

# Building a Stochastic Convection Scheme

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## Stochastic Good, Deterministic Bad

The vast majority of parameterization schemes make a basic but false assumption:

they are deterministic. ie, the tendency of the resolved flow due to a subgrid process is supposedly unique for a particular realization of the resolved scale.

For example then, spatially uniform convective forcing automatically produces a spatially uniform cumulus cloud field. Particularly for grid boxes of mesoscale size, this is quite unrealistic. Variability arises very naturally because the instability is released by discrete clouds with some distribution of sizes. But, it's worse than that:

determinism eliminates important physical interactions between resolved motions and subgrid fluctuations.

These interactions need not be negligible. For many diverse physical systems they are crucial; eg, they may cause phase changes to self-organized states.

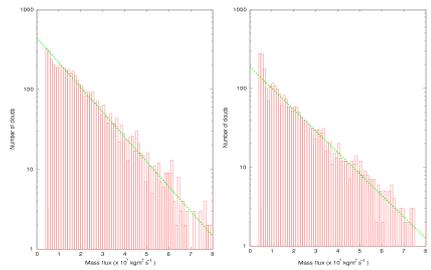
## A Cumulus Spectrum

In convective equilibrium, a random distribution of the total mass flux amongst a random number of clouds implies a probability distribution that is exponential (Cohen and Craig, 2002),

$$dn = \frac{\langle N \rangle}{\langle m \rangle} \exp\left(-\frac{m}{\langle m \rangle}\right) dm$$

Here  $m$  is the mass flux of a single cloud,  $N$  is the total number of clouds and the angled brackets denote ensemble averages.

An exponential distribution is found at various heights and with various forcings in simulations of radiative-convective equilibrium with a cloud-resolving model (CRM).



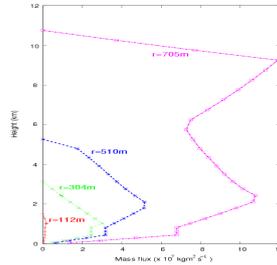
Log-linear histograms of cloud mass fluxes in CRM simulations. On the left at 1.5km for an imposed cooling rate of 8K/d, and on the right at 1.0km for 16K/d. Best fit lines for exponential distributions have been added.

## Implementing the PDF

We are developing a stochastic convection scheme based on the exponential distribution. Leaving behind the traditional mesoscale approach of a "representative plume," we consider instead the probability of finding a cloud of particular mass flux at a particular height. The stochastic scheme has a variable number of clouds with varying sizes.

A key property of each cloud is its radius (proportional to the square root of the randomly-chosen mass flux). The radius controls the rate of mixing between environmental and cloudy air. Thus, individual clouds have their own distinct characteristics, calculated by a suitable 1D plume model. In principle,

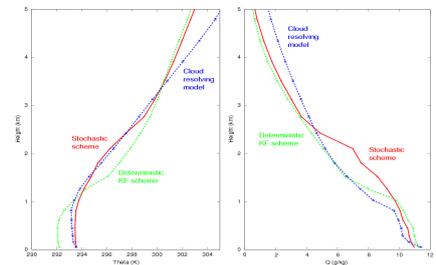
any plume model could be used, but in practice we have used the plume model of the popular Kain-Fritsch (1990) scheme.



Examples of mass flux profiles for clouds of different radii launched into the same environment.

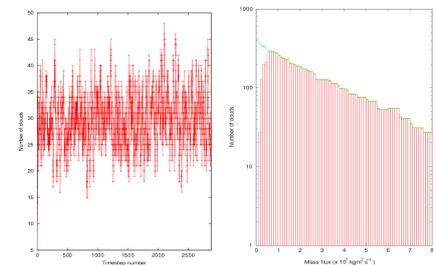
## The Scheme in Action

We have tested the scheme in radiative-convective equilibrium using the UM single column model. A box of  $(128\text{km})^2$  is used, corresponding to the CRM domain size. The aim is to replicate in a statistical sense the behaviour of the CRM under the same conditions. Our scheme is contrasted with the standard, deterministic implementation of the Kain-Fritsch (KF) scheme, which has only a single cloud type with a single radius. The equilibrium state achieved is similar in the three cases.



Equilibrium profiles of potential temperature (left) and water vapour (right) in radiative-convective equilibrium.

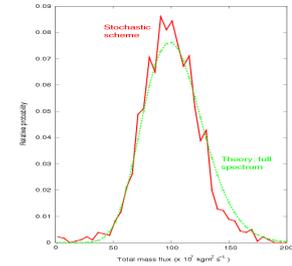
The number of cumulus clouds fluctuates significantly around a mean of 30, demonstrating the impact of a stochastic scheme. By contrast, the KF parameterization gives a single plume, which is present around 50% of the time.



Number of cumulus clouds present (left) in a box of size  $(128\text{km})^2$ , using the stochastic scheme. The log-linear mass flux spectrum of clouds produced by the stochastic scheme at the reference height of 1.8km is shown on the right. The green line is for clouds selected for launching from the pdf and the red histogram is for the clouds actually launched.

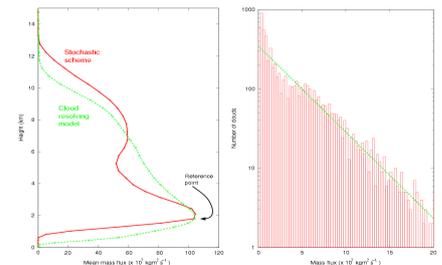
The spectrum of clouds actually produced by our scheme does not include the very smallest clouds. These correspond to very weak plumes which cannot penetrate even a single model level, and can safely be neglected. To demonstrate this, we show

the pdf for the total mass flux. The pdf produced by the stochastic scheme is in good agreement with both the theoretical result (derived using the complete spectrum) and the pdf found in the CRM (not shown). By contrast, the pdf for a deterministic scheme has a spike at zero flux (no plume present), together with an extremely narrow spike at some fixed value of flux (plume present, with weak spread from variations in the environmental conditions).



Pdf for the total mass flux; theoretical result and distribution from the stochastic scheme.

For this test the stochastic scheme is closed by using the CRM result for the ensemble mean mass flux at a reference height of 1.8km. This enables us to apply the exponential distribution at that height. In a deterministic scheme, the flux at one height defines the flux profile for all heights, under given environmental conditions. However, in a stochastic scheme, the flux profile depends upon the distribution of clouds present at any moment. There is no reason a priori to expect the time-mean profile from the stochastic scheme to agree with that from the CRM. Also, while we have imposed a mean exponential distribution at the reference height, the distributions for other heights need not be exponential. In practice, however, the mean flux profiles are in good agreement and the stochastic scheme does indeed generate exponential distributions for other heights.



Mean mass flux profile (left) in radiative-convective equilibrium. The log-linear mass flux spectrum of clouds produced by the stochastic scheme at 3.1km is shown on the right. An exponential best fit line has been added.

## Future Development

These observations give us confidence that the stochastic scheme is producing a good representation of both the mean convective state and the statistical variations in that state. We are currently investigating a generic closure method for the stochastic scheme so that it can be used in full model simulations.

## References

Cohen and Craig (2002). JCOMM Internal Report 137.  
Kain and Fritsch (1990). *J. Atmos. Sci.*, **47**, 2784.