Stochastic Parameterisation Based on Physical Principles

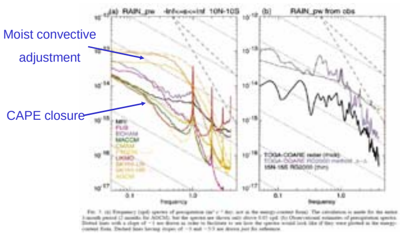
G. C. Craig, C. Keil DLR

Institut für Physik der Atmosphäre

DLR

Motivation

Variability in atmospheric models often depends crucially on upscale energy transfer associated with small-scale processes such as cumulus convection. These processes are typically represented by highly nonlinear parameterisation schemes, which generate noise through interaction with model numerics. The figure at right shows how different tropical rainfall variability is in models with different parameterisations. It is clearly desirable to replace this uncontrolled noise with a physically based representation of the unresolved variability. Considerations of fluctuations based on equilibrium statistical mechanics can a provide a basis for a parameterisation that generates appropriate variability independent of model resolution.



Horinouchi et al. (2003)

Implementation of a Stochastic Convective Parameterisation

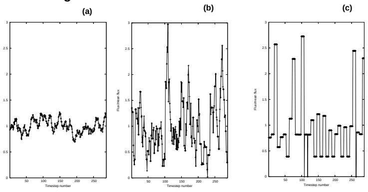
As described by Plant and Craig (2005), we are in the process of building a stochastic convection scheme based on the statistical theory described above. The scheme follows the mass flux formalism (based on Kain and Fritsch 1993 and Kain 2004), and has the following main ingredients:

- no trigger function – the presence/absence of convection in equilibrium is due to subgrid variability, represented implicitly by the assumption of a random spatial distribution
- Cloud model – an ensemble of plumes with an exponential distribution of cloud base mass flux m; each plume acts as representative cloud of given m
- CAPE Closure - CAPE determined from mean sounding (space-time averaging over the scales defined in section 3 to remove convective variability); total mass flux scaled to remove CAPE over timescale proportional to forcing (see section 2)

The scheme has been implemented in the single column version of the Met Office Unified Model (SCM). The large averaging time interval used in the CAPE closure in the single column tests is to replace the spatial averaging that is not possible in this framework. The results shown here are based on:

- parameterizations for boundary layer transport, stratiform cloud
- forced as in CRM experiment (fixed tropospheric cooling)
- 20 min timestep
- CAPE closure based on sounding averaged over 100 timesteps

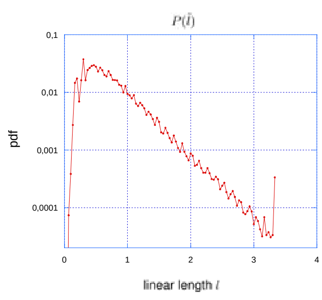
A first impression of the behaviour of the parameterisation can be seen in the mass flux time series in the figure. If the column is set to represent a large horizontal area, the fluctuations about the mean value are small (Fig. 6a), while for a smaller area the amplitude increases as expected (Fig. 6b). It is instructive to contrast this behaviour with that of the original Kain-Fritsch scheme (Fig. 6c) where the mass flux tends to oscillate between a value above and a value below the mean, with occasional excursions to very large values or to zero. This “deterministic” scheme produces a large amount of random noise, but of the wrong distribution.



Time series of total convective mass flux for the stochastic parameterisation for a column of (a) area (400 km)2, and (b) area (64 km) 2, and (c) for the Kain-Fritsch scheme.

An Analogous Problem in Turbulence

Wang and Peters (2005) suggest that dissipation elements (bounded by extrema, saddle points and zero gradient surfaces) in a scalar field advected by a turbulent flow can be modeled as being stochastically generated by eddy turnover. The requirements that the elements are space-filling and that the mean length scale for the elements is given by the scalar Taylor length are analogous to the two parameters governing the convective statistics described on this poster. The resulting prediction that the size distribution of the dissipation elements is exponential has been verified by direct numerical simulation, as shown in the figure (Wang and Peters 2005).



A Theory for Convective Variability based on Statistical Mechanics

For convection in equilibrium with a given forcing, the mean mass flux should be well-defined. But at a particular time, this mean value would only be measured in an infinite domain. For a region of finite size, we ask what is the magnitude and distribution of the variability, and what scale must one average over to reduce it to a desired level?

Craig and Cohen (2005) describe a theory for convective statistics based on the Gibbs canonical ensemble. The key assumptions are:

1. Large-scale constraints - mean mass flux within a region: $\langle M \rangle$

   - mean mass flux per cloud: $\langle m \rangle$

2. Scale separation - environment sufficiently uniform in time and space to average over a large number of clouds
3. Weak interactions - clouds feel only mean effects of total cloud field (no organisation)
4. Equal *a priori* probabilities - all locations and mass fluxes for a cloud are equally probable

A straight-forward calculation shows that the most probable distribution subject to these constraints has the frequency of clouds with a given mass flux following a Boltzman distribution:

$$d\bar{n}(m) = \frac{\langle N \rangle}{\langle m \rangle} e^{-m/<m>} dm \qquad (1)$$

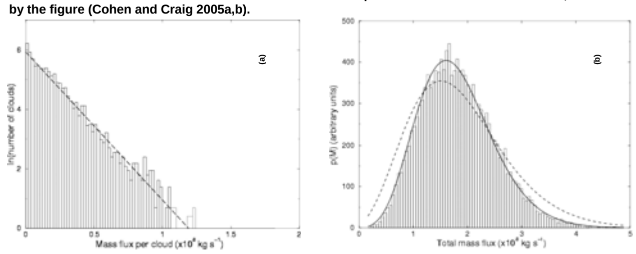
Where <N> = <M>/<m> is the mean number of clouds per unit area. The total mass flux within a region is given by:

$$p(M) = \left(\frac{\langle N \rangle}{\langle m \rangle} \right)^{1/2} e^{-\langle N \rangle} M^{-1/2} e^{-M/\langle m \rangle} I_1 \left(2 \left(\frac{\langle N \rangle}{\langle m \rangle} M \right)^{1/2} \right) \qquad (2)$$

which has variance:

$$\frac{\langle (\delta M)^2 \rangle}{\langle M \rangle^2} = \frac{2}{\langle N \rangle} \qquad (2a)$$

The variance is inversely proportional to the cloud number density, as expected for objects randomly distributed in space, but is a factor of two larger because of the variable (exponentially distributed) mass flux of the individual clouds. These distributions are well-reproduced in the CRM simulations, as shown by the figure (Cohen and Craig 2005a,b).



Comparison of theory with a cloud-resolving model simulation (128x128km doubly periodic domain, dx=2km, 50 levels, fixed SST, uniform tropospheric cooling of -2 K/day), (a) histogram of log of mass flux of individual clouds with dashed line indicating a best fit to Eq. (1) ,and (b) histogram of total mass flux in domain with solid line a best fit to Eq. (2), and dashed line the fit obtained with <m> taken from Panel (a) .

Possible Future Work that would benefit from collaboration in METSTROEM

- set up and integrate a *simplified version* of a stochastic convection scheme to test upscale effects
- diagnose information on *cloud interactions* from the simulations with a view to constructing an appropriate model, this might be as simple as a virial expansion on our equation of state, but could take the form of a lattice model or something analogous to a collision kernel
- derive and test *fluctuation-dissipation relations* to probe timescales of evolution of the system (preliminary results are very promising)
- test the parameterisation in atmospheric *models with variable resolution*

References:

Cohen, B. G. and G. C. Craig, 2004: The response time of a convective cloud ensemble to a change in forcing. *Quart. J. Roy. Meteor. Soc.,* 130, 933-944.

Craig, G. C. and B. G. Cohen, 2005a: Fluctuations in an equilibrium convective ensemble. Part I: Theoretical basis. *J. Atmos. Sci.,* submitted.

Cohen, B. G. and G. C. Craig, 2005b: Fluctuations in an equilibrium convective ensemble. Part II: Numerical experiments. *J. Atmos. Sci.,* submitted.

Horinouchi, T. et al., 2003: Tropical Cumulus Convection and Upward-Propagating Waves in Middle-Atmospheric GCMs. *J. Atmos. Sci.*, 60, 2765-2782.

Kain, J. S., and J. M. Fritsch, 1993: Convective parameterization in mesoscale models: The Kain-Fritsch scheme. *Meteorological Monographs,* 24. Amer. Meteor. Soc., 165--170.

Kain, J. S., 2004: The Kain-Fritsch convective parameterization: An update. *J. Appl. Meteorol.,* 43, 170-181.

Plant, R. S. and G. C. Craig, 2005: A stochastic parameterisation for deep convection based on equilibrium statistics. In preparation.

Wang, L. and N. Peters, 2005: The length scale distribution function of the distance between extremal points in passive scalar turbulence. *J. Fluid Mech.,* submitted.

Possible Partners

This work would fit into a broad stochastic parameterisation consortium, or a more narrowly focused project based on application of ideas from statistical physics. The work contributes to an overall goal of parameterisations that can be applied consistently at any resolution, and thus are suitable for adaptive grid models.

Meteorology
- Stochastic parameterisation in weather forecasting and climate models.
- Practical use of adpative grid dynamical cores in models with full physics

Fluid Mechanics
- Other parameterisation problems with equilibrium constraints
- Turbulent backscatter

Mathematics
- Numerical methods for stochastic differential equations
- Asymptotic approximations to stochastic equations, central limit theorem

Contact: George Craig, DLR Oberpfaffenhofen, Institut für Physik der Atmosphäre, D-82234 Weßling Phone: +49 8153 28-2581 Fax: +49 8153 28-1841 e-mail: George.Craig@dlr.de http://www.dlr.de/ipa