Dynamic diagnoses of turbulent mixing lengths in shallow convection

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Navigating the Turbulence Grey Zone in Numerical Weather Prediction
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Aims

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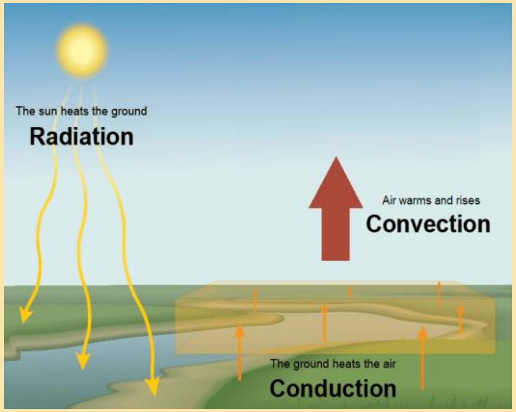
- Offline dynamic-model calculations of flow-dependent Smagorinsky mixing length
- What are its dependencies in terms of the variable, location within the flow, scale of analysis?
- Implement dependencies into the MONC LES model and test their effect on shallow Cu test cases

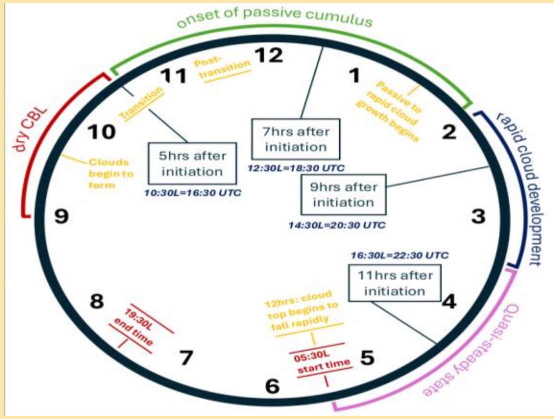
Case Studies

Dry CBL	BOMEX	ARM



- Idealised dry case
- Strong temperature inversion imposed at 1km
- $\Delta x = \Delta z = 20$ m

- Marine boundary layer, quasi steady
- $\Delta x = \Delta z = 20$ m



- Diurnal cycle of deepening cloud, reaching up to ~3km by 1630
- $\Delta x = 25$, $\Delta z = 10$ m

Smagorinsky scheme

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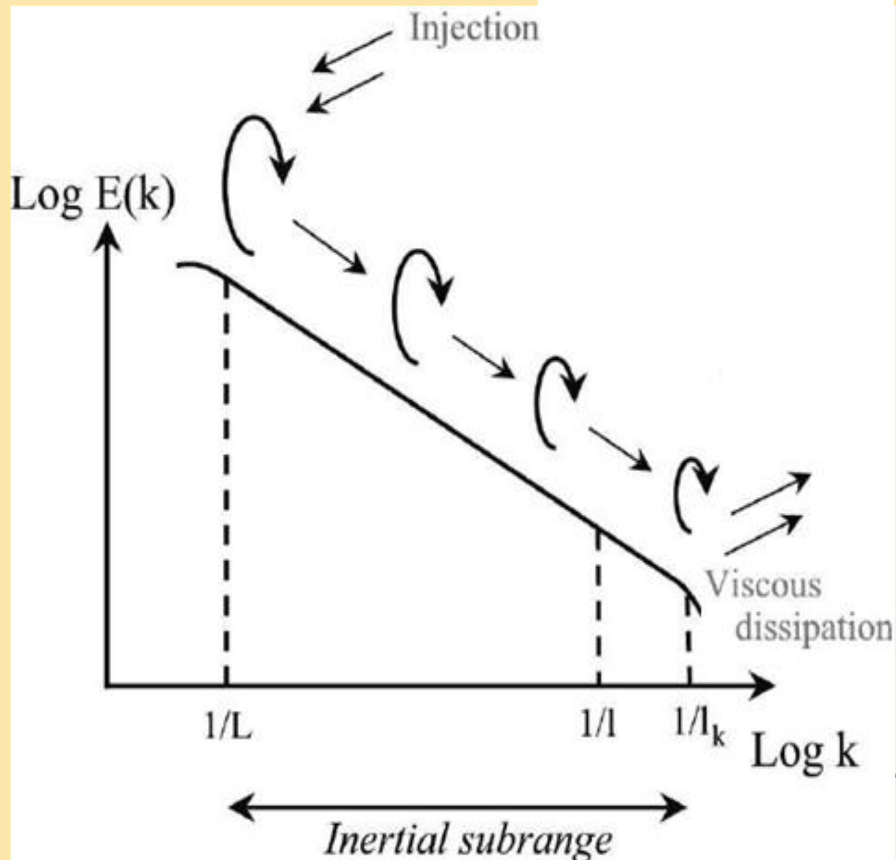
- Dissipation estimated using a mixing length
- Smagorinsky sets C_s to a constant, typically ~ 0.2.
- Scalar mixing is assumed to be related to momentum mixing via a Prandtl number Pr_ψ typically 0.7

$$l_{\mathrm{mix}} = C_s \Delta$$

$$l_\psi = C_\psi \Delta$$

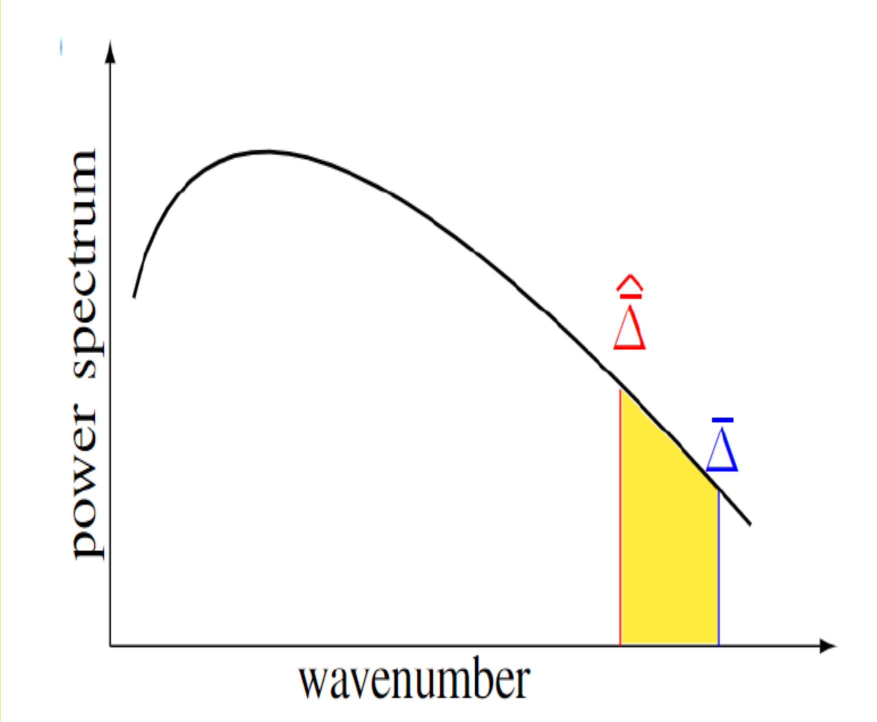
$$C_\psi^2 = \frac{C_s^2}{Pr_\psi}$$



Dynamic method

- Compares flow filtered at two scales, $\bar{\Delta}$ and $\hat{\bar{\Delta}}$

- L_{ij} is difference in stress
- M_{ij} is Smagorinsky estimate of the same
- Minimizing L-M leads to an estimate of C_s



Momentum:

$$L_{ij} = \hat{\bar{u}}_i \hat{\bar{u}}_j - \widehat{\bar{u}_i \bar{u}_j}$$

$$M_{ij} = \hat{\bar{\Delta}}^2 |\hat{\bar{S}}| \hat{\bar{S}}_{ij} - \bar{\Delta}^2 \widehat{|\bar{S}| \bar{S}_{ij}}$$

$$C_s = \sqrt{\frac{1}{2}\left(\frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{kl}^2 \rangle}\right)}$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

$$|\bar{S}| = \sqrt{2 \bar{S}_{kl} \bar{S}_{kl}}$$

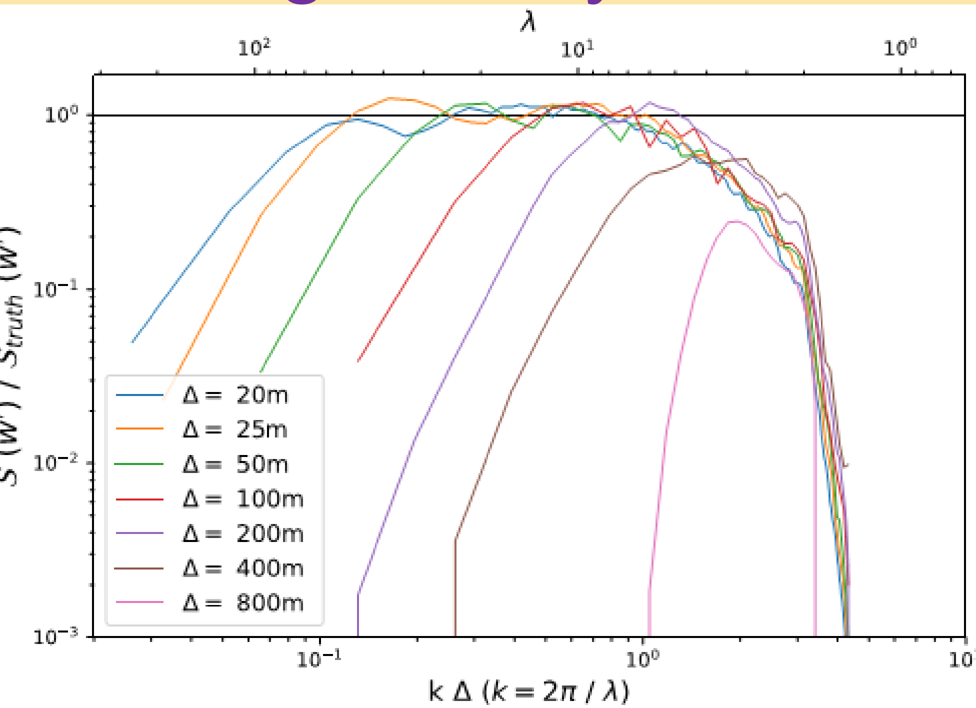
Scalars:

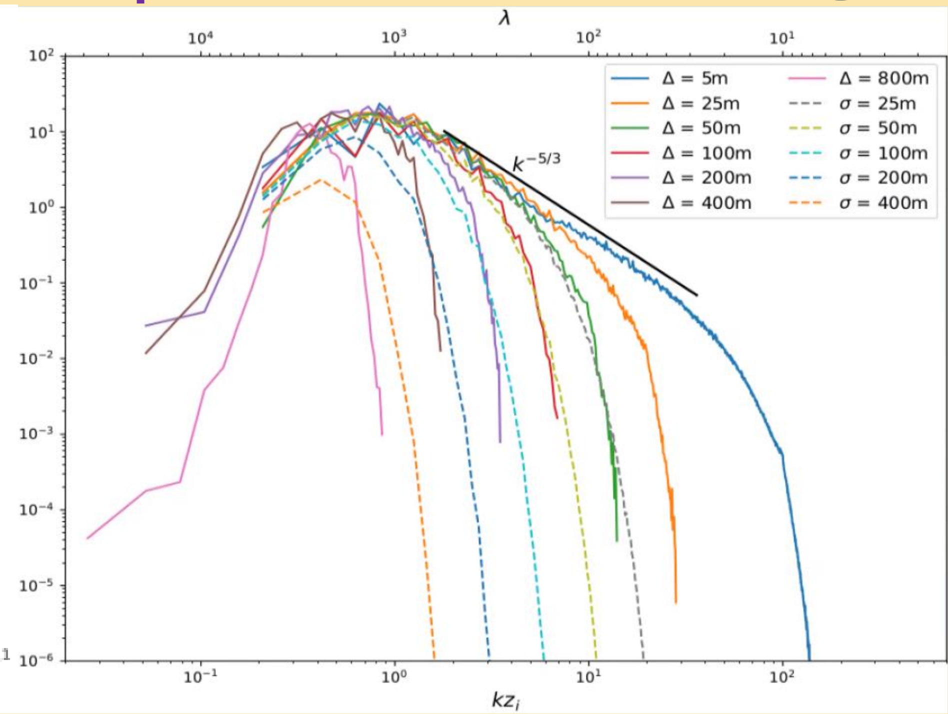
$$H_j = \hat{\bar{u}}_j \hat{\bar{\psi}} - \widehat{\bar{u}_j \bar{\psi}}$$

$$R_j = \hat{\bar{\Delta}}^2 |\hat{\bar{S}}| \widehat{\frac{\partial \bar{\psi}}{\partial x_j}} - \bar{\Delta}^2 \widehat{|\bar{S}| \frac{\partial \bar{\psi}}{\partial x_j}}$$

$$C_\psi = \sqrt{\frac{1}{2}\left(\frac{H_j R_j}{R_k^2}\right)}$$

Smagorinsky as a filter operation



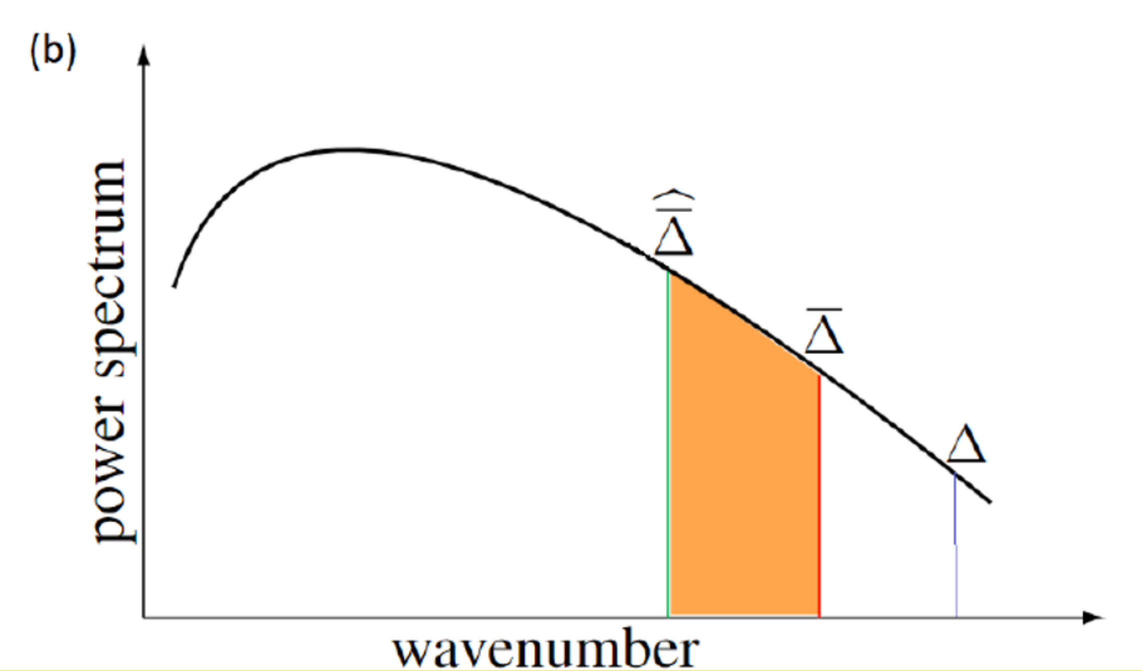


- Departures from $k^{-5/3}$ for dry CBL run at different resolutions
- Smagorinsky (and numerics) act as a filter at small scales
- Similar to a Gaussian

- Gaussian filter applied to fine-resolution data with a width $\sigma = \Delta/2$ can well approximate model spectrum with grid length Δ

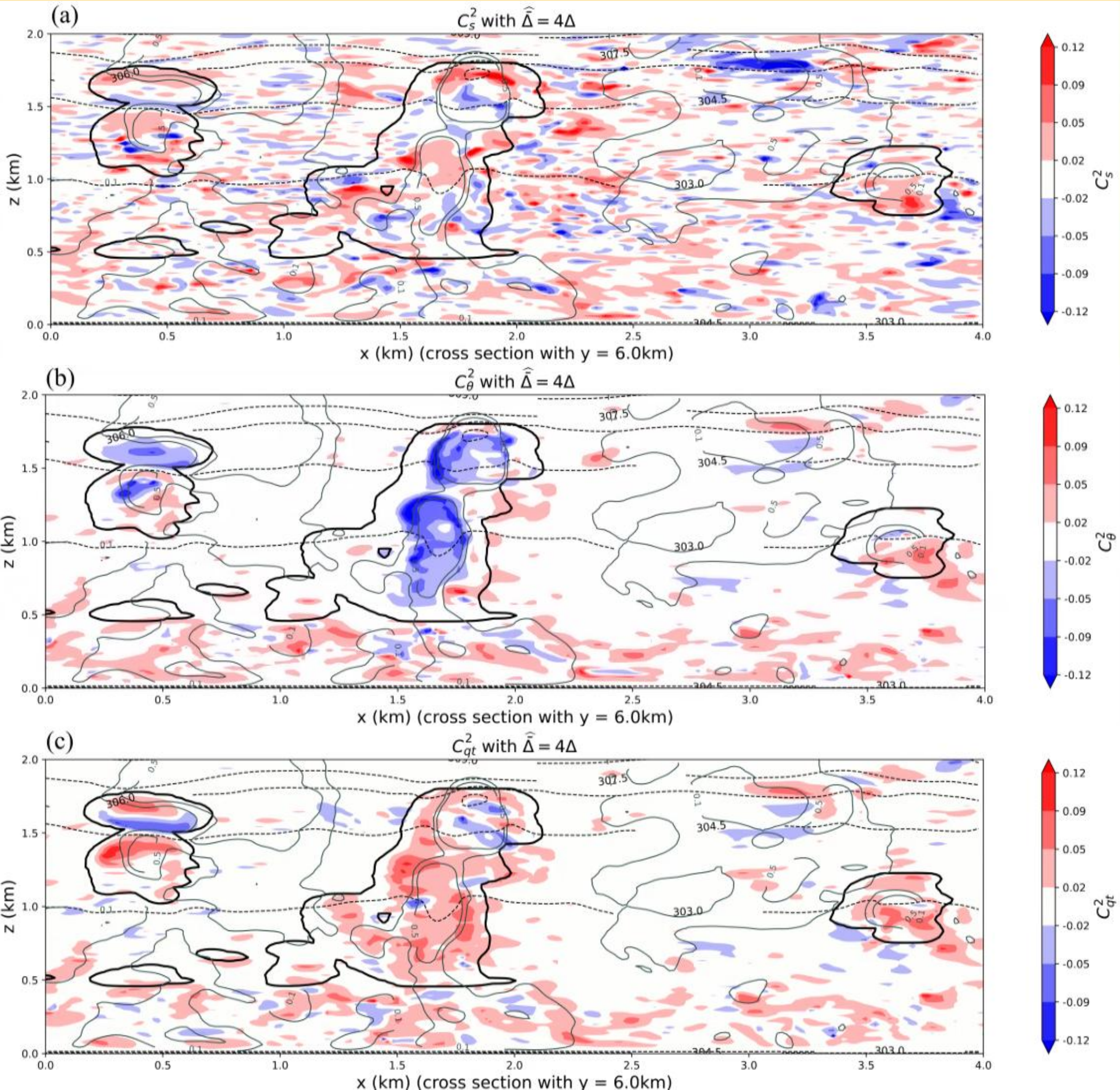
Filtering approach

- Filter the raw LES data with grid length Δ so as to approximate model spectrum with grid length $\overline{\Delta} = 2\Delta$
- And filter again to be appropriate for $\widehat{\overline{\Delta}} = 4\Delta$



- Apply dynamic mixing based on stress difference in range $\overline{\Delta} = 2\,\Delta \rightarrow \widehat{\overline{\Delta}} = 4\,\Delta$
- Similarly to consider the range $4\,\Delta \rightarrow 8\,\Delta$, $8\,\Delta \rightarrow 16\,\Delta$ etc

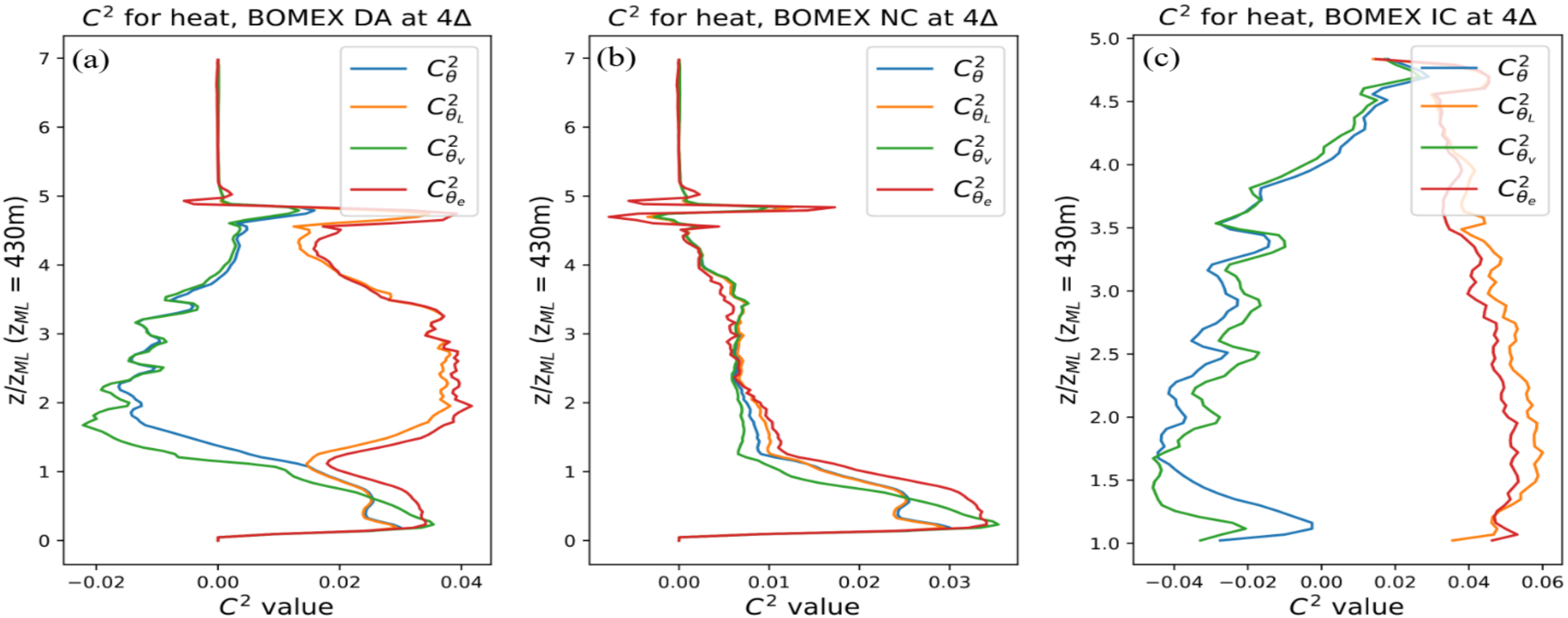
Snapshots of C_s^2, C_θ^2,and $C_{q_T}^2$



- Strong variations in space on fine scales
- Systematic differences between variables and between mixed-layer, in-cloud and cloud-free parts

BOMEX, xz section

Different scalar variables

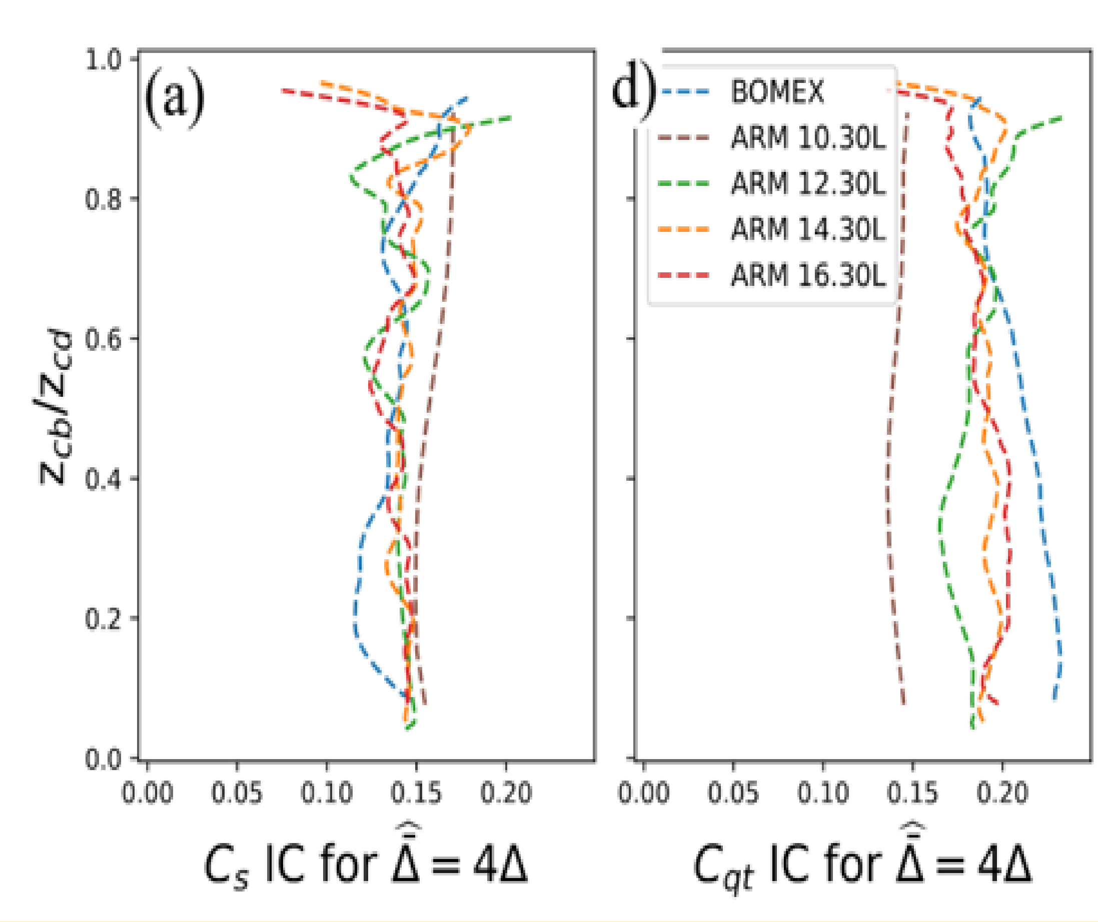


Domain average | No-cloud | In-cloud (truncated)

- Average C^2 values agree in the absence of cloud
- Similar mixed-layer and in-cloud values but smaller outside cloud
- Within cloud, cloud-conserved variables θ_L and q_T have C^2 +ve but θ and θ_v have C^2 -ve

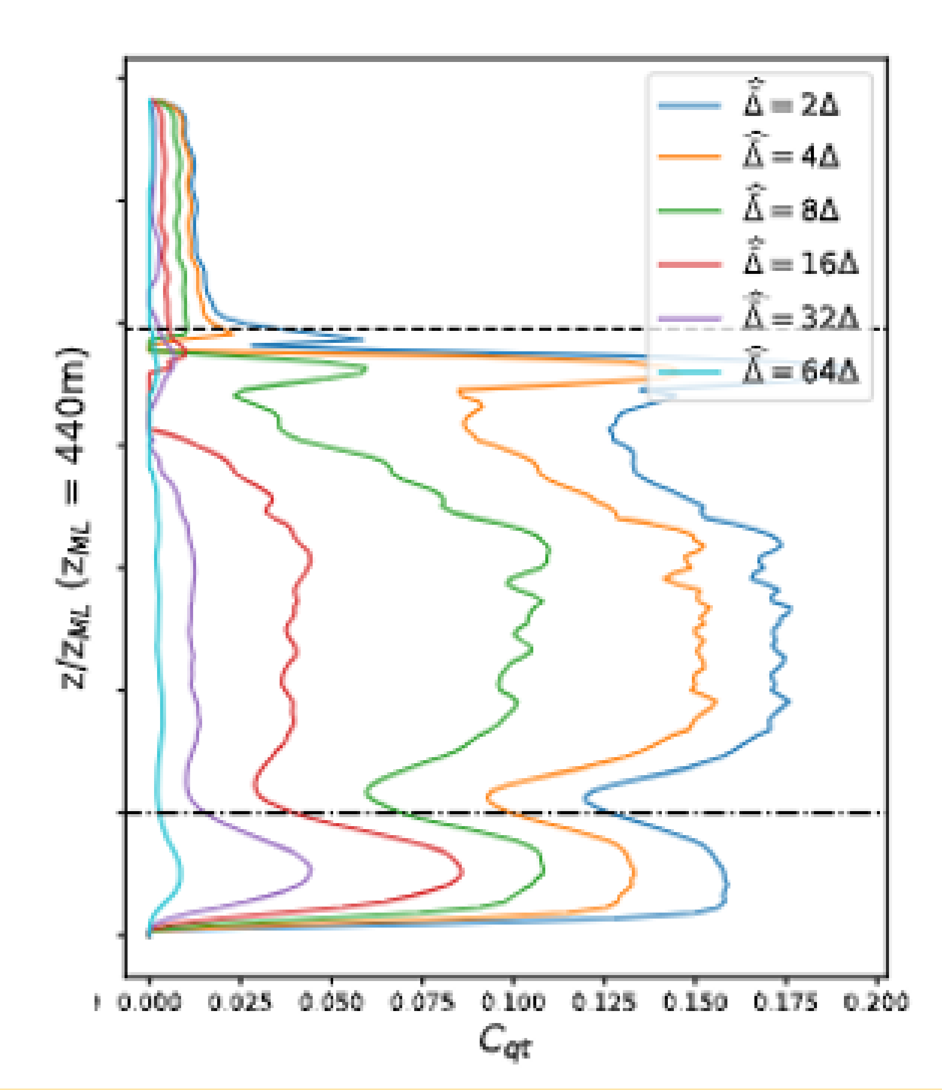
Variations with cloud-depth

- In-cloud C_s (left) and C_{q_T} (right) profiles as a function of normalized height within the cloud layer
- Modest differences between BOMEX and different times during ARM



Variations with filter scale

- C_{q_T} in ARM at 1630
- Mixing lengths reduced near surface, near inversion at cloud top but approx. constant within mixed layer and cloud layer
- C_{q_T} reduces at larger scales
- i.e $l_{mix} = C_{q_T}\Delta$ increases sub-linearly with Δ



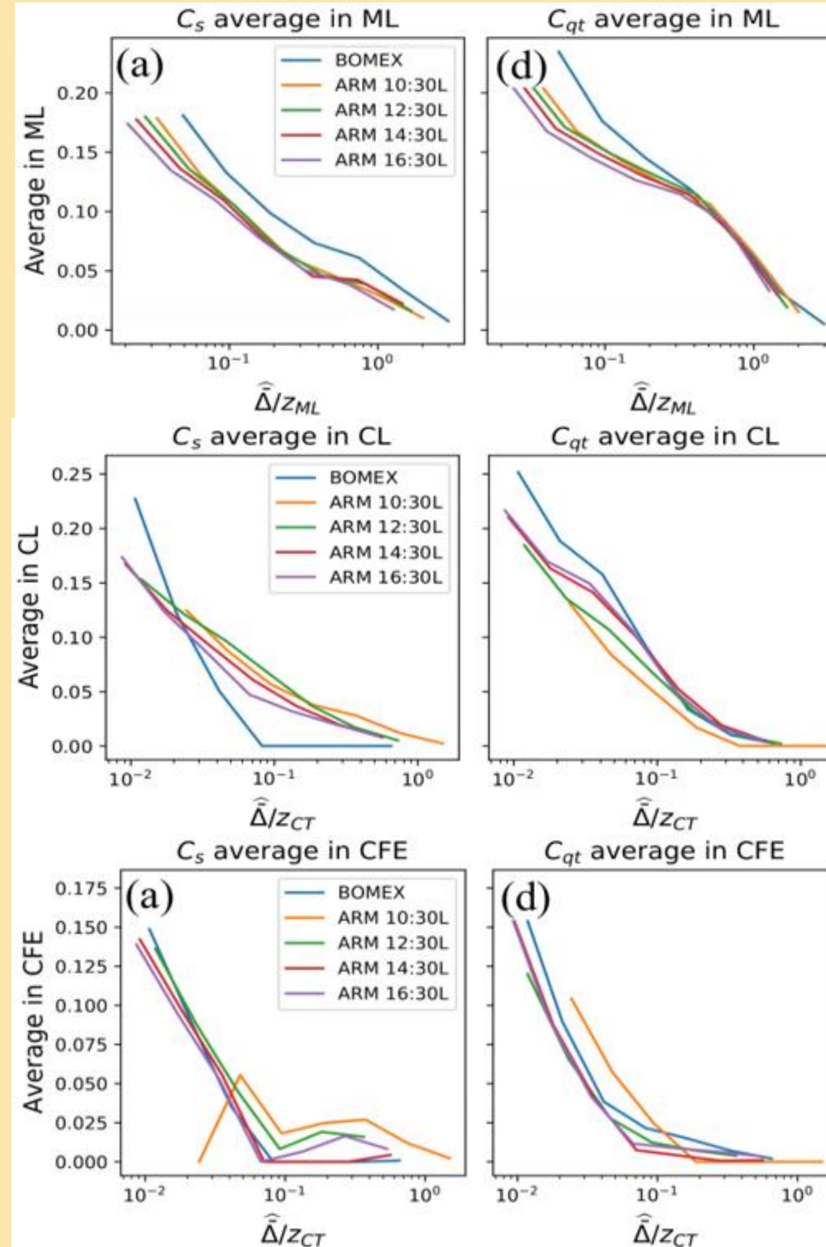
Variations with filter scale

- Clear decrease in C_s (left) and C_{q_T} (right) as filter scale increases
- Especially in the cloud-free environment
- (i.e. mixing there is on small scales only, < 10% cloud depth)

Mixed layer

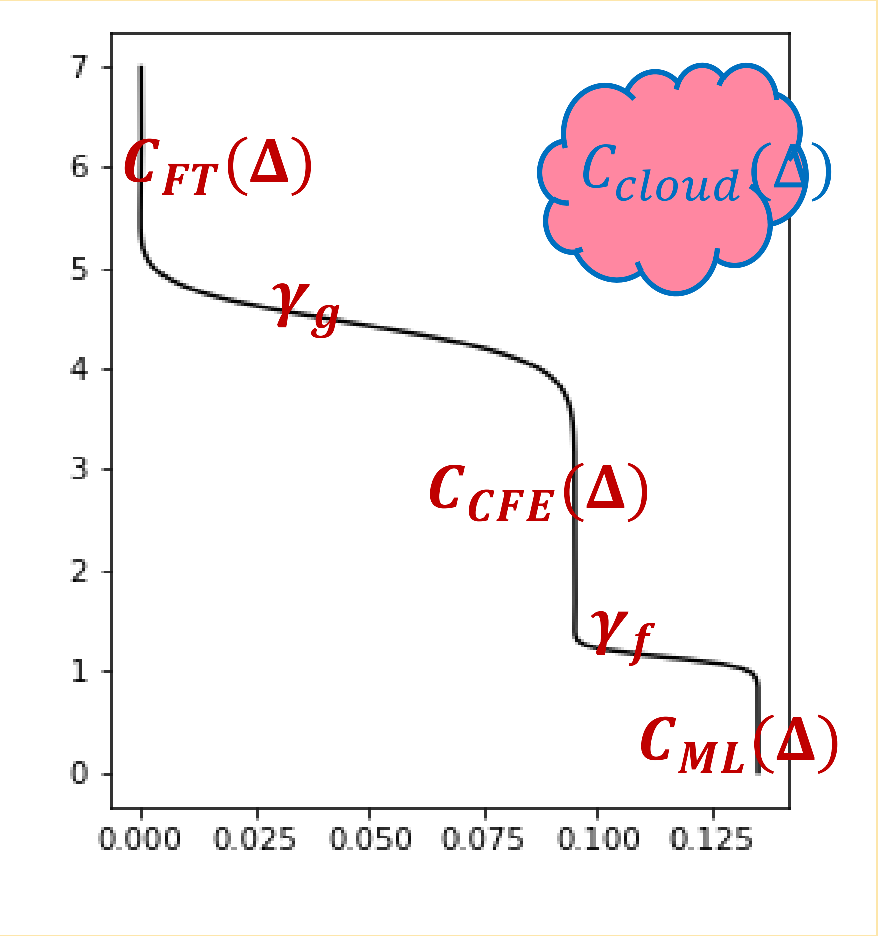
Within cloud

Cloud-free environment



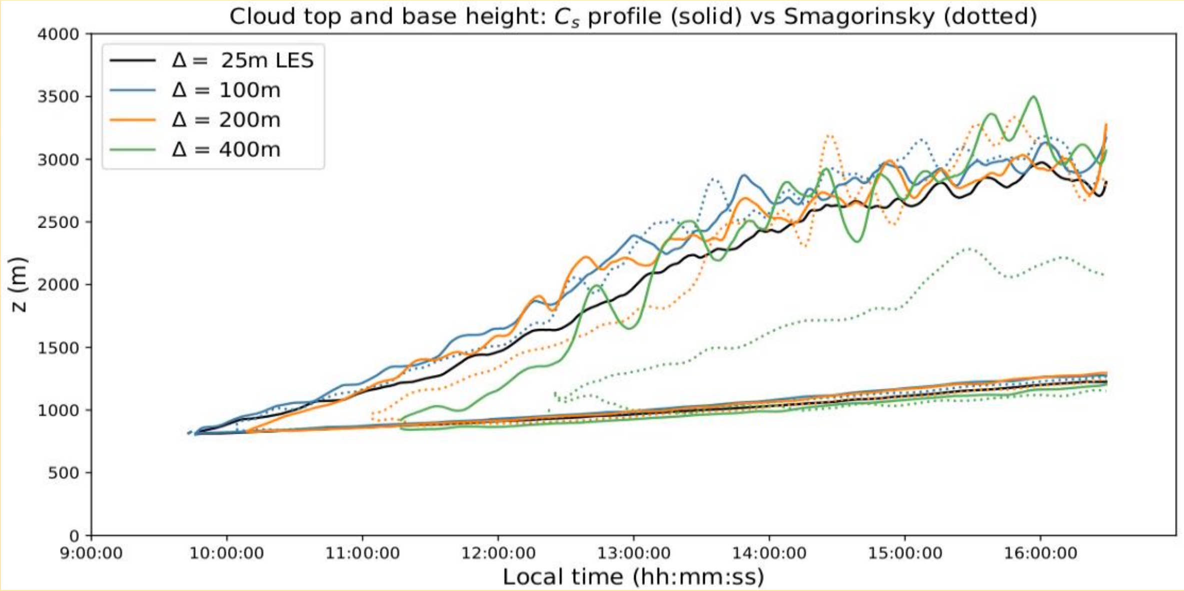
Simple parameterization

- Capture mean dependencies on scale and location within the flow
- Run cases in the grey zone
- Either for C_s alone with fixed Pr
- Or with a separate parameterization for the scalars



Parameterization results

- ARM, evolution of cloud base and top
- 25m LES
- 100m
- 200m
- 400m



- Apply C_s profile diagnosed for 100m. Fixed Pr.
- Solid C_s=0.23. Dotted for the parameterization
- Improves cloud initiation
- C_s is reduced from 0.23, with more motions represented explicitly, larger resolved fluxes etc
- A global reduction to a constant C_s is somewhat helpful but not as effective

Parameterization results

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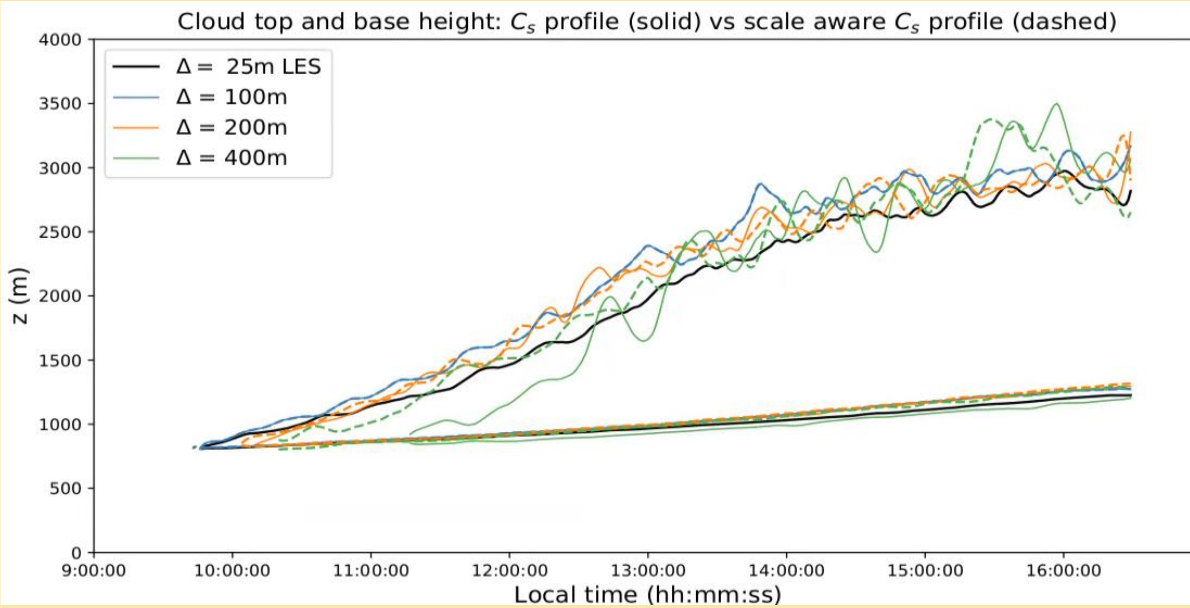

- Add scale awareness to Cs profile
- Improves timing further
- And improves rate of cloud growth
- Distinction between in-cloud and cloud-free mixing is valuable for reproducing the cloud-top height evolution

Conclusions

- Momentum, potential temperature and moisture are mixed differently
- C is significantly higher in-cloud compared to “clear sky” areas in the cloud layer
    - Different areas within clouds do not seem to affect the mean C
- Scale dependence of C_s
- This was the most important ingredient to enable accurate cloud layer formation and development
- Vertical profile of C_s improves cloud-top height, especially later in the diurnal cycle