1 The Surface Energy Balance

The exchange of energy between the Earth's surface and the overlying atmosphere involves four important processes, namely:

- *Absorption* and *Emission* of 'natural' electromagnetic radiation by the surface.
- *Thermal Conduction* of heat energy *within the ground.*
- *Turbulent transfer* of heat energy towards or away from the surface *within the atmosphere.*
- *Evaporation* of water stored in the soil or *Condensation* of atmospheric water vapour onto the surface.

Each of these processes can be associated with an *energy flux density.*

Definition: *Energy Flux Density* The rate of transfer of energy normal to a surface of unit area. The SI unit is J m^{-2} s^{-1} which is equivalent to W m^{-2}.

The energy balance of a surface layer of finite depth and unit horizontal area can be written as,

$$\frac{dQ}{dt} = R_n - G - H - \lambda E \tag{1}$$

- Q is the total heat energy stored in the surface layer.
- R_n is the **net surface irradiance** (commonly referred to as the *net radiation*). It represents the gain of energy by the surface from radiation. It is a positive number when it is towards the surface.
- G is the **Ground Heat Flux**. It is the loss of energy by heat conduction through the lower boundary. It is a positive number when it is directed away from the surface into ground. The value at the surface is denoted G_0.
- H is the **Sensible Heat Flux**. It represents the loss of energy by the surface by heat transfer to the atmosphere. It is positive when directed away from the surface into the atmosphere.
- λE is the **Latent Heat Flux**. It represents a loss of energy from the surface due to evaporation. (λ is the *specific latent heat of evaporation*, units J kg^{-1} and E is the evaporation rate, with units kg m^{-2} s^{-1}).

For an infinitely thin surface layer the heat storage in Eq. 1 is zero and reduces to,

$$R_n - G_0 - H - \lambda E = 0 \tag{2}$$

or

$$R_n - G_0 = H + \lambda E \tag{3}$$

The quantity $R_n - G_0$ is known as the *available energy*. In modelling the surface energy balance we need to be able to calculate the *available energy* and partition it between the sensible and latent heat fluxes.

The way in which the *available energy* is partitioned between the sensible and latent heat flux can be quantified by taking the ratio of the sensible to latent heat flux, which is known as the *Bowen ratio*,

$$B_0 = \frac{H}{\lambda E} \tag{4}$$

The *Bowen ratio* is non-dimensional and depends on the availability of water at the surface.

- For surfaces where water is freely available B_0 is small, and most of the available energy is transferred to the atmosphere in the form of latent heat.
- For arid surfaces (e.g. deserts) B_0 is large, and most of the available energy is transferred to the atmosphere in the form of sensible heat, which warms the air close to the surface.
- Vegetation is a significant influence on the Bowen ratio.

In terms of the Bowen ratio the surface energy balance can be written as,

$$R_n - G_0 = H\frac{(1 + B_0)}{B_0} \tag{5}$$

or

$$R_n - G_0 = \lambda E(1 + B_0) \tag{6}$$

2 Heat Conduction

Conduction of heat within the soil satisfies *Fourier's Law of Heat Conduction*

$$F_h = -k\frac{dT}{dz} \tag{7}$$

- F_h is the heat flux in the z-direction. Units W m^{-2}.
- k is the thermal conductivity. Units J m^{-1} s^{-1} K^{-1}.

If the temperature gradient in the soil is not constant with depth then the heat flux F_h must also vary with depth. Conservation of energy requires,

$$\rho c\frac{d\overline{T}}{dt}\Delta z = F_h(z) - F_h(z + \Delta z) \tag{8}$$

- ρ is density.
- c is the specific heat capacity of the soil.
- $\overline{T}$ is the average temperature of the soil layer between z and $z + \Delta z$.

The effects of any horizontal fluxes have been neglected, which is usually a good approximation.

Using Eq. 7 for $F_h(z)$ and $F_h(z + \Delta z)$ and taking the limit $\Delta z \to 0$ gives,

$$\frac{dT}{dt} = \frac{k}{\rho c}\frac{d^2T}{dz^2} = \kappa_s\frac{d^2T}{dz^2} \tag{9}$$

where it has also been assumed that the soil properties do not vary with depth. The quantity $\kappa_s = k/(\rho c)$ is the *Thermal Diffusivity*, with units m^2 s^{-1}.

Note: Equation 8 can also be applied to the atmosphere by replacing the specific heat capacity, c, by the specific heat of air at constant pressure, c_p.

3 The Diurnal Temperature Wave in the Soil

The ground heat flux at the surface, G_0, varies approximately sinusoidally through the day, in response to diurnal variations in the net radiation, i.e.,

$$G_0 \approx G_{0,max} \sin(\Omega t) \tag{10}$$

where $\Omega = 2\pi/(24 \times 3600) \approx 7.3\times\ 10^{-5}\ \mathrm{s}^{-1}$. From the dimensions of the thermal diffusivity, κ_s and Ω the combination,

$$D = \left(\frac{\kappa_s}{\Omega}\right)^{1/2} \tag{11}$$

has dimensions of a length. The full solution of Eq. 9 shows that at a depth z the ground flux also varies sinusoidally, but with the amplitude reduced by a factor of $\exp(-z/\sqrt{2}D)$, The amplitude of variations in temperature also decreases with depth in the same way. In addition to the changes in amplitude, the phase of the diurnal variation in temperature and flux also changes with depth.

The ground heat flux is determined by measuring the temperature difference between the faces of a plastic disc buried in the soil. The thermal conductivity of the disc is known so the heat flux through the disk can be calculated. The thermal properties of the plastic are chosen to be similar to soils, so that the heat flux passing through the disc should be close to the ground heat flux.

The disc must be put at some depth in the soil. Clearly the depth needs to be small compared to D, or the measured flux will not be representative of the flux at the surface.

4 Atmospheric Radiation

Solar Radiation refers to electromagnetic radiation emitted by the sun.

- The sun emits approximately as a black body (energy emitted per unit area=σT^4) with a temperature of 6000 K. Most of the emitted radiation has wavelengths between 0.3 to 2.0 μm, with the maximum around 0.5 μm. Visible radiation has wavelengths between 0.4 and 0.7 μm. Solar radiation is often referred to as *short-wave* radiation.
- At the top of the atmosphere the *direct solar beam* can be considered to be formed of parallel rays. The *irradiance* of a surface normal to the beam at the top of the atmosphere is about 1380 W m^{-2}.
- The direct solar beam is attenuated as it passes through the atmosphere due to absorption and scattering, so the irradiance at the surface (for the sun directly overhead in cloud-free conditions) is reduced to about 1000 W m^{-2}.
- Some of the radiation scattered from the direct beam by molecules and aerosols reaches the surface as *diffuse solar radiation*

Terrestrial radiation refers to electromagnetic energy emitted by the Earth's surface and the atmosphere.

- The Earth's surface emits like a blackbody with a temperature of about 290 K. From *Wien's displacement law* ($\lambda_{max} \propto 1/T_s$) the wavelength of terrestrial radiation will be much larger than for solar radiation. Most of the energy is in the wavelength range 3 to 30 μm, with a maximum around 10 μm. Terrestrial radiation is often referred to a *long-wave* radiation.
- For most surfaces the long-wave radiation emitted per unit area is given by $\epsilon_s \sigma T_s^4$, where ϵ_s is an *effective emissivity*. From *Kirchoff's law* the surface will absorb a fraction ϵ_s of any incident long-wave radiation and reflect a fraction $(1 - \epsilon_s)$.
- The emissivity of the atmosphere varies strongly with wavelength, so the atmosphere does not behave like a black body. For wavelengths between 8 and 12 μm the atmosphere is almost transparent so the emissivity is very small. This wavelength range is known as a *window region.* For wavelengths between 3 and 8 μm and for wavelengths greater than 12 μm the atmosphere is nearly opaque, i.e. the emissivity is close to one.
- The emission and absorption characteristics of the atmosphere vary with the concentrations of *green-house gases* such as *water vapour* and *carbon dioxide.* Variations in water vapour are responsible for the short term variability in the radiative characteristics of the atmosphere.

- Liquid water has an emissivity close to one, so that thick clouds act like black body radiators.

5 Irradiance at the Earth's surface

The *net solar irradiance* of the surface can be represented as,

$$S_n = S \downarrow - S \uparrow \tag{12}$$

where $S \downarrow$ is the total down-welling short-wave radiation and $S \uparrow$ is the total up-welling short-wave radiation. Since the Earth's surface does not emit short-wave radiation $S \uparrow$ is entirely associated with reflection of some of the down-welling radiation. It follows that,

$$S_n = (1 - \alpha) \times S \downarrow = (1 - \alpha) \times (S_b + S_d) \tag{13}$$

where $\alpha = S \uparrow / S \downarrow$ is the *surface albedo*, S_b is the *direct-beam solar irradiance* and S_d is the *diffuse solar irradiance*

- The direct beam irradiance of the surface is,

  $$S_b = S_p \cos(\theta) \tag{14}$$

  where S_p is the is the irradiance normal to the direct beam and θ is the *solar zenith angle.*

- Under clear sky conditions the irradiance S_p normal to the direct beam can be related to the irradiance normal to the direct beam at the top of the atmosphere, S_{ext}, by,

  $$S_p = S_{ext} \exp(\tau / \cos(\theta)) \tag{15}$$

  where τ is the *atmospheric turbidity coefficient* which will depend on such things as the aerosol content of the atmosphere.

- The magnitude of the diffuse solar irradiance depends on atmospheric aerosol and clouds.

The out-going long-wave radiation from the surface is

$$L \uparrow = \epsilon_s \sigma T_s^4 + (1 - \epsilon_s) L \downarrow \tag{16}$$

The first term on the right-hand side of this equation is the radiation emitted by the surface and the second term is the reflected down-welling long wave radiation incident on the surface.

The *net long-wave irradiance* of the surface is,

$$L_n = L\downarrow - L\uparrow = \epsilon_s \times (L\downarrow - \sigma T_s^4) \tag{17}$$

In the field the net longwave irradiance will be measured directly and the surface temperature will also be estimated. With these measurements Eq. 17 can be used to estimate the downward longwave irradiance as,

$$L\downarrow = \sigma T_s^4 + \frac{L_n}{\epsilon_s} \tag{18}$$

The net surface irradiance is just,

$$R_n = S_n + L_n \tag{19}$$

6 Turbulent Flows

From experience we know that wind near the surface is not steady but fluctuates in time and space. It is often possible to consider the windspeed to consist of a steady, or slowly varying part with fluctuations superimposed, so that instantaneously the windspeed can be written as,

$$U = \overline{U} + u' \tag{20}$$

Here $\overline{U}$ is the *mean windspeed* (which we will consider to be constant) and u' is the fluctuating or *turbulent* component. Partitioning the wind into a mean and fluctuating part is known as a *Reynolds decomposition.*

Other meteorological parameters, such as vertical velocity, temperature, vapour density can also be decomposed into a mean and fluctuating part, i.e.

$$\begin{aligned} w &= \overline{w} + w' \\ T &= \overline{T} + T' \\ \chi &= \overline{\chi} + \chi' \end{aligned}$$

where the vapour density χ has units of kg m^{-3}. Close to the surface $\overline{w} \approx 0$ so that $w \approx w'$.

Note: from the definition of the mean, the mean value of the fluctuations is zero.

In general the *turbulent fluctuations* in different parameters are correlated. It is through these correlations that turbulence is responsible for the transport heat, moisture and momentum in the atmosphere.

7 Turbulent Eddies

The turbulent fluctuations have a spatial extent and it is useful to think about these fluctuations as 'blobs' of air with properties that differ from the mean properties of the atmosphere. These 'blobs' are usually known as *turbulent eddies.* The most important characteristics of these *eddies* are,

- They are three dimensional.
- The fluctuations in the three components of the velocity are of similar magnitude, when averaged over many eddies.
- Turbulent eddies can be characterised by a length, representing the typical size of the eddy, and typical magnitude of the velocity fluctuation. These are usually termed the *turbulent length scale* and the *turbulent velocity scale.*

8 The Turbulent Heat Flux and Eddy Covariance

Consider a series of N turbulent eddies with vertical velocities w_i and temperatures $T_i = \overline{T} + T_i'$. The sensible heat flux per unit area associated with the ith eddy is,

$$F_{hi} = \rho c_p w_i T_i \tag{21}$$

(*you might like to verify the units of this are* $W\ m^{-2}$)

The average flux associated with the N eddies is,

$$\overline{F_h} = \frac{1}{N}\sum_{i=1}^{N} F_{hi} = \rho c_p \times \frac{1}{N}\sum_{i=1}^{N}(w_i T_i) \tag{22}$$

Using $w_i = \overline{w} + w_i'$, $T_i = \overline{T} + T_i'$, and recalling that $\overline{w} \approx 0$ and $\overline{T}$ is constant, leads to

$$\overline{F_h} = \rho c_p \sum_{i=1}^{N}(w_i' T_i') = \rho c_p \overline{w'T'} \tag{23}$$

Although $\overline{w'}$ and $\overline{T'}$ are both zero, fluctuations in the vertical velocity and temperature may be correlated (for example, if when $w' > 0$, on average $T' > 0$ and vice-versa, $\overline{w'T'}$ would be positive) so that the covariance $\overline{w'T'}$ is not zero.

The same reasoning can be applied to moisture to give,

$$\overline{F_\chi} = \overline{w'\chi'} \tag{24}$$

$$\overline{F_{\lambda\chi}} = \lambda\overline{w'\chi'} \tag{25}$$

$\overline{F_\chi}$ is an estimate of the surface evaporation rate (E, kg m^{-2} s^{-1}) while $\overline{F_{\lambda\chi}}$ is an estimate of the surface latent heat flux (λE, W m^{-2}).

- This technique of estimating turbulent fluxes is known as the *eddy correlation technique*
- Quantities such as $\overline{w'T'}$ and $\overline{w'\chi'}$ are known as *eddy covariances*, or less correctly as *eddy correlations.*
- Strictly $\rho c_p \overline{w'T'}$ is a statistical estimate of the turbulent flux, if we sampled a different series of eddies we would get a different answer. How different depends on how large our sample of eddies is, the larger N the smaller the differences.

- To estimate fluxes using the eddy correlation technique, measurements must be made at some height above the surface. The fluxes are only the same as the surface flux if they don't vary with height. In the atmosphere fluxes usually vary with height (*what tells us this must be the case, at least for the sensible heat flux ?*), however the variation is is sufficiently gradual that the difference between the flux at 100m and the surface flux is typically less that 10%. The layer between the surface and about 100m is known, somewhat incorrectly, as the *constant flux layer* or the *surface layer*

In practice we don't try to isolate individual eddies. Outputs from fast response anemometers and thermometers are sampled (just like music samples) and means, eddy covariances are calculated by averaging over these samples. By doing this we average over the eddies, but also average fluctuations within the eddies.

9 The Momentum Flux

Defining the momentum per unit volume as $\mu = \rho U$ exactly the same reasoning as above can be applied to the windspeed to give the momentum flux as,

$$\overline{F_\mu} = \rho \overline{w'u'} \tag{26}$$

$\overline{F_\mu}$ is the flux of horizontal momentum along the vertical direction. The flux of a component of momentum in a direction normal to that component is known as a *shear stress*. It has units of kg m^{-1} s^{-2} which is equivalent to N m^{-2} or Pascals, i.e. the same units as pressure.

Near the surface $\overline{F_\mu}$ is usually negative (*why ?*), so that momentum is being transported from the atmosphere to the surface. This acts to decelerate the flow, and exerts a force on the surface in the direction of the mean wind.

10 K-Theory, and the Mixing Length

An eddy with a positive vertical velocity will have originated below the measurement height. If we assume it originates a distance l below the measurement height the fluctuation in the horizontal windspeed is,

$$u' = -l\frac{d\overline{U}}{dz} \tag{27}$$

similarly an eddy with a negative vertical velocity will have originated above the measurement height and be associated with a fluctuation in the horizontal windspeed of $u' = l d\overline{U}/dz$. The momentum flux becomes,

$$\overline{F_\mu} = -\rho\overline{|w'|}l\frac{d\overline{U}}{dz} = -\rho K_\mu \frac{d\overline{U}}{dz} \tag{28}$$

(taking the modulus of the fluctuations in vertical velocity accounts for the signs in the u' and w'. Note, that although the mean of the vertical velocity is zero, the mean of the absolute value isn't, and can be taken as a measure of the magnitude of a typical turbulent fluctuation in velocity).

Equation 28 is known as a *flux-gradient relationship*. It is analogous to the Eq. 7 for heat conduction.

- K_μ is known as the *eddy diffusivity* (units m^2 s^{-1}). However unlike the diffusivity in Eq. 7 which is a property of the material, the eddy diffusivity is a property of the turbulent flow (so there is no such thing as the eddy diffusivity for air).
- l is known as the *mixing length*. The *mixing length* is related to the *turbulent length scale*.
- $\overline{|w'|}$ is a measure of the strength of the turbulence. This is usually known as the *turbulent velocity scale*.
- We need to understand what determines the *mixing length* and *turbulent velocity scale*.

The vertical velocity must be zero at the surface, this means that,

$$\left|\frac{dw'}{dz}\right| \sim \frac{\overline{|w'|}}{z} \tag{29}$$

Also,

$$\left|\frac{du'}{dx}\right| \sim \frac{\overline{|w'|}}{L} \tag{30}$$

where it has been assumed that the magnitude of the fluctuations in the horizontal windspeed is similar to the *turbulent velocity scale* $\overline{|w'|}$. From the continuity equation $|du'/dx| \sim |dw'/dz|$ which means that,

$$L \propto z \tag{31}$$

i.e., the size of turbulent eddies increases with distance from the surface.

The mixing length is usually taken to be

$$l = \kappa z \tag{32}$$

where the proportionality constant κ is known as the von Karman constant, and has a value of about 0.4.

If there is no potential temperature gradient then conditions are termed *neutral* and the turbulence is referred to as *shear driven* or *mechanical.* Under these circumstances it turns out that,

$$\overline{|w'|} = \left(-\overline{w'u'}\right)^{1/2} = u_* \tag{33}$$

where u_* is known as the *friction velocity.*

Combining these results gives,

$$K_\mu = \kappa u_* z \tag{34}$$

and

$$\overline{w'u'} = -\kappa u_* z \frac{d\overline{U}}{dz} \tag{35}$$

As it stands Eq. 35 may not seem very useful, since in order to calculate the momentum flux using the eddy diffusivity it is necessary to know the momentum flux to calculate the eddy diffusivity, which means it is time to introduce :

11 The Logarithmic Wind Profile

From the definition of u_*, Eq. 35 can be rearranged to give,

$$\frac{d\overline{U}}{dz} = \frac{u_*}{\kappa z} \tag{36}$$

which can be integrated to give,

$$\overline{U}(z) = \frac{u_*}{\kappa}\left(\ln(z) - \ln(z_o)\right) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) \tag{37}$$

Notice that $\overline{U}(z_o) = 0$.

The height z_o is known as the *roughness length.* It depends on the nature of the surface. For smooth surfaces the *roughness length* is small, while for rough surfaces it is large.

12 K-Theory and Other Fluxes

The arguments given above can be applied to any atmospheric property Q, with volumetric concentration q to give,

$$\overline{F_q} = -K_q \frac{d\overline{q}}{dz} = -\kappa u_* z \frac{d\overline{q}}{dz} \tag{38}$$

This turns out to be only approximately true, and then only for *near-neutral conditions* ($d\theta/dz$ close to zero). The reason for the differences between K_μ and K_q is where the problem of turbulence becomes difficult (*it is related to an important difference between momentum and other quantities such as temperature and humidity*). Equation 38 suggests that within the constant flux layer the profile of $\overline{q}(z)$ should also satisfy a *logarithmic profile law.*

13 The Aerodynamic Method for determining fluxes

Equations 36 and 38 can be integrated between two heights, z_1 and z_2 in the constant flux layer to give,

$$\overline{U}(z_2) - \overline{U}(z_1) = \frac{u_*}{\kappa} \ln\left(\frac{z_2}{z_1}\right) \tag{39}$$

$$\overline{q}(z_2) - \overline{q}(z_1) = -\frac{\overline{F_q}}{\kappa u_*} \ln\left(\frac{z_2}{z_1}\right) \tag{40}$$

which can be re-arranged to give,

$$\overline{F_q} = -u_*^2 \times \frac{(\overline{q_2} - \overline{q_1})}{(\overline{U_2} - \overline{U_1})} = -\frac{(\overline{q_2} - \overline{q_1})}{r_a} \tag{41}$$

where $r_a = (\overline{U_2} - \overline{U_1})/u_*^2$ is known as the *aerodynamic resistance* of the layer. Notice the similarity between Eq. 41 and Ohm's law in electricity. The difference in the mean value of q at the two heights playing the role of the potential difference and the flux the role of the current.

For $z_1 = z_o$,

$$r_a = \frac{\overline{U}(z)}{u_*^2} = \frac{[\ln(z/z_o)]^2}{\kappa^2 \overline{U}(z)} \tag{42}$$

Equation 41 shows that we can estimate surface fluxes by making measurements at two heights within the constant flux layer. This is known as an *aerodynamic method* for estimating fluxes. So for example, the heat flux can be estimated by measuring the temperature and winds at two heights, so that

$$\overline{F_h} = H = -\rho c_p \frac{(\overline{T}_2 - \overline{T}_1)}{r_a} \tag{43}$$

14 So what if $d\theta/dz$ is not equal to zero.

In several places it has been assumed that $d\theta/dz$ is small or zero. Although it is possible to deal with situations where this is not so, how this is done is beyond the scope of this course. However, it is possible to get a qualitative idea of the effects of non-zero $d\theta/dz$ on the eddy diffusivity. We'll consider $d\theta/dz < 0$, which is usually the case during the day.

With $d\theta/dz < 0$, if a turbulent eddy is moving upwards past the measurement level, it will in general be warmer than average. This means that the eddy is positively buoyant and experiences and upward acceleration. The reverse is true for downward moving eddies. This suggests that the typical vertical velocity scale in the presence of a potential temperature gradient will be larger than if the atmosphere is neutral ($d\theta/dz = 0$). We might, therefore expect that,

$$\overline{|w'|} > u_* \tag{44}$$

You should be able to show from this that for a given value of u_*, when $d\theta/dz < 0$, the eddy diffusivity will be greater, the windshear smaller and the aerodynamic resistance smaller than when $d\theta/dz = 0$.

As an additional complication the approximation, $K_\mu \approx K_q$ becomes less accurate when $d\theta/dz < 0$, although the reason for this involves a more detailed understanding of turbulence. Generally $K_q > K_\mu$ in the daytime, convective boundary layer.

Although it is possible to account for these effects, in the analysis of the data collected in this course they will be ignored (which might affect the accuracy of your surface energy balance results).

15 The Bowen Ratio revisited.

As $d\theta/dz$ becomes more negative, taking the eddy diffusivities for heat and moisture to be the same as the diffusivity for momentum becomes increasingly inaccurate. However, it is still a good approximation to take the eddy diffusivities for heat and moisture to be the same.

The *aerodynamic method* (Eq. 41) applied to heat and moisture gives,

$$\overline{F_h} = -\rho c_p \frac{(\overline{T}_2 - \overline{T}_1)}{r_H} \tag{45}$$

$$\lambda\overline{F_\chi} = -\lambda \frac{\overline{\chi}_2 - \overline{\chi}_1}{r_\chi} \tag{46}$$

where r_H and r_χ are the aerodynamic resistances for heat and moisture.

Since $r_H \approx r_\chi$ is a reasonable approximation, the ratio of these two equations gives,

$$\frac{\overline{F_h}}{\lambda\overline{F_\chi}} = \frac{\rho c_p(\overline{T}_2 - \overline{T}_1)}{\lambda(\overline{\chi}_2 - \overline{\chi_1})} = \frac{H}{\lambda E} = B_0 \tag{47}$$

From Eq. 47 the *Bowen ratio*, B_0 can be estimated from measurements of temperature and humidity at two levels without having to know K_μ. Knowing the *Bowen ratio* and the *available energy* the sensible and latent heat fluxes can be estimated.