Theory of moist convection in statistical equilibrium

By analogy with Maxwell-Boltzmann statistics

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With thanks to: George Craig, Brenda Cohen, Laura Davies, Michael Ball, Richard Keane

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Outline

An overview of Craig and Cohen (2006) theory, plus extensions/follow-ups and some speculations

- Assumptions made
- Results of theory
- CRM results
- From theory to parameterization
- Where next for the theory?

Assumptions Made

Validity of Statistics

- Assume convection can be characterised by ensemble of cumulus elements/ clouds / plumes
- These are the microscopic components (cf. gas particles)
- By “large-scale state” we mean a region containing many such elements
- This is the macroscopic state (cf. thermodynamic limit)
- Statistical equilibrium: the statistical properties of the cumulus ensemble are to be treated as a function of the large-scale state (as in gas kinetics)

Existence of large-scale state

- Assume a scale separation in both space and time between individual cumulus elements and the large-scale
- i.e., that a “large-scale-state” exists in practice, and not just as a theoretical abstraction
- In statistical equilibrium with a fixed forcing the ensemble mean mass flux $\langle M \rangle$ is well defined (cf. total energy of gas particles)

Another important variable

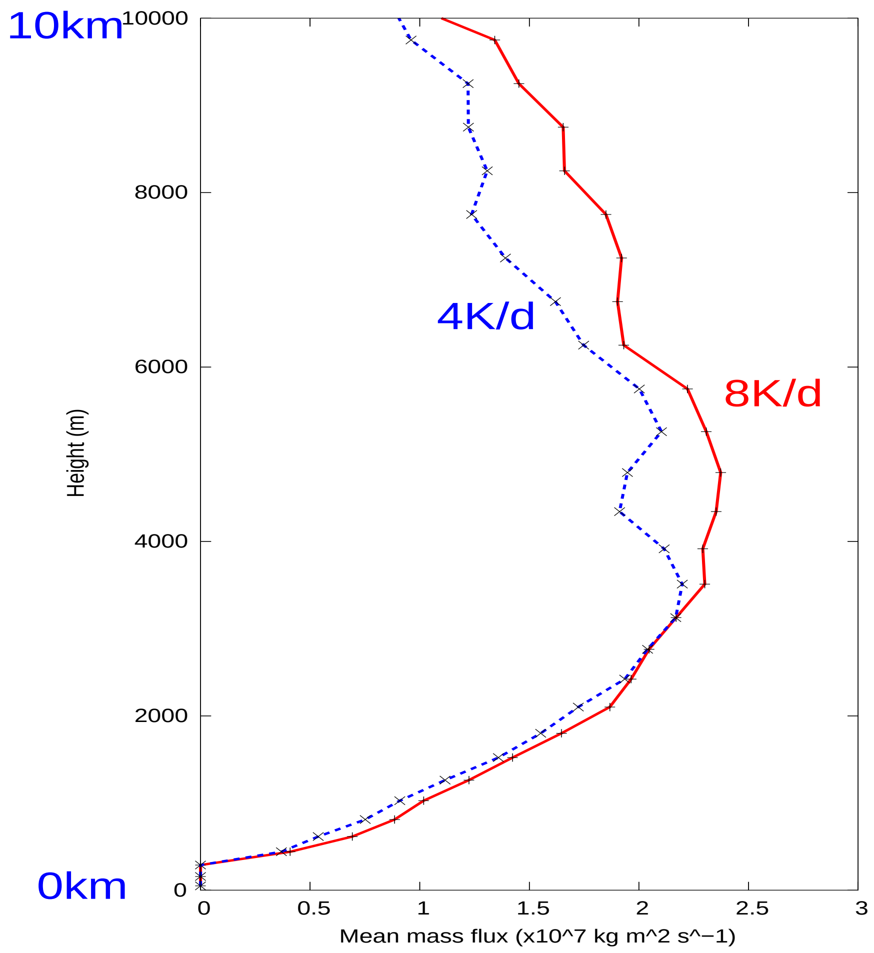
- The ensemble mean mass flux of an individual convective element, $\langle m \rangle$ (cf. the temperature of a gas)
- Must imagine the ensemble average to extend over elements and over the element's life cycle
- Mean number of elements present is simply

$$\langle N \rangle = \frac{\langle M \rangle}{\langle m \rangle} \tag{1}$$

This would be constant for a gas of course.

Mean mass flux per cloud

$\langle m \rangle$ can depend on large-scale state, but weakly in practice



Increased forcing predominantly affects cloud number $\langle N \rangle$:

- not the mean w (scalings of Emanuel and Bister 1996; Grant and Brown 1999)
- nor the mean size (Robe and Emanuel 1996; Cohen 2001)

More assumptions

- Cumulus elements are point-like and non-interacting
- So, no organization and elements randomly distributed
- Equal a priori probablities: all states with all distributions of cloud locations and mass fluxes are equally likely

Results of Theory

Boltzmann distribution

- pdf of mass flux per cumulus element is exponential,

$$p(m)dm = \frac{1}{\langle m \rangle} \exp\left(\frac{-m}{\langle m \rangle}\right) dm \tag{2}$$

- cf. Boltzmann distribution of energies in a gas
- Applies to fixed (but undefined) level in the atmosphere.

Variable number of elements

- Cumulus elements subject to Boltzmann pdf, but their number is not fixed, unlike number of gas particles
- If clouds randomly distributed in space, number in a finite region given by Poisson distribution
- pdf of the total mass flux is convolution of this with the Boltzmann,

$$p(M) = \frac{1}{\langle M \rangle} \sqrt{\frac{\langle M \rangle}{M}} \exp\left(-\frac{M + \langle M \rangle}{\langle m \rangle} \right) I_1 \left(\frac{2}{\langle m \rangle} \sqrt{\langle M \rangle M} \right)$$

(3)

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Variance of M

- The spread of the M distribution can be characterised by the variance,

$$\frac{\langle(\delta M)^2\rangle}{\langle M\rangle^2} = \frac{2}{\langle N\rangle} \tag{4}$$

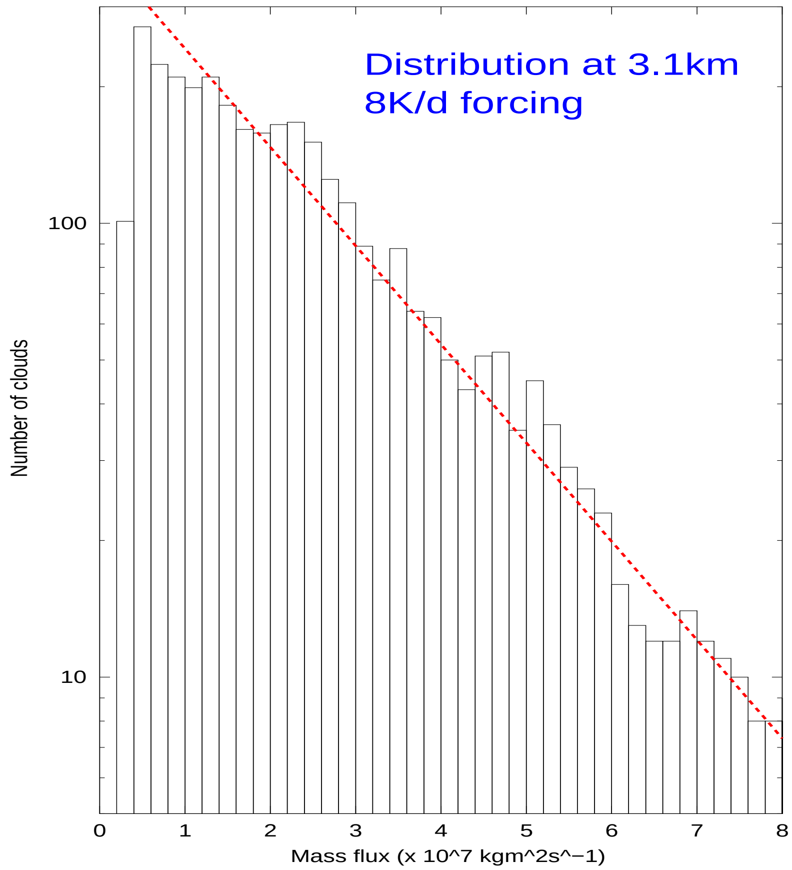
- Fluctuations diminish for larger $\langle N\rangle$, as region size and/or forcing increases

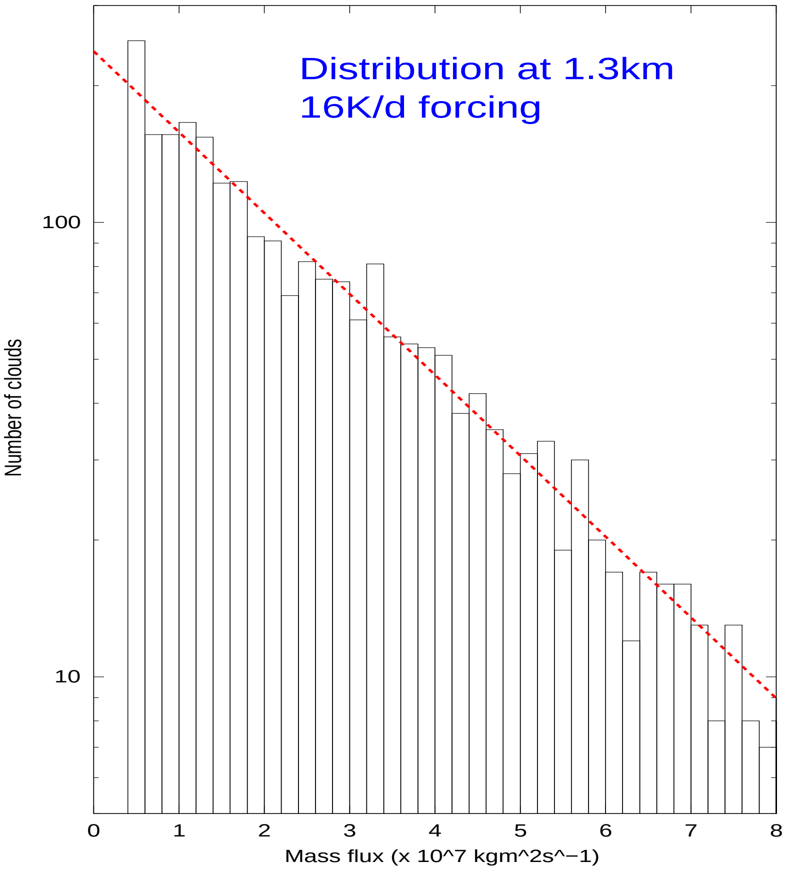
CRM Results

Exponential Distributions

- Cohen and Craig (2006) showed exponential works well for radiative-convective equilibrium just above cloud base
- For two definitions of cumulus elements
- Exponential is remarkably robust
- Some other examples from Cohen's simulation data at other heights and for other strengths of forcing...

Exponential Distributions



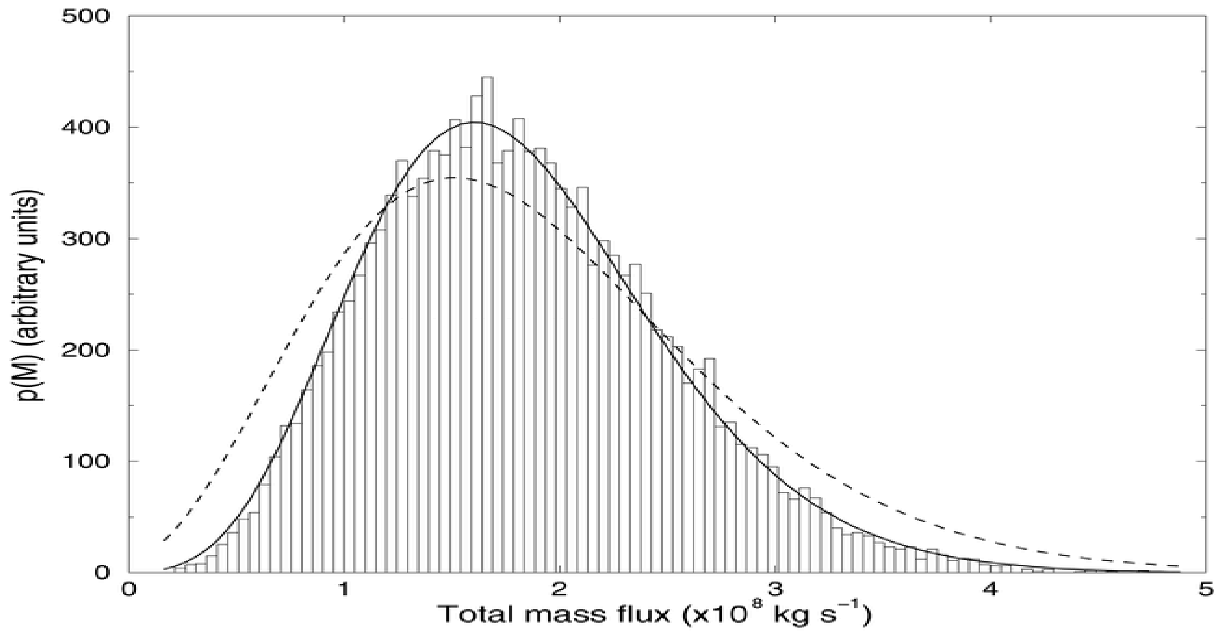


Other tests

- With constant surface fluxes (rather than constant SST) (Davies and Plant 2007)
- Large-domain CRM with spatially-varying SST and imposed large-scale wind forcing (Shutts and Palmer 2007) and theory reinterpreted for convective precipitation rates

Total mass flux distribution

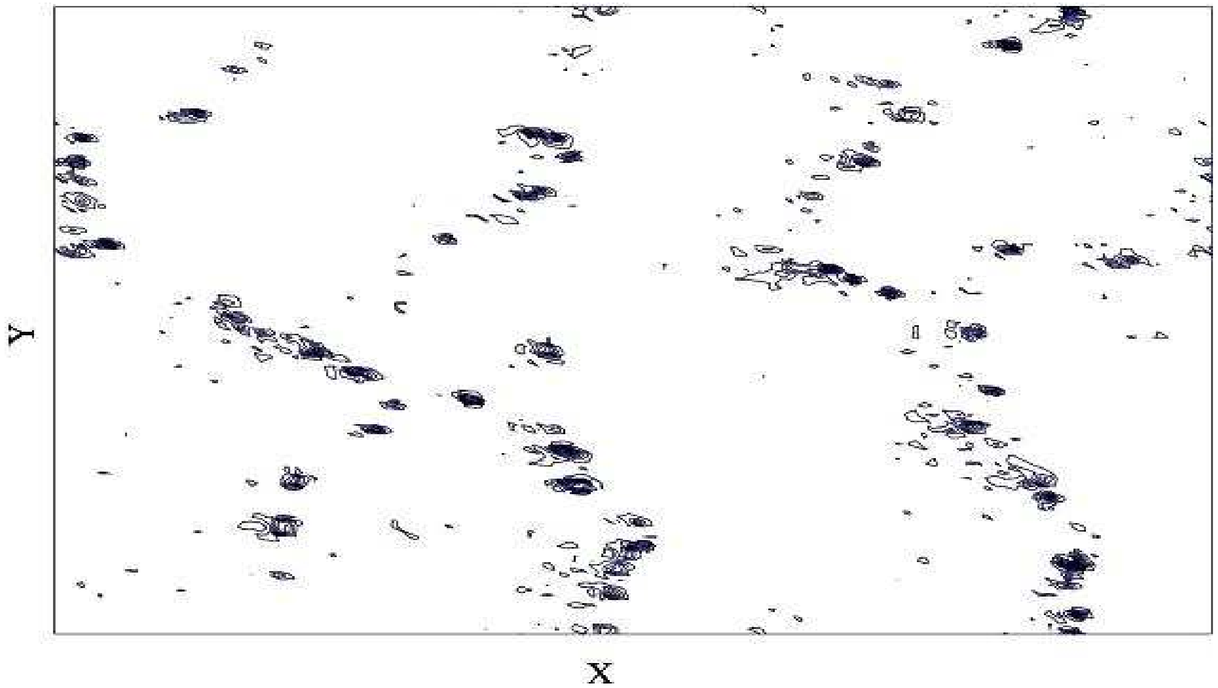
Histogram for M from Cohen and Craig (2006). Solid line is curve-fit with parameters free, and dashed line is theory.



- Cohen and Craig (2006) argue that artificial deviations due to finite CRM domain
- Smallest variance deviations found of $\sim$ 10%

With wind shear imposed

Snapshot of *w* at 2.8km



- Organization increased variance by $\sim 10\%$
- Actually get very close agreement with theory (!)

From Theory to Parameterization

Plant and Craig parameterization

Mass-flux formalism...

1. average in the horizontal to determine the large-scale state
2. evaluate properties of equilibrium statistics: $\langle M \rangle$ and $\langle m \rangle$
3. draw randomly from the equilibrium pdf to get number and properties of cumulus element in the grid box
4. compute convective tendencies from this set of cumulus elements

Some details

- $\langle M \rangle$ from CAPE closure, with adjustment timescale that depends on forcing
- Each element based on modified Kain-Fritsch plume model
- Link from Boltzmann distribution of m to an ensemble of plumes is provided by

$$m = \frac{\langle m \rangle}{\langle r^2 \rangle} r^2 \qquad ; \qquad \varepsilon \sim \frac{1}{r} \tag{5}$$

with r the plume radius

Deterministic limit

- In deterministic limit, parameterization corresponds to a spectrum of plumes with varying entrainment rates in the Arakawa and Schubert (1974) tradition
- Deterministic limit for very large $\langle N \rangle$ in the grid box
  - or very small $\langle m \rangle$
- In ideal gas terms, limit is when grid box is big enough to be a well-defined large-scale state
  - or the case of $T \longrightarrow 0$
- competitive with other deterministic parameterizations

Away from the limit

- parameterization is stochastic
- the character and strength of the noise has a physical basis
- physical noise $>>$ numerical noise from scheme

Why allow stochastic fluctuations?

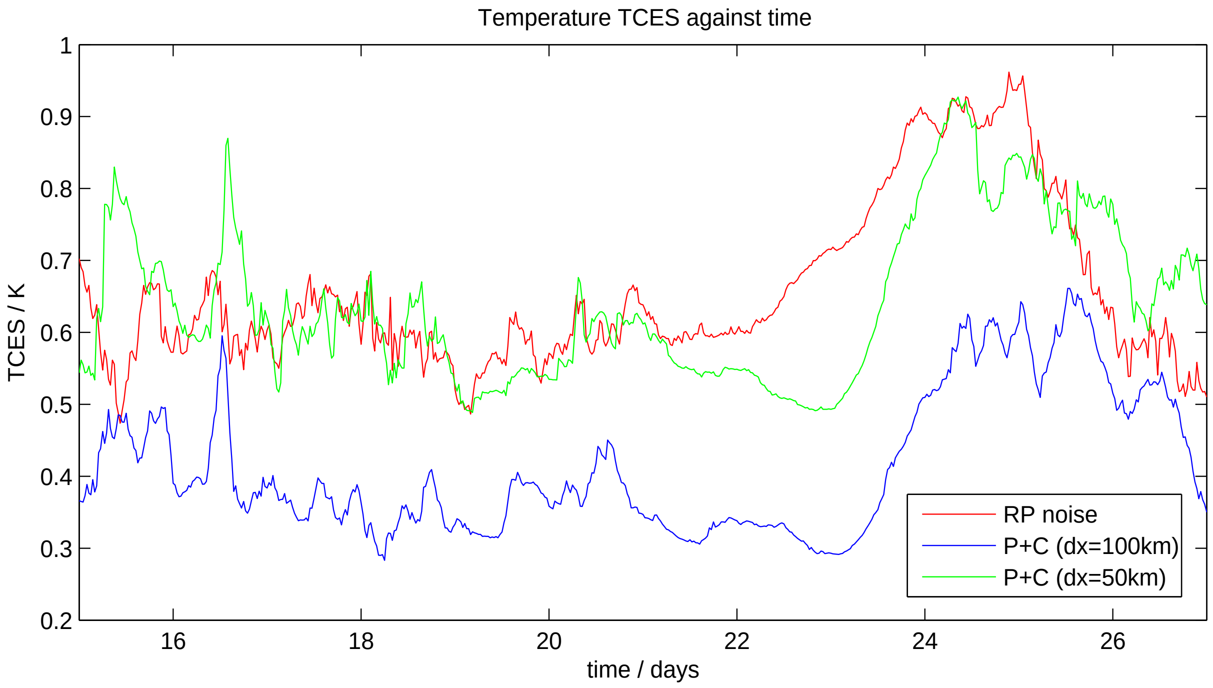
1. Convective instability is released in discrete events
2. Discrete character can be a major source of variability
3. The number of events in a GCM grid-box is not large enough to produce a steady response to a steady forcing
   The grid-scale state is not necessarily a large-scale state
4. Fluctuating component of sub-grid motions may have important interactions with grid-scale flow

NB: we are considering statistical fluctuations about equilibrium, not systematic deviations away from equilibrium

Status of this scheme

- SCM tests of radiative-convective equilibrium (Plant and Craig 2007)
  - fully consistent with Craig and Cohen theory and replicates behaviour of their CRM simulations
- SCM tests of GCCS case 5 (Ball and Plant 2007),
  - comparable to other parameterizations
  - for grid-box area of 50km, the variability is similar to that from
    - a poor-man's ensemble of deterministic parameterizations
    - generic methods designed to represent model uncertainty

GCSS Case 5 Test

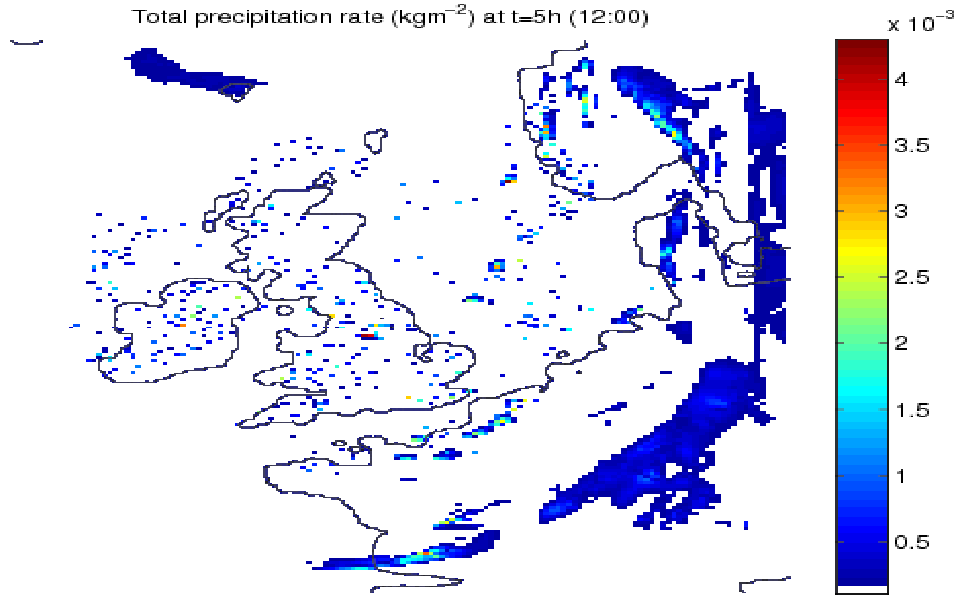


Near-future tests of impact

- Idealized work in 3D to define the averaging needed to obtain “large-scale state”
- Tests in aqua-planet GCM
- Case study tests in DWD Lokal Modell for COSMO-LEPS regional ensemble system
- Statistical tests in Met Office short-range ensemble (MOGREPS)

Near-future tests of impact

3D implementation finally debugged last week!



24/08/05, 06Z, precipitation rate at T+5

Where Next for the Theory?

Departures from this theory?

- Exponential distribution rather robust
- But some departures seen in total mass flux variance
- Suggests assumption that may be breaking down is that of random distribution of elements
- Not larger than $\sim 10\%$ in radiative-convective equilibrium
- Organization in sheared cases has up to $\sim 10\%$ effect

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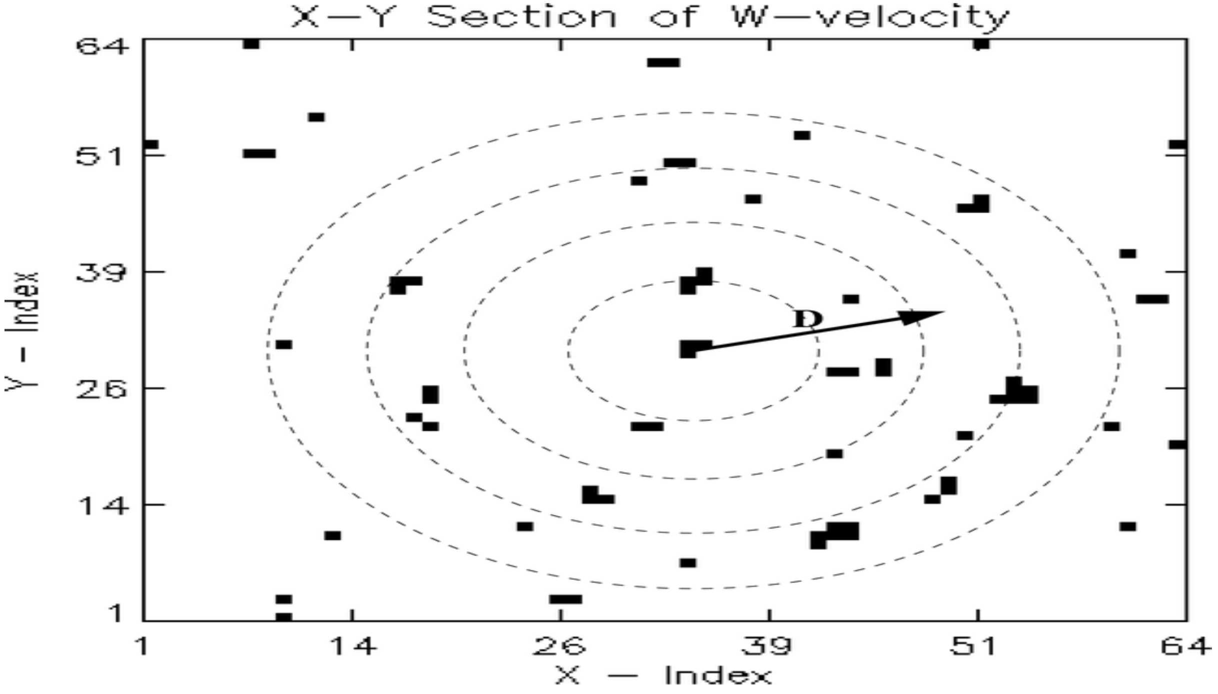
Non-random distribution?

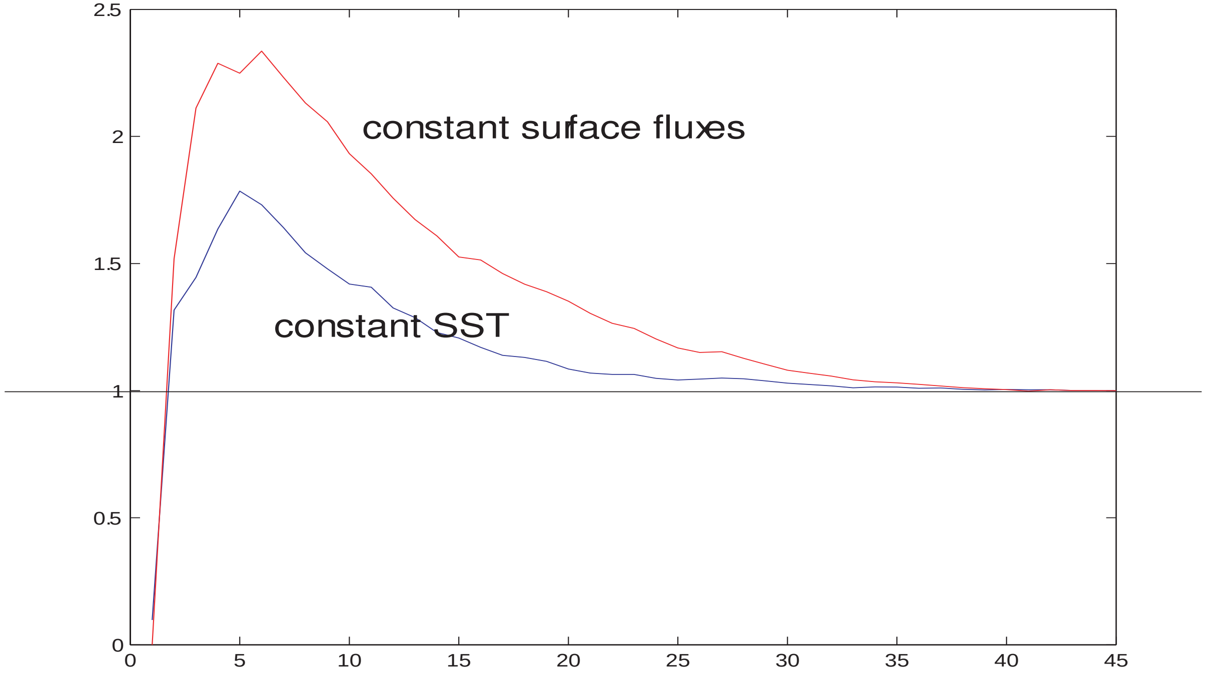
Compute pdf of all of the cloud separations



Normalize to 1 for a random spatial distribution

Non-random distribution

In radiative-convective equilibrium, for different surface conditions



- Short-range repulsion; clumping at intermediate scales
- Self-organization (cold-pool dynamics?) in uniform, unsheared forcing

Plume interactions

- How to generalize to allow for interactions between cumulus elements?
- Statistical mechanics methods allow for interactions
- But we need to characterise the interactions...
- Through a potential, $V(\underline{r})$?
- Are interactions two-body?

Proposal for interactions

- Assuming interactions are weak, treat perturbatively
- First approximation is mean-field theory / Hartree / tree-level
- Each element is subject to mean interaction potential due to all other elements
- In gas particle terms this leads to a non-ideal gas equation
- Boltzmann distribution becomes

$$\exp(-(\mathrm{KE}+nv)/kT) \tag{6}$$

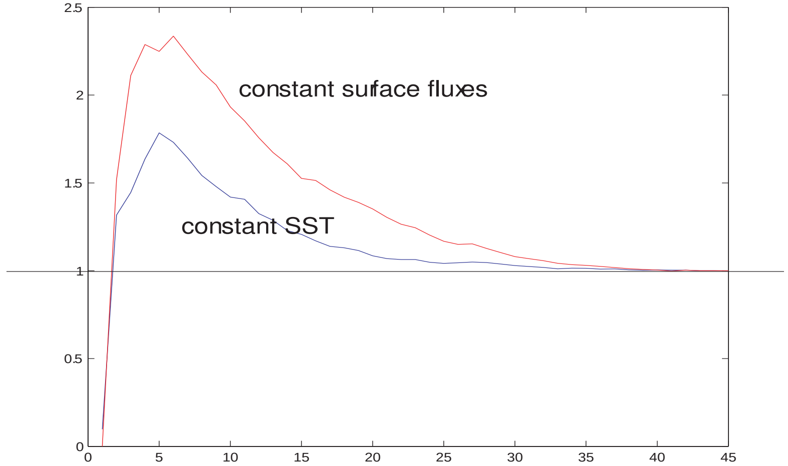
where KE is kinetic energy, n is particle density and $v = \int V(\underline{r})d\underline{r}$

Proposal for interactions

- May be difficult to test for altered distribution in cumulus terms
- But can be used to predict spatial correlation functions
- We could try to derive correct potential by examining such functions
- Would be valuable to be able to test results by varying the character of the interaction

Varying plume interactions

- This part is certianly possible...
- Perform radiative-convective equilibrium CRM runs
- Compute surface fluxes as $\sim (\theta_1 - \theta_0)^\alpha$
- Allowing α to vary suppresses or exaggerates cold pool outflows



- red is $\alpha = 0$
- blue is $\alpha = 1$

A very speculative look ahead

- Hartee is formally the first order perturbation expansion of a Lagrangian field theory of interacting cumulus elements
- Straightforward in principle to calculate higher order effects
- Field theory can account for a non-trivial environment / vacuum/ ground-state with non-zero ensemble mean plume density, corresponding to the large-scale forcing for convection
- Quasi-equilibrium holds if environment is time-invariant, and stable against plume perturbations
- Could extend by accounting for discrete nature of plumes, producing a quantum field theory for convection (yikes!)

Conclusions / Speculations

- Statistical mechanics approach of Cohen and Craig predicts cumulus ensemble properties
- A successful theory, with analogies to ideal gases
- Can be directly incorporated into parameterization
- A theoretical framework exists to build a comprehensive statistical theory of convection (i.e., field theory)
- The big difficulty is to write down the Lagrangian / the plume interaction potential
- First tests of the basic idea would build analogies to non-ideal gases