Structure of Light Mesons.

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Introduction.

The strong interaction is believed to be completely described by the theory of quantum chromodynamics (QCD). This theory is written in terms of coloured quarks and gluons, yet, due to the property of confinement, direct observations can only ever be made of colourless bound states. It is the properties of such states that we would like to understand and predict. Unfortunately, at low energies the coupling strength between the quarks and gluons becomes large, so that ordinary perturbation theory breaks down. In practice, this means that we are unable to calculate hadronic properties directly from QCD.

Instead, we must turn to a variety of approaches, which are simple enough to handle and which mimic many of the features of QCD. In this way, we might hope to gain some appreciation of how these features influence observed hadron properties.

I have been exploring one such model, which is based on a non–local four–quark interaction. The character of the interaction is constrained through chiral symmetry, which is an approximate symmetry of QCD, being broken only by the small current masses of the up and down quarks. The form of the non–locality is chosen to simplify the calculations, but may be motivated through a picture of the QCD vacuum.

The particles of interest are the lowest lying $J^\pi = 0^+, 1^\pm$ states, namely π, ρ, ω and a_1. We can use the model to calculate the masses of these mesons and interactions between them, such as their decay widths and electromagnetic properties.

Constructing The Mesons.

The theory is attacked through the formalism of the Dyson–Schwinger (DSE) and Bethe–Salpeter (BSE) integral equations. They are treated in the so–called ladder approximation, which corresponds to taking the first term of an expansion in $1/N_c$, where N_c is the number of colours.

The DSE tells us how a quark interacts with the many–body vacuum state, which is a condensate of $\overline{q}q$ pairs. Movement of a single quark within such a ground state is represented by the generation of an effective mass. The mass is $\sim$ 300 to 400 MeV at zero momentum, which makes contact with the simple non–relativistic quark model, long used to estimate mass splittings and magnetic moments of the hadrons. For large spacelike momenta, the constituent mass vanishes, in agreement with the successful application of perturbation theory in QCD in this regime. That the effective mass varies with momentum is a direct consequence of the non–local nature of our interaction.

The momentum variation of the quark mass is an essential part of our method for confining the quarks. If the mass were a constant, M, then it would be possible to create an unbound $\overline{q}q$ pair at an energy of $2M$. Therefore, it would be impossible to describe mesons which have a mass of more than $2M$. But, in our model there is no value of p^2 for which $p^2 = M^2(p^2)$, so free quarks cannot occur.

We must next construct bound states from the quarks. To do so, we represent the mesons as a chain of quark loops (see fig. 1) in the BSE, which is the relativistic analogue to the Schrodinger equation, for a two–body state. Solution of the BSE gives the meson masses.

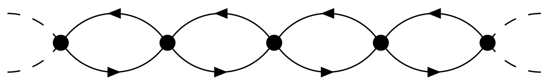


Fig. 1. Mesons described as a chain of constituent quark bubbles.

Meson Properties.

We are now in a position to calculate properties of the mesons. For example, we can look at their decay modes. Matrix elements for $1 \to 2$ decays are obtained by evaluating diagrams like that shown below. The blobs V we get from the BSE solutions. They describe the coupling of each meson involved to the quarks in the loop.

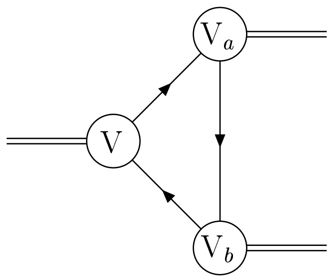


Fig. 2. $1 \to 2$ amplitudes. There is also a similar diagram where $a \leftrightarrow b$.

We can also calculate the meson couplings to external vector and axial currents. This gives us information about weak and electromagnetic decays. We hit upon a complication here because in the presence of non–local interactions, the usual local current is not conserved. So, we need to include appropriate non–local contributions to the currents in order to satisfy the conservation equations. These non–local pieces involve four–quarks and lead to extra diagrams of the type shown on the right in the figure below.

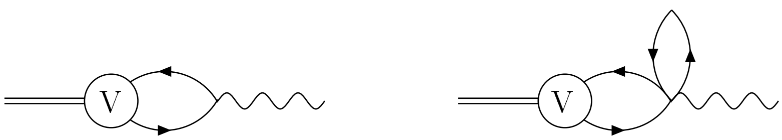


Fig. 3. Coupling to external currents.

Results.

In the following table, numerical results for various physical quantities are listed. One model parameter has been left free and the errors quoted on the model results indicate the range of values obtained when this parameter is allowed to vary.

Meson masses.		
Particle	Model	Experiment
π	Fit	140MeV/c^2
ρ	Fit	770MeV/c^2
a_1	$1007 \pm 55\text{MeV/c}^2$	$1230 \pm 40\text{MeV/c}^2$
ω	Fit	783MeV/c^2
Decay widths.		
Decay	Model	Experiment
$\rho \to \pi\pi$	$112 \pm 9\text{MeV}$	$151 \pm 1\text{MeV}$
$a_1 \to \rho\pi$	$104 \pm 84\text{MeV}$	$\sim 400\text{MeV}$
$\pi \to \mu\nu_\mu$	Fit	25.3meV
$\tau \to \rho\nu_\tau$	$0.297 \pm 0.037\text{meV}$	$\leq 2.23 \pm 0.02\text{meV}$
$\tau \to a_1\nu_\tau$	$0.216 \pm 0.123\text{meV}$	$\leq 0.187 \pm 0.07\text{meV}$
$\rho \to e^+e^-$	$3.33 \pm 0.82\text{keV}$	$6.77 \pm 0.32\text{keV}$
$\omega \to e^+e^-$	$0.38 \pm 0.05\text{keV}$	$0.60 \pm 0.02\text{keV}$
$\pi^0 \to \gamma\gamma$	$8.62 \pm 0.06\text{eV}$	$7.74 \pm 0.55\text{eV}$
D/S wave in $a_1 \to \rho\pi$.		
–	Model	Experiment
Ratio	-0.068 ± 0.020	-0.11 ± 0.02
Vacuum condensate.		
–	Model	Experiment
$\sqrt[3]{-\langle\overline{q}q\rangle}$	$198 \pm 9\text{MeV}$	$230 \pm 20\text{MeV}$
RMS pion radius.		
–	Model	Experiment
$\sqrt{\langle r_\pi^2\rangle}$	$0.587 \pm 0.004\text{fm}$	$0.663 \pm 0.006\text{fm}$

Since the approximation used is equivalent to working at leading order in a $1/N_c$ expansion, we might expect $\sim 30\%$ accuracy in masses and amplitudes. Thus, the results are not expected to be in perfect agreement with experiment. Nevertheless, they should be qualitatively reasonable if we are to have any confidence in the model. This is generally the case. An obvious exception is the decay width for $a_1 \to \rho\pi$, the reason being that it suffers from too small a phase space : $m_\rho + m_\pi = 910$MeV is quite close to the model a_1 mass.

Electromagnetic interactions.

As discussed earlier, we need an electromagnetic current with local and non–local pieces to satisfy current conservation. The full γqq vertex, Γ, consists of the bare vertex, non–local parts and vector–dominance parts, where the photon couples via a ρ or ω particle. Putting together these contributions, we calculate some electromagnetic processes. For example, we have looked at the pion form factor. This is of interest because it is sensitive to the internal structure of the pion. It therefore provides a useful test of our approach, which treats mesons as extended objects, unlike many models where the mesons are described by point–like fields. The relevant diagrams are shown below.

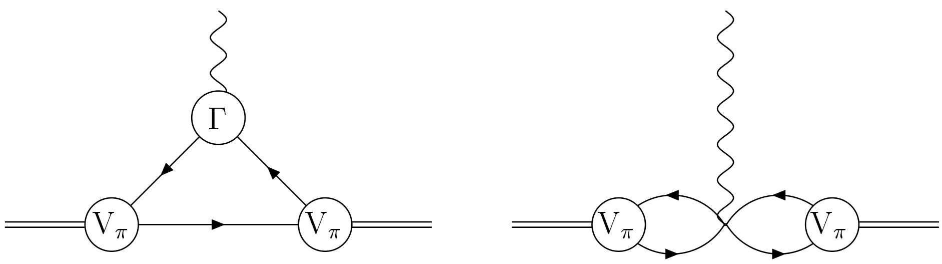


Fig. 4. Spacelike pion form factor. There is also another triangular diagram, with the fermion arrows in the opposite sense.

The numerical results are displayed in the graph and compare favourably with the experimental data points[1]. I have shown the modulus of the form factor against the q^2 of the current, in GeV^2. There is a gap from zero to $4m_\pi^2$ since that region is kinematically inaccessible. An imaginary part of the rho meson propagator may be generated through pion loops. These are not

[1]The data has been extracted from Amendolia et al. Nucl. Phys. B227 (1986),168 and Bebek et al. Phys. Rev. D13 (1975),25.

included in our calculations, being N_c suppressed. Hence, the peak in the data at the rho mass shows up as a pole in the plot.

The pion form factor yields information about the internal structure of $\pi^{\pm}$, but, by C parity invariance, the form factor for π^0 is identically zero. In order to probe the structure of this particle, we must consider transition form factors, such as the amplitude for $\gamma^{\star}(q)\pi^0 \to \gamma$, as a function of the momentum of the excited photon. A closely related process is $\pi^0 \to \gamma\gamma$, the rate for which is given by a low–energy theorem (the Adler anomaly). Our model should reproduce the theorem, which provides a useful check on our calculations. We check this by evaluating the following diagrams.

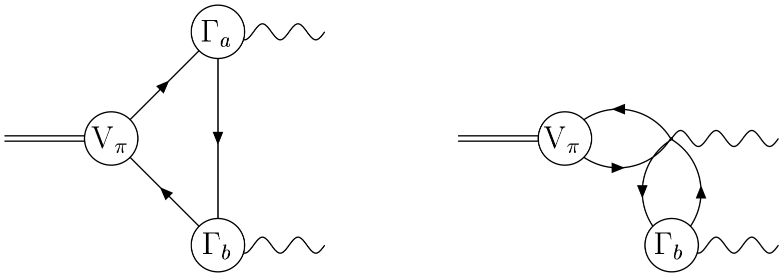
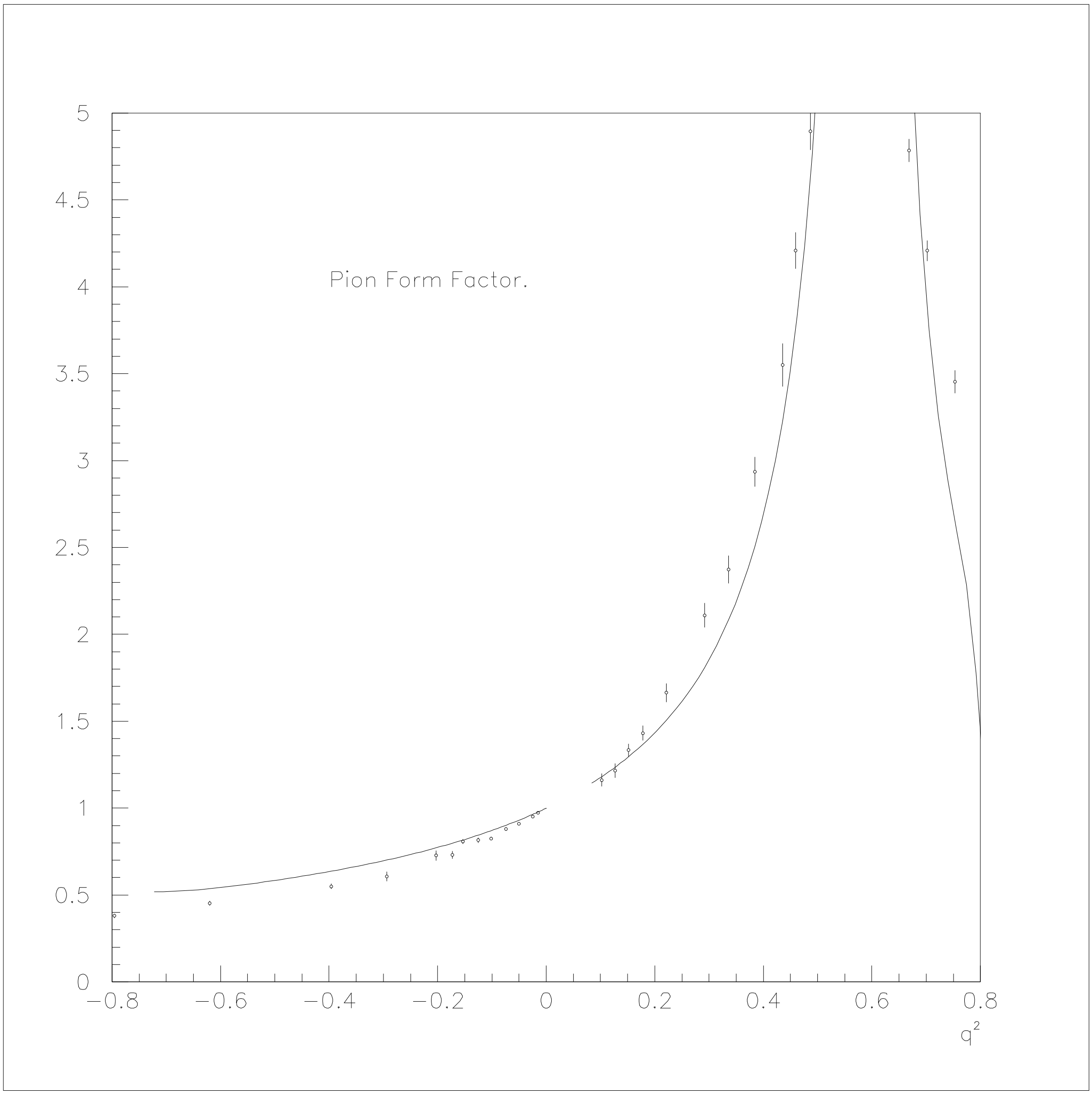
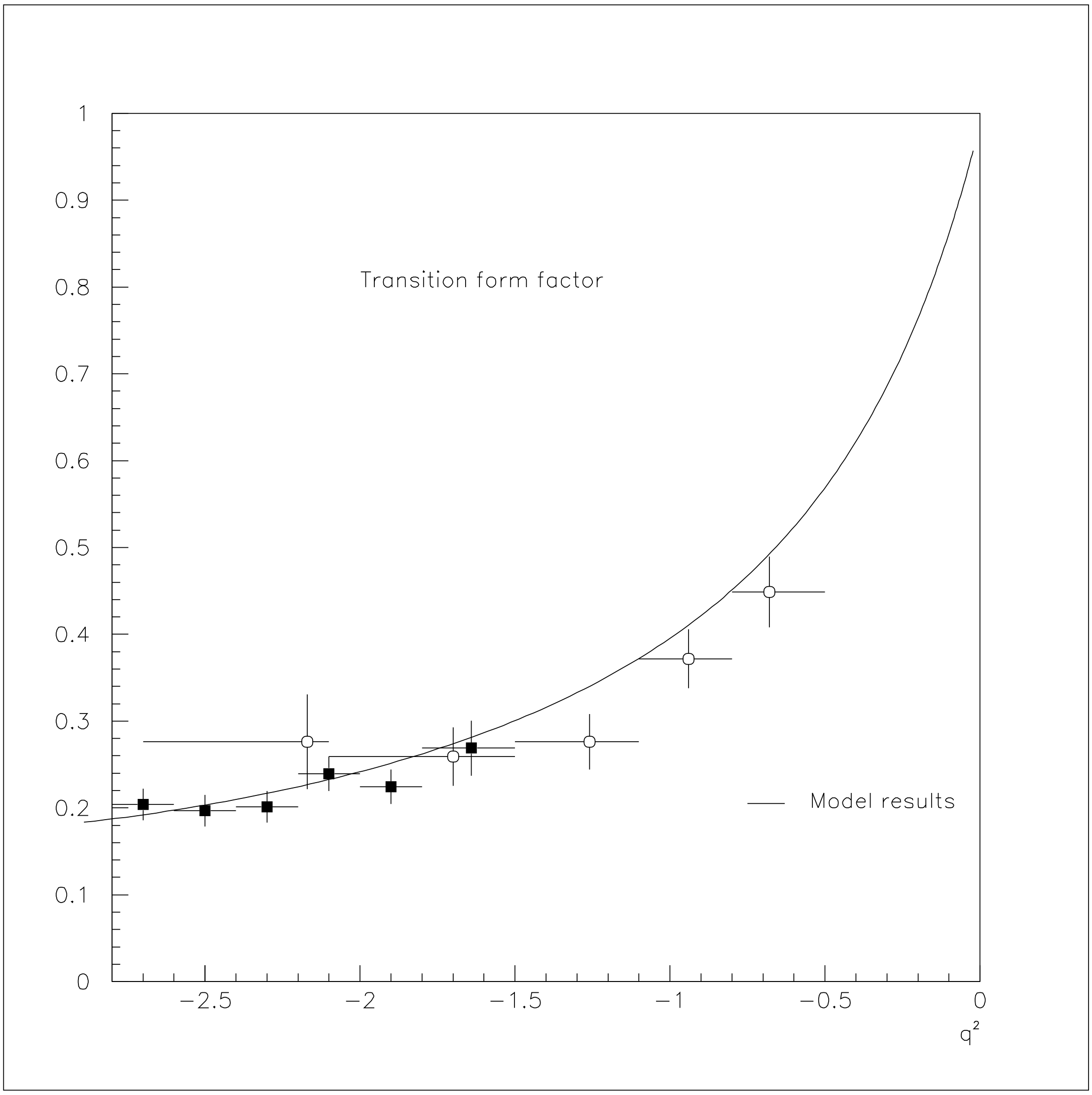


Fig. 5. $\pi^0 \to \gamma\gamma$. There are similar diagrams to the above, where $a \leftrightarrow b$.

The results for $\gamma^{\star}(q)\pi^0 \to \gamma$ are shown in the graph, with q^2 in GeV^2 and the form factor normalized to unity at $q^2 = 0$. They compare fairly well to a monopole fit of the data measured by the CELLO collaboration[2]. Unfortunately, there is very little data available at present, which makes it impossible to distinguish between competing descriptions of the process. It is hoped the situation will improve in the future, with the possibility of high–precision data from CEBAF experiments.

[2] As given in Z. Phys. C49 (1991),401.


Pion Form Factor.
q²
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8



Transition form factor
Model results
q²
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−2.5
−2
−1.5
−1
−0.5
0