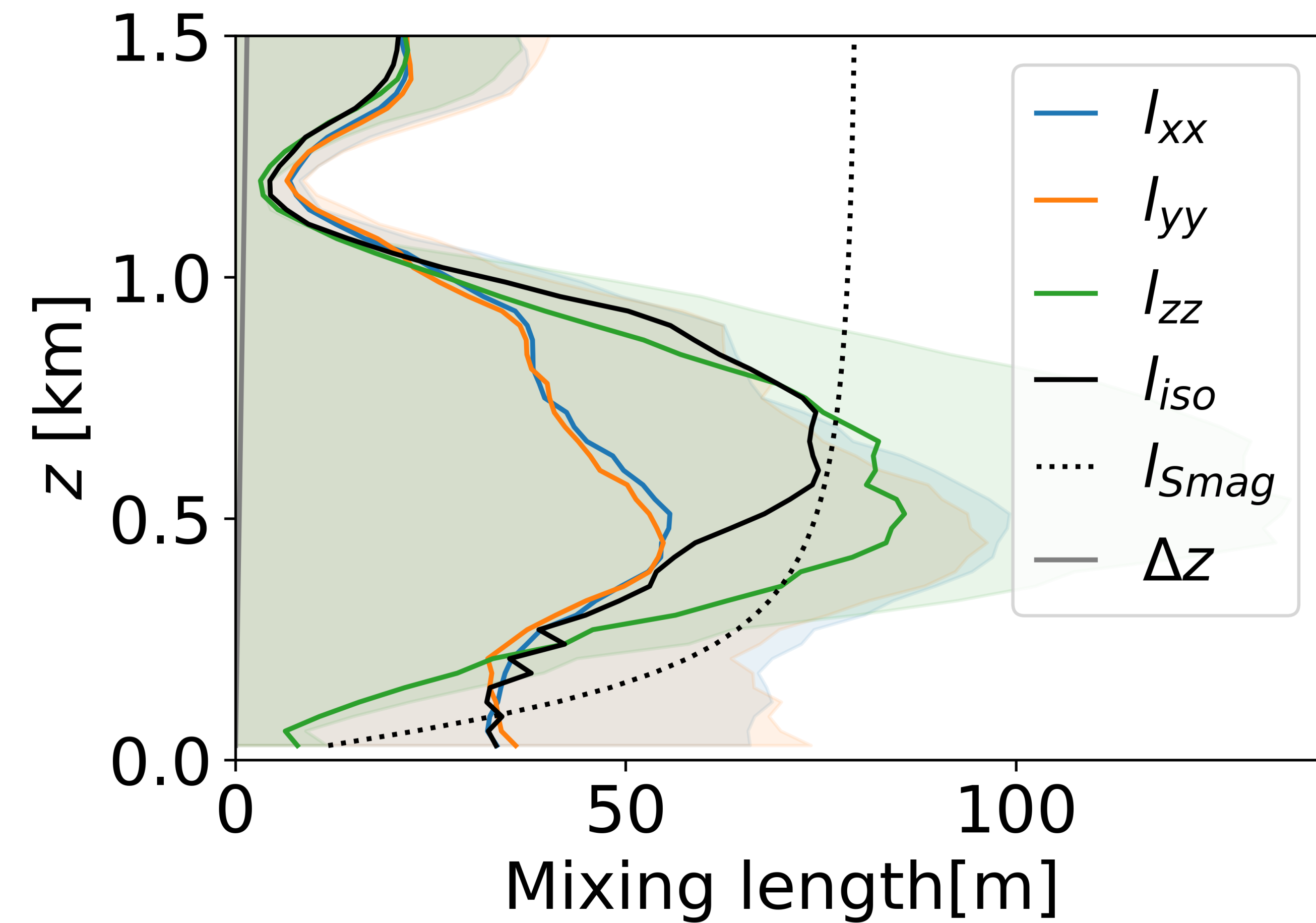
A posteriori evaluation

$$\Delta x = 400 \text{ m}, \Delta z = 30 \text{ m}$$



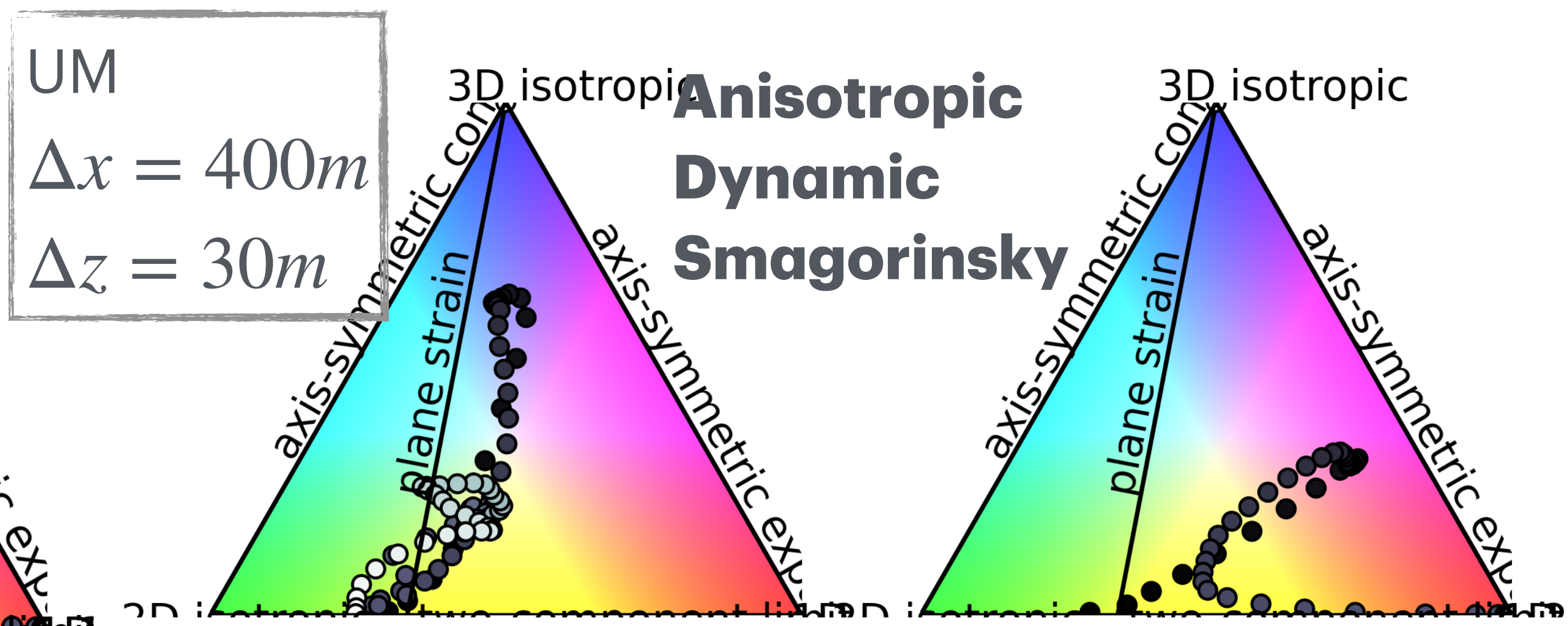
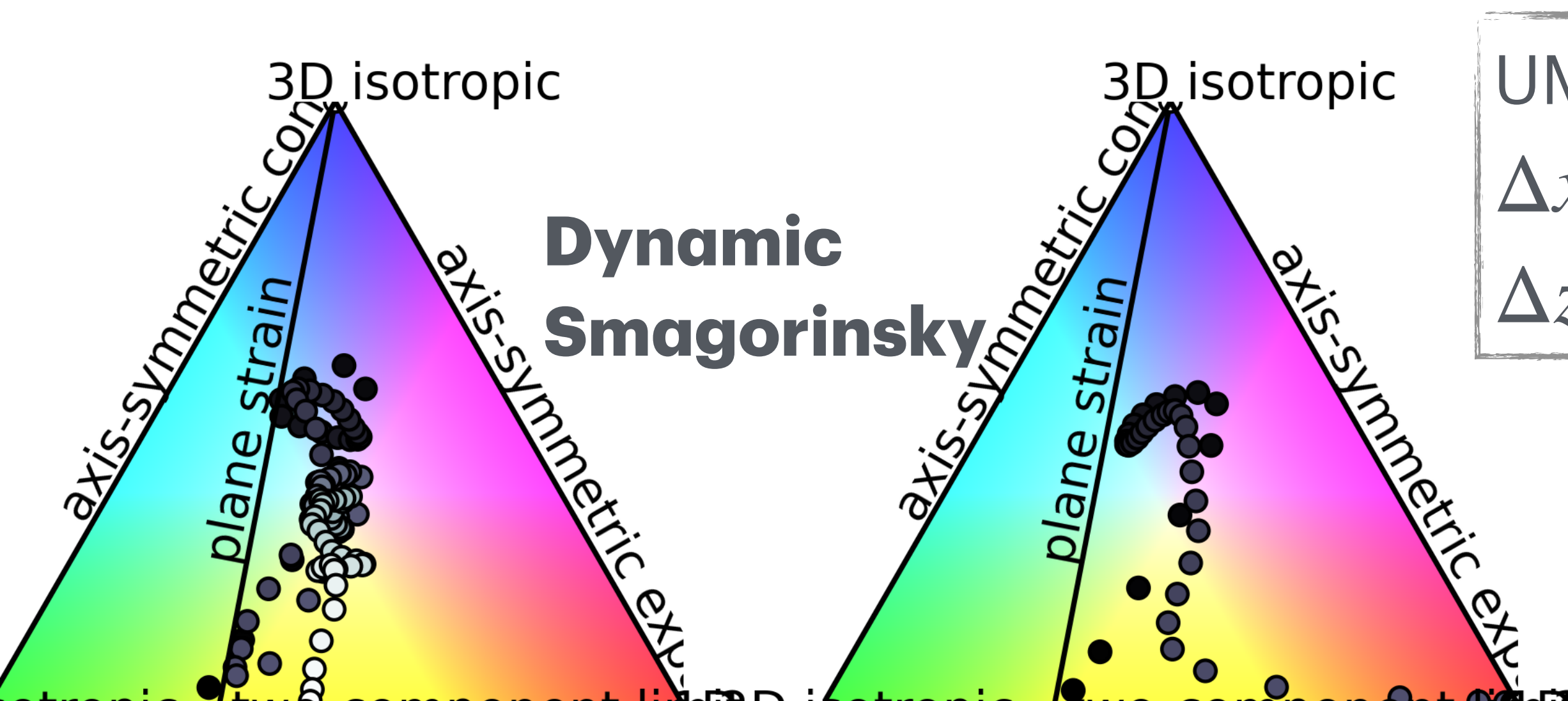
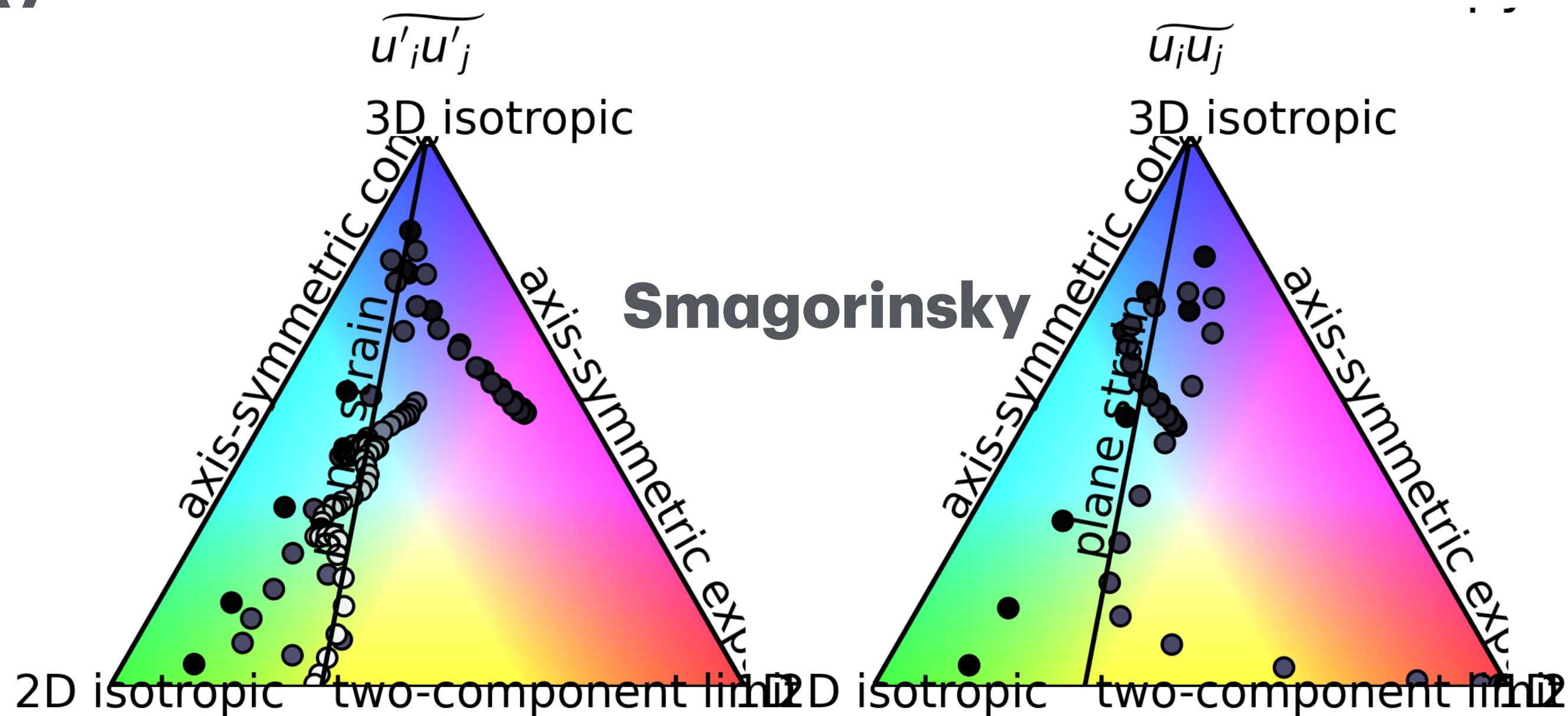
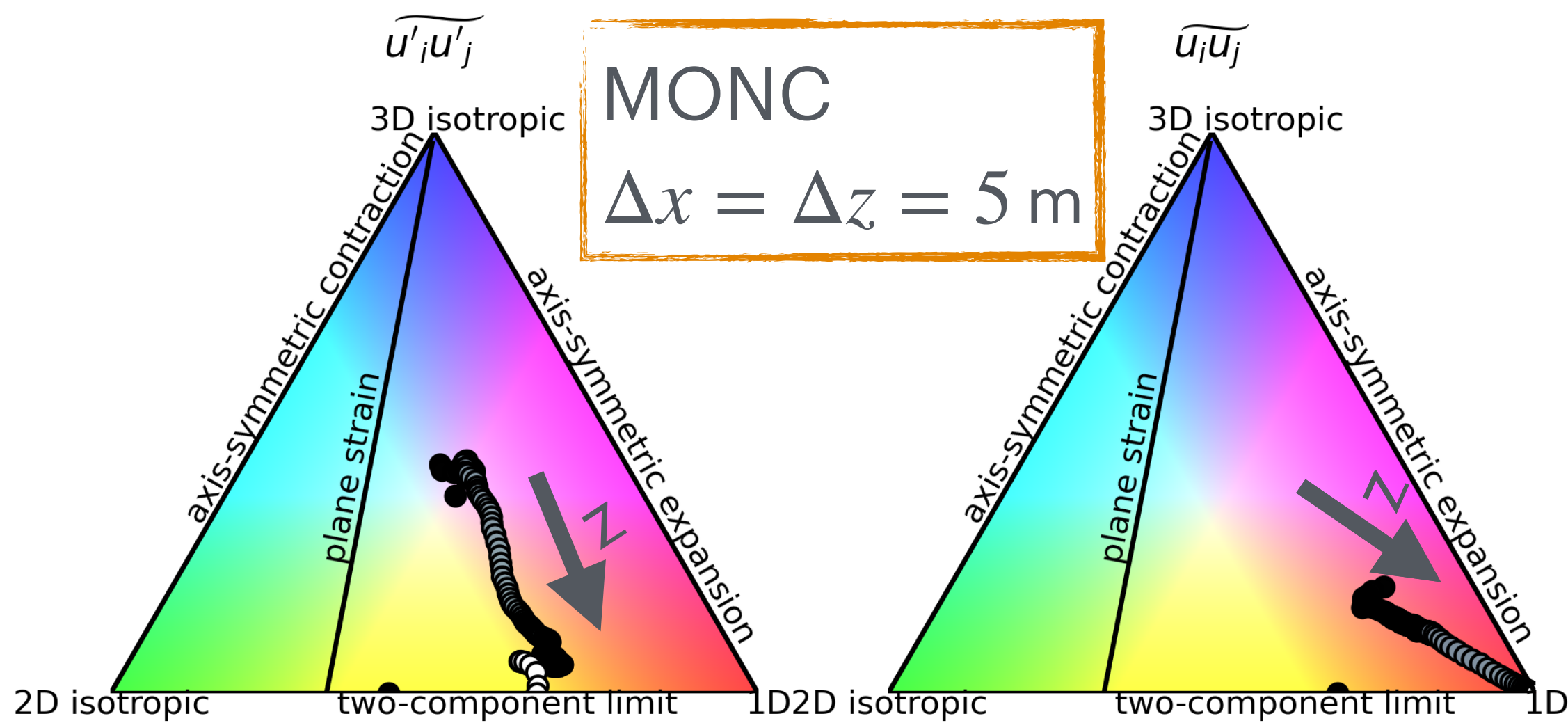
Mixing lengths and viscosity

$$\Delta x = 400 \text{ m}, \Delta z = 30 \text{ m}$$



Dynamic anisotropy

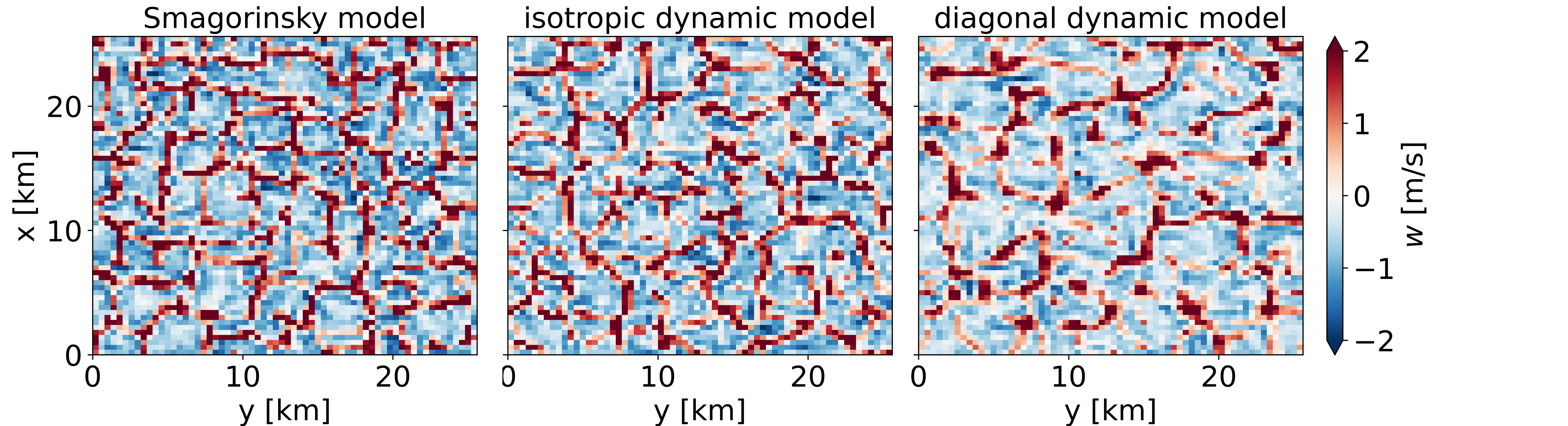
Coarse Grain: 16Δ



Field Morphology

Vertical wind at mid-BL, $z = 500m$

$$\Delta x = 400\text{ m}, \Delta z = 30m$$



Summary

- High aspect ratio grid in turbulent grey zone → anisotropic diffusivities
- LES evidence of anisotropy of momentum stresses
- Dynamic model with simple tensorial diffusivity
$$\nu_{ij} \sim \text{diag}(l_1^2, l_2^2, l_3^2)_{ij} \Delta^2 |\tilde{S}|$$
- Encouraging indications from UM simulations of dry CBL

Field Morphology

Vertical wind at mid-BL, $z = 500m$

$\Delta x = 400\text{ m}$, $\Delta z = 5m - 50m$, shear-driven

