

A method for estimating the turbulent kinetic energy dissipation rate from a vertically-pointing Doppler lidar, and independent evaluation from balloon-borne in-situ measurements

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ABSTRACT

A method of estimating dissipation rates from a vertically-pointing Doppler lidar with high temporal and spatial resolution has been evaluated by comparison with independent measurements derived from a balloon-borne sonic anemometer. This method utilises the variance of the mean Doppler velocity from a number of sequential samples and requires an estimate of the horizontal wind speed. The noise contribution to the variance can be estimated from the observed signal-to-noise-ratio and removed where appropriate. The relative size of the noise variance to the observed variance provides a measure of the confidence in the retrieval. Comparison with in-situ dissipation rates derived from the balloon-borne sonic anemometer reveal that this particular Doppler lidar is capable of retrieving dissipation rates over a range of at least three orders of magnitude.

This method is most suitable for retrieval of dissipation rates within the convective well-mixed boundary-layer where the scales of motion that the Doppler lidar probes remain well within the inertial sub-range. Caution must be applied when estimating dissipation rates in more quiescent conditions. For the particular Doppler lidar described here, the selection of suitably short integration times will permit this method to be applicable in such situations but at the expense of accuracy in the Doppler velocity estimates, therefore optimisation of certain instrument parameters may be required for specific implementations.

1. Introduction

Turbulent properties of the boundary layer can be measured by aircraft (e.g. Fairall *et al.*, 1980) and vertical profiles of the turbulent kinetic energy dissipation rate have been obtained from balloon-borne turbulence probes in the convective boundary layer (Caughey and Palmer, 1979), nocturnal boundary layer (Caughey *et al.*, 1979), and cloudy boundary layers (Hignett, 1991; Siebert *et al.*, 2003). Such in-situ observations, however, are necessarily restricted both spatially and temporally. An active remote sensing approach is required to achieve routine, continuous coverage with simultaneous measurement at all altitudes across a significant part of the lower atmosphere, including the full depth of the boundary layer. Doppler radars and lidars can provide the necessary high resolution velocities and there are a number of methods currently available for estimating the dissipation rate. These broadly fall into 3 categories; Doppler spectral width, temporal spectra or structure function methods, and conical scanning. An example of a method from each of these categories, as applied

to Doppler lidar, is given by Banakh *et al.* (1999). The methods in these categories may be applicable to both Doppler radar (Brewster and Zrnić, 1986; Cohn, 1995; Doviak and Zrnić, 1993; Chapman and Browning, 2001) and Doppler lidar (Gal-Chen *et al.*, 1992; Banakh and Smalikho, 1997; Davies *et al.*, 2004). Other possibilities include dual-Doppler lidar (Davies *et al.*, 2005) and radars sensitive to clear-air echoes, which can use the returns arising from turbulent mixing across atmospheric refractive index gradients to estimate dissipation rates (Cohn, 1995).

Evaluation of the various methods for estimating dissipation rate from a Doppler lidar or radar is usually performed by comparison with ground-based or tower-based sonic anemometers and sodars (Drobinski *et al.*, 2004; Calhoun *et al.*, 2005; Sjöholm *et al.*, 2008). Comparisons with other instruments have also been carried out, such as a lightweight three-dimensional magnetometer carried on a radiosonde (Harrison *et al.*, 2009).

The method of estimating the dissipation rate, ϵ , from the Doppler spectral width assumes that turbulence is entirely responsible for the spectral broadening; in practice there are additional sources of spectral broadening that must be accounted for (Hocking, 1985; Doviak and Zrnić, 1993), such as wind-shear.

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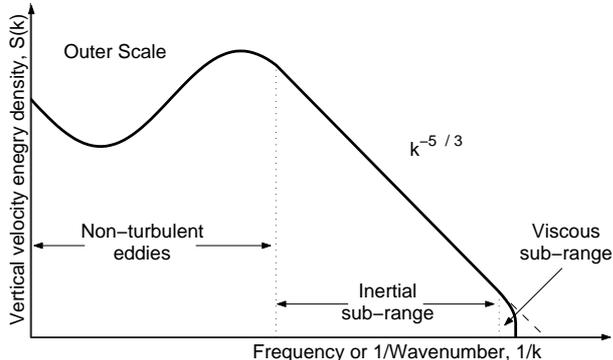


FIG. 1: Schematic of vertical velocity energy density spectra versus 1/wavenumber conforming to Kolmogorov’s hypothesis.

Not all Doppler lidars provide the full Doppler spectrum, so in this paper we utilise a method which requires only the mean Doppler velocity. We make the assumption that the variations of the mean Doppler velocity over a short sampling time are entirely due to turbulence. Using the variance of a number of samples of the mean Doppler velocity sidesteps most of the issues involved in correcting the various sources of additional spectral broadening associated with the Doppler spectral width method, but, since a longer integration time is required, care must be taken that the scales of turbulent motion now encompassed still remain within the inertial sub-range (Frehlich and Cornman, 2002).

In this paper we outline a simple method for estimating dissipation rate from unattended continuously operating Doppler lidars. In section 2 we present the method for estimating dissipation rate from the variance of the mean Doppler velocity, with corrections for the expected uncertainty in the observed Doppler velocities for a heterodyne system. An error analysis is given in section 3 and validation of the method by comparison with balloon-borne in situ data is presented in section 4.

2. Estimating dissipation rate

a. Standard method from velocity power spectra

Standard methods for estimating dissipation rate from high-frequency measurements of turbulent velocities typically involve the transformation of the velocity spectra into the frequency domain (e.g. by the use of Fast Fourier Transforms, FFTs). In theory, with sufficient resolution, these vertical velocity spectra are presumed to have a form similar to that shown in 1, when plotted versus frequency (or 1/wavenumber). Production of turbulent kinetic energy is dominated by large eddies (length scales greater than 1 km), which then decay into smaller and smaller eddies (the inertial sub-range) until the length scales

are small enough for the kinetic energy to be dissipated into heat by molecular diffusion in the viscous sub-range (scales on the order of centimeters or less).

If it is assumed that turbulence is a homogeneous and isotropic process of energy dissipation then the Kolmogorov hypothesis (1941) states that within the inertial sub-range the statistical representation of the turbulent energy spectrum $S(k)$ is given by

$$S(k) = a\epsilon^{2/3}k^{-5/3}, \quad (1)$$

where $a = 0.55$ is the Kolmogorov constant for 1-dimensional wind spectra (Paquin and Pond, 1971), ϵ is the dissipation rate and k is the wavenumber, which can be related to a length scale, L ($k = 2\pi/L$) by invoking Taylor’s hypothesis of frozen turbulence (Taylor, 1935). If observed spectra fit the form shown in figure 1, a $-5/3$ power-law can be fitted to the portion of the spectrum that lies within the inertial sub-range and thus, ϵ can be estimated.

b. Variance of mean Doppler velocity

We now introduce a new parameter σ_v^2 , which is the variance of the observed mean Doppler velocity over a defined number of sequential samples, N (O’Connor *et al.*, 2005). Initially, we consider the case where the observed variance is dominated by the turbulent processes in the vertical and there are no significant contributions from other sources. The velocity variance is then equivalent (Bouniol *et al.*, 2003) to integrating (1) so that

$$\sigma_v^2 = \int_k^{k_1} S(k) dk, \quad (2)$$

$$= -\frac{3}{2}a\epsilon^{2/3} (k_1^{-2/3} - k^{-2/3}), \quad (3)$$

$$= \frac{3a}{2} \left(\frac{\epsilon}{2\pi}\right)^{2/3} (L^{2/3} - L_1^{2/3}), \quad (4)$$

where the wavenumber $k_1 = 2\pi/L_1$ corresponds to the length scale describing the scattering volume dimension for the dwell time of the lidar for a single sample, and $k = 2\pi/L$ relates to the length scale of the large eddies travelling through the lidar beam during the N sampling intervals.

The length scale is given by

$$L = Ut + 2z \sin\left(\frac{\theta}{2}\right), \quad (5)$$

where θ is the half-angle divergence of the lidar beam, U is the horizontal wind, t is the observation time and z is the height in m. Usually, the second term in (5) is negligible as Doppler lidars typically have a very small divergence, < 0.1 mrad. If the lidar instrument is set to acquire one profile of velocity measurements

every 4 seconds, the length scales for a typical wind speed of $U = 10 \text{ m s}^{-1}$ in the boundary layer are $L_1 = 40 \text{ m}$ and, if 10 samples are used to calculate $\sigma_{\bar{v}}^2$, $L = 400 \text{ m}$. The length scales for horizontal wind speeds as low as 0.25 m s^{-1} , with an integration time of 4 seconds or greater, should still be much larger than the expected cut-off in the viscous sub-range. We can now write

$$\epsilon = 2\pi \left(\frac{2}{3a} \right)^{3/2} \sigma_{\bar{v}}^3 (L^{2/3} - L_1^{2/3})^{-3/2} \quad (6)$$

and hence estimate ϵ directly from $\sigma_{\bar{v}}^2$ without the need to calculate FFTs.

c. Noise contribution to variance

So far we have assumed that turbulence is the only source of variance. We now consider the influence of noise on the Doppler velocity measurement. The error in an individual Doppler lidar velocity estimate is dependent on the signal to noise ratio (SNR), of the measurement. For a heterodyne Doppler lidar, Pearson *et al.* (2009) have shown that when many pulses have been averaged, the theoretical standard deviation of the Doppler velocity estimate, σ_e , for weak signals can be reliably approximated by (Rye and Hardesty, 1993)

$$\sigma_e = \left(\frac{\Delta v^2 \sqrt{2}}{\alpha N_p} (1 + 1.6\alpha + 0.4\alpha^2) \right)^{1/2}, \quad (7)$$

$$\alpha = \frac{\text{SNR}}{(2\pi)^{1/2} (\Delta v/B)}, \quad (8)$$

where α is the ratio of the lidar detector photon count to the speckle count (Rye, 1979), Δv is the signal spectral width and B is the receiver bandwidth (both expressed here in m s^{-1}), and N_p is the accumulated photon count. Both N_p and α are determined from the instrument characteristics and the SNR of the target return for a single point sample:

$$N_p = \text{SNR } n M, \quad (9)$$

where n is the number of pulses averaged per profile and M is the number of points sampled within a specified range gate to obtain a raw velocity. Note that, due to oversampling and subsequent averaging, the final range gate length does not necessarily coincide with the pulse length.

For a direct detection system, the theoretical minimum for the standard deviation of the Doppler velocity estimate is given by $\sigma_e = \Delta v / (N_p^{0.5})$, although in practice there are additional factors to consider (McKay, 1998).

Data from two coherent heterodyne Doppler lidars are presented here. The instruments are very

TABLE 1: Doppler lidar specifications. The instrument deployed during the 2nd REPARTEE experiment in London is essentially the same as the one deployed at Chilbolton, but with certain parameters adjusted to maximise the measurement capabilities within the boundary layer. Where particular parameters differ, the REPARTEE instrument parameters are given in brackets. Both instruments were built by Halo Photonics.

Wavelength	1.5 μm
Pulse repetition rate	15 kHz (20 kHz)
Nyquist velocity	10 m s^{-1} (14 m s^{-1})
Sampling frequency	50 MHz (30 MHz)
Points per range gate	12 (6)
Pulses averaged	20000
Range resolution	36 m (30 m)
Integration time	30 s (4 s)
Pulse duration	0.2 μs
Lens diameter	6 cm (8 cm)
Divergence	33 μrad
Focus	∞ (801 m)
Antenna	monostatic optic-fibre coupled

similar in design but have had certain parameters optimised for different objectives. The instrument at Chilbolton has a longer integration time to improve sensitivity as it is configured for a primary function of observing liquid and ice cloud at all heights up to 10 km. The instrument deployed for the 2nd REPARTEE campaign in central London (Martin *et al.*, 2009) is optimised for boundary-layer studies and has achieved the required sensitivity with a shorter integration time by having the telescope focus set to approximately 800 m (note that this reduces the instrument sensitivity dramatically above 2 km). The specifications of the two Doppler lidar instruments are given in Table 1. The pulse length for both instruments is the same, 30 m, but the signals are oversampled (by a factor of 10 for the instrument at Chilbolton and by 6 for the REPARTEE instrument). These high-resolution samples, or points, are then averaged up to yield the raw velocity estimates at the selected range gate length; the number averaged is given by the number of points per range gate parameter in Table 1. The acquisition time for a single profile obtained from 20,000 pulses is dependent on the pulse repetition rate of the instrument and additional time is then required for real-time computation of the velocities; Chilbolton requires approximately 1.33 s for acquisition and 4 s for computation per profile, REPARTEE requires 1 s for acquisition plus 3 s for computation per profile. The Chilbolton instrument then performs additional averaging of 5 profiles to give a total integration time of about 30 s.

We first consider the REPARTEE instrument as

it is potentially more suitable for estimating dissipation rate because of its much shorter integration time. The theoretical standard deviation of the Doppler

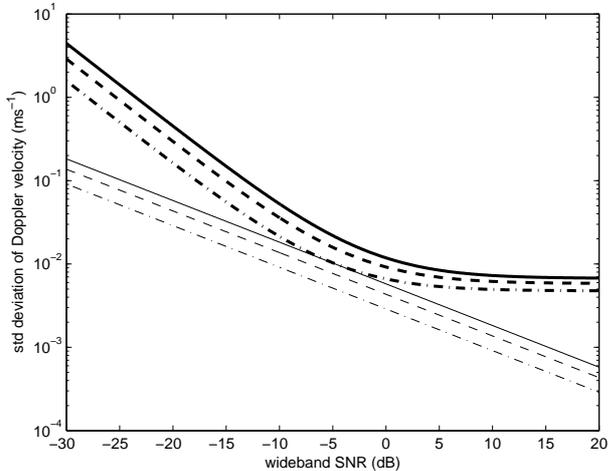


FIG. 2: Theoretical standard deviation of Doppler velocity estimate for the REPARTEE heterodyne Doppler lidar (thick lines) for three signal spectral widths, equivalent to 1 (dot-dashed), 1.5 (dashed) and 2 (solid) m s^{-1} . Also shown are the theoretical standard deviation of Doppler velocity estimates for direct detection lidar systems (thin lines). The wideband SNR refers to the SNR of the target return for a single pulse.

velocity estimate as a function of wideband SNR is given in figure 2 for the REPARTEE instrument. It is immediately obvious that the relationship between σ_e and wideband SNR for a heterodyne system is not the same as that for a direct detection system. In fact, at -25 dB, σ_e is an order of magnitude higher. It is also apparent that, for a heterodyne system, once the wideband SNR has reached 0 dB, increasing the SNR further does not improve σ_e . The choice of signal spectral width, Δv , also has some influence on estimating σ_e . Here, we follow Pearson *et al.* (2009) and select the value of 1.5 m s^{-1} as a typical value for the rest of the paper.

We first investigate observed vertical-velocity energy density spectra to confirm that they have the same shape as the idealised form given in Fig. 1, to examine the noise contribution, and to note whether the spectra contain a sufficient portion within the inertial sub-range for (6) to be valid. Data from the REPARTEE instrument has been selected because the shorter integration time allows the spectra to encompass smaller scales. Figures 3 - 5 display vertical-velocity energy density spectra at three different heights (90, 630 and 1170 m) calculated from 60 minutes of data over 3 adjacent gates (approximately 2700 individual velocity estimates). The targets in all cases are aerosol particles in the boundary layer. If velocity measurements and their random estimation error are uncorrelated, then Frehlich

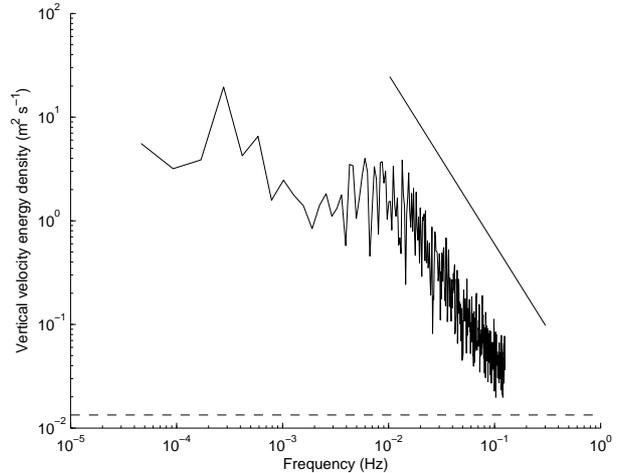


FIG. 3: Power spectra of 60 minutes of data with a time resolution 4 s, centred on 0800 UTC, 29 October 2007, at a height of 90 m (taken from 3 adjacent range gates at 60-120 m). Also shown is the expected noise contribution (dashed line) calculated from the mean SNR of -10 dB, and a reference $-5/3$ power law (solid line). Data is from the REPARTEE instrument.

(2001) states that the temporally uncorrelated estimation error will appear as a constant-amplitude high-frequency region in the velocity spectra. The noise contribution is computed as the ensemble mean, $\langle \sigma_e \rangle$, of the individual estimates of σ_e calculated for each individual measurement using (7). In terms of vertical-velocity energy density spectra, the levels of the theoretical noise contribution $\langle \sigma_e \rangle^2$ are then scaled by 2π to obtain the noise variance displayed as dashed lines in figures 3 - 5.

The vertical-velocity energy density spectrum in figure 3 was obtained from data with a mean SNR close to -10 dB and there is no indication of noise at the high end of the frequency spectrum, which is consistent with a very low theoretical noise level of $0.016 \text{ m}^2 \text{ s}^{-1}$ (in terms of wavenumber). This spectrum has the same form as the idealised version given in Fig. 1 and a $-5/3$ power-law can be fitted to the high frequency end of the spectrum (from approximately 1×10^{-2} to 1.5×10^{-1} Hz). The horizontal wind speeds given by the Doppler lidar in scanning mode, and the Met Office North Atlantic and European (NAE) operational numerical weather prediction model, were both close to 2 m s^{-1} at 90 m and 0800 UTC. This implies that the transition from the inertial sub-range to the outer scale for non-turbulent eddies occurs at length scales of 200 m.

The energy density spectrum in figure 4 was obtained from data with a mean SNR of -15 dB, for which the theoretical noise level is $0.12 \text{ m}^2 \text{ s}^{-1}$. This agrees very well with the observed spectrum, which abruptly flattens out above 10^{-2} Hz, and is consis-

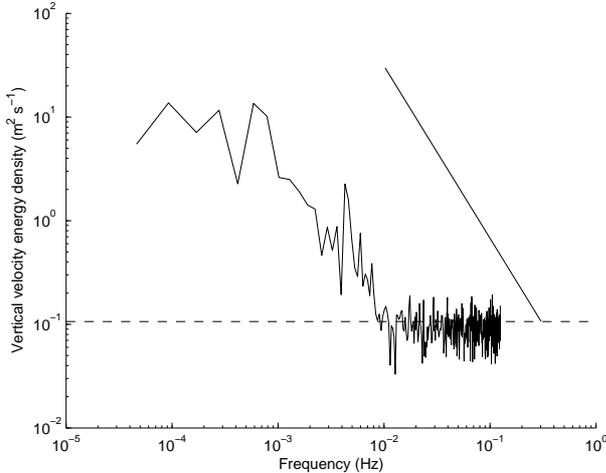


FIG. 4: Same as Figure 3 except at a height of 630 m. The mean SNR is -15 dB.

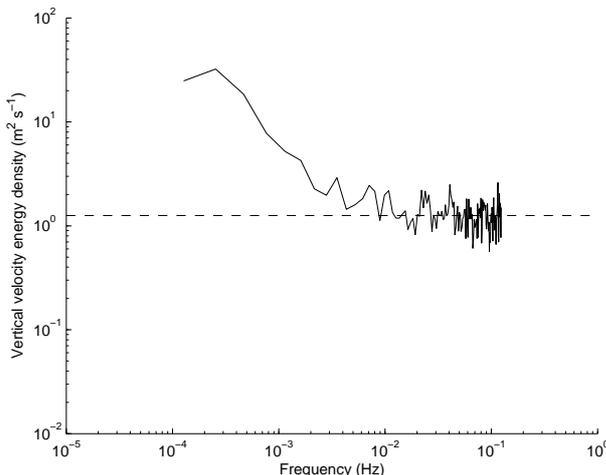


FIG. 5: Same as Figure 3 except at a height of 1170 m. The mean SNR is -20 dB.

tent with the explanation given by Frehlich (2001). A $-5/3$ power-law can still be fitted to the portion of the spectrum from 4×10^{-3} to 1×10^{-2} Hz). The horizontal wind speed at 630 m is about 5 m s^{-1} and so the transition from inertial sub-range to outer-scale is at length scales of about 1250 m.

In figure 5, the energy density spectrum was derived from data with a mean SNR of about -20 dB. The noise dominates the spectrum above 10^{-3} Hz for this case, and is consistent with a theoretical noise level of $1.3 \text{ m}^2 \text{ s}^{-1}$. Again, the observed spectrum above 10^{-3} Hz has a constant amplitude characteristic of temporally uncorrelated estimation noise. It is not reasonable to attempt to fit a $-5/3$ power-law to any portion of this particular spectrum.

If it is assumed that the sources of variance have a Gaussian distribution and are independent of one

another, the observed variance, $\sigma_{\bar{v}}^2$, is the sum of the variances from each source (Doviak and Zrnić, 1993; Frehlich *et al.*, 1998) such that

$$\sigma_{\bar{v}}^2 = \sigma_{\bar{w}}^2 + \sigma_e^2 + \sigma_d^2, \quad (10)$$

where $\sigma_{\bar{w}}^2$ is the contribution from air turbulence that we are interested in, the contribution from noise is $\sigma_e^2 \approx \langle \sigma_e \rangle^2$, and σ_d^2 is the contribution from the variation in still-air terminal fall speeds of particulates within the measurement volume from one sample to the next. Aerosol and liquid cloud droplets have negligible terminal fall speeds and σ_d^2 can be safely ignored for returns from these targets. Figures 3 - 5 show that the variance arising from the uncertainty in the Doppler velocity measurements can be estimated reliably and that it is valid to assume that the two sources of variance, turbulence and estimator noise, are independent. Thus, given an observed total variance and a calculated noise variance, ϵ can be derived using (6) by replacing the theoretical $\sigma_{\bar{v}}^2$ with $\sigma_{\bar{w}}^2 = \sigma_{\bar{v}}^2 - \sigma_e^2$. Targets such as rain or ice particles will have significant terminal fall speeds; therefore it may be necessary to quantify σ_d^2 when attempting to calculate ϵ in such situations.

3. Error in derived dissipation rates

To estimate the error in ϵ we first assume that $L_1 \ll L$ in (6), so that $\epsilon \propto \sigma_{\bar{w}}^3/L$, and through the propagation of errors, the fractional error in ϵ is

$$\frac{\Delta \epsilon}{\epsilon} = \frac{3\Delta \sigma_{\bar{w}}}{\sigma_{\bar{w}}} + \frac{\Delta L}{L}. \quad (11)$$

Radiosonde, tower measurements or wind profilers can be used to estimate the horizontal length scale, as can the output from an operational forecast model. In this study, we use model winds from an operational forecast model to derive L and, since the horizontal winds from the Met Office mesoscale model are generally accurate to $1\text{-}2 \text{ m s}^{-1}$ (Panagi *et al.*, 2001), we estimate the fractional error in L to be about 10% for a typical horizontal wind speed of 10 m s^{-1} . The extremely small divergence of the Doppler lidar in this study (0.033 mrad) means that the second term in (5) can be ignored even at very low U and short observation times.

Following Lenschow *et al.* (2000) we can estimate the measurement error in a variance as follows,

$$\Delta \sigma_{\bar{w}}^2 \approx \sigma_{\bar{w}}^2 \sqrt{\frac{4}{N} \frac{\sigma_e^2}{\sigma_{\bar{w}}^2}}, \quad (12)$$

and therefore provide the fractional error for each individual estimate of ϵ . It should be noted that (12) assumes that each velocity sample used to calculate the variance has a similar error to the ensemble mean

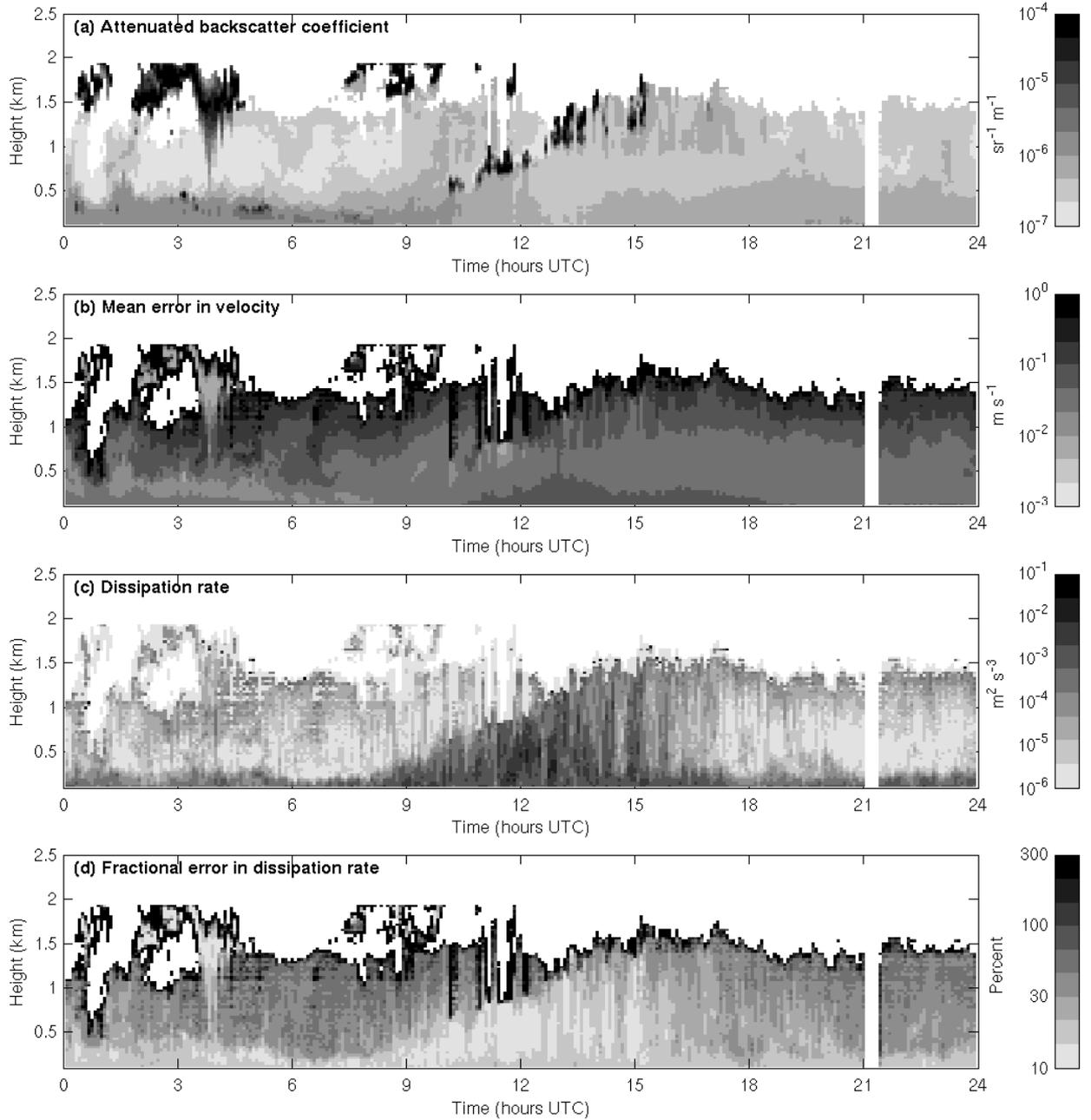


FIG. 6: Top panel (a); attenuated backscatter coefficient from the Doppler lidar during the REPARTEE campaign in London on 29 October 2007. Panel (b) displays the ensemble mean of the theoretical uncertainty in observed velocities for each sample used to derive (c) the dissipation rate. The estimated fractional error in dissipation rate is given in panel (d).

error, $\sigma_e \approx \langle \sigma_e \rangle$. This assumption can be tested and those variances for which this is no longer approximately true should be flagged as unreliable.

Estimates of ϵ derived from the REPARTEE Doppler lidar data are shown in Fig. 6, along with the derived $\langle \sigma_e \rangle$ and fractional error in dissipation rate, for the same day as in Figs. 3 - 5. The REPARTEE instrument has a maximum range of about 2 km and, as Fig. 6a indicates, is sufficiently sensitive to

detect aerosol (or clouds) at almost all ranges, potentially providing an estimate of dissipation rate throughout most of the lower atmosphere. The convective boundary layer is clearly visible in Fig. 6c from 0900 to 1600 UTC where ϵ is high, in this case $> 10^{-4} \text{ m}^2 \text{ s}^{-3}$, and the convective boundary layer top reaches 1.5 km at 1400 UTC. Maximum values of ϵ approach $5 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}$.

Within the convective boundary layer, the limit-

ing factor in providing accurate estimates of ϵ is the uncertainty in the horizontal winds used to estimate the length scales. Outside this region however, uncertainty in the velocity variance estimates is much more likely to be the dominant source of error; a smaller velocity variance due to less turbulent conditions is compounded by more uncertainty in the variance estimate due to low SNR. Figures 6b and 6d corroborate this. Within the convective boundary layer, where ϵ is high, the fractional error in ϵ is estimated to be as low as 10-15%. For quiescent conditions in early morning or late evening with similar mean errors in velocity, the derived fractional error in ϵ is closer to 30-45%. Fractional errors $> 100\%$ indicate where noise is the dominant source of variance and provide a quality flag for removal of unreliable ϵ estimates. Such values are found, for example, for locations within the time-height period (0730-0830 UTC and 1125-1215 m) used for generating Fig. 5, whose noisy spectrum does not show any sign of an inertial sub-range.

In providing these error estimates, we have also implicitly assumed that the sampling time is sufficiently short to ensure that we remain in the inertial sub-range, where (1) applies. If the sampling time is too long then the observed velocity variance will include contributions from the outer scales of turbulence, where (1) no longer applies, and ϵ will be underestimated. For this case, the horizontal winds used to derive ϵ were taken from the Met Office NAE model and ranged from 2-6 m s⁻¹. With $N = 10$ consecutive 4-second samples used to calculate σ_v^2 , this corresponds to length scales for L of 80-240 m, which, according to Figs. 3 and 4, suggests that the assumption is reasonable at ranges close to the surface and is valid at greater ranges. The length scales for L_1 , 8-24 m, are substantially greater than the transition from the inertial sub-range to the viscous sub-range. At high levels of turbulence within the well-mixed boundary-layer the sampling time of this Doppler lidar is sufficiently fast to acquire enough samples while remaining in the inertial sub-range, but this may no longer be true in very quiescent conditions above the well-mixed boundary-layer. In these cases the value of ϵ can be severely underestimated.

4. Balloon-borne in-situ evaluation

We now present estimates of the dissipation rate for data taken from the Doppler lidar at Chilbolton on 22 April 2008. Appropriate parameters for this day are given in Fig. 7. Low cloud or fog at 300 m in height is present from 0400 to 0800 UTC and completely attenuates the lidar signal (Fig. 7a) but otherwise there is potential coverage throughout most of the lowest 2 km. A convective boundary layer is again

evident in Fig. 7c, from about 1000 to 1600 UTC, which grows to over 1.5 km with ϵ values reaching $5 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$. Similar to the REPARTÉE case, the horizontal winds taken from the Met Office NAE model again ranged from 2-6 m s⁻¹, but, since the Chilbolton instrument has a longer integration time to improve the sensitivity, with $N = 10$ consecutive 30-second samples used to calculate σ_v^2 , the length scales for L are somewhat larger, at 600-1800 m.

Since these length scales may now incorporate unwanted contributions from the outer scale as well as the inertial sub-range, especially when close to the surface, we also performed calculations using $N = 12$ samples taken from 4 consecutive rays in time and from 3 adjacent gates in height. The length scales for L using this approach are potentially more applicable, at 240-720 m, although again probably too large close to the surface. The absolute values of ϵ are not exactly the same as those in Fig. 7c but the pertinent features of such a figure are very similar and so not included here.

During April 2008 the University of Leeds UFAM SkyDoc balloon was flown at Chilbolton in close proximity to the Doppler lidar carrying a turbulence sonde and mean meteorology instrumentation. The turbulence sonde is a prototype instrument incorporating the sensing head and control electronics of a Gill Windmaster 3-axis sonic anemometer in a compact aerofoil housing. The sonic anemometer measures the 3 components of the turbulent wind at 40 Hz and internally averages the measurements to 10 Hz before outputting the data via a serial interface. The data stream was logged via a compact embedded linux computer installed in the housing along with a 12V battery pack. A separate enclosure housed a mean meteorology package to measure air pressure, mean temperature and relative humidity, and mean wind speed; a compact aerosol probe (CLASP, Hill et al. 2008) was also included to measure aerosol size spectra for a related study.

For calculation of the dissipation rate we select the portion of the power spectrum at frequencies greater than 2 Hz; this limit is chosen to avoid that part of the spectra contaminated by the motion of the tether-sonde; this spans a frequency range of approximately 0.08-0.2 Hz.

The flight track of the balloon is superimposed on the plots in Fig. 7 and the weight of the turbulence sonde limited the maximum height to about 600 m. Lidar estimates of ϵ closest to the location of the balloon in height and time were selected for comparison. The balloon is obviously not co-located exactly with the lidar beam, and depending on wind conditions, may be as much as 400 m away in the horizontal. We consciously used model forecast horizontal winds

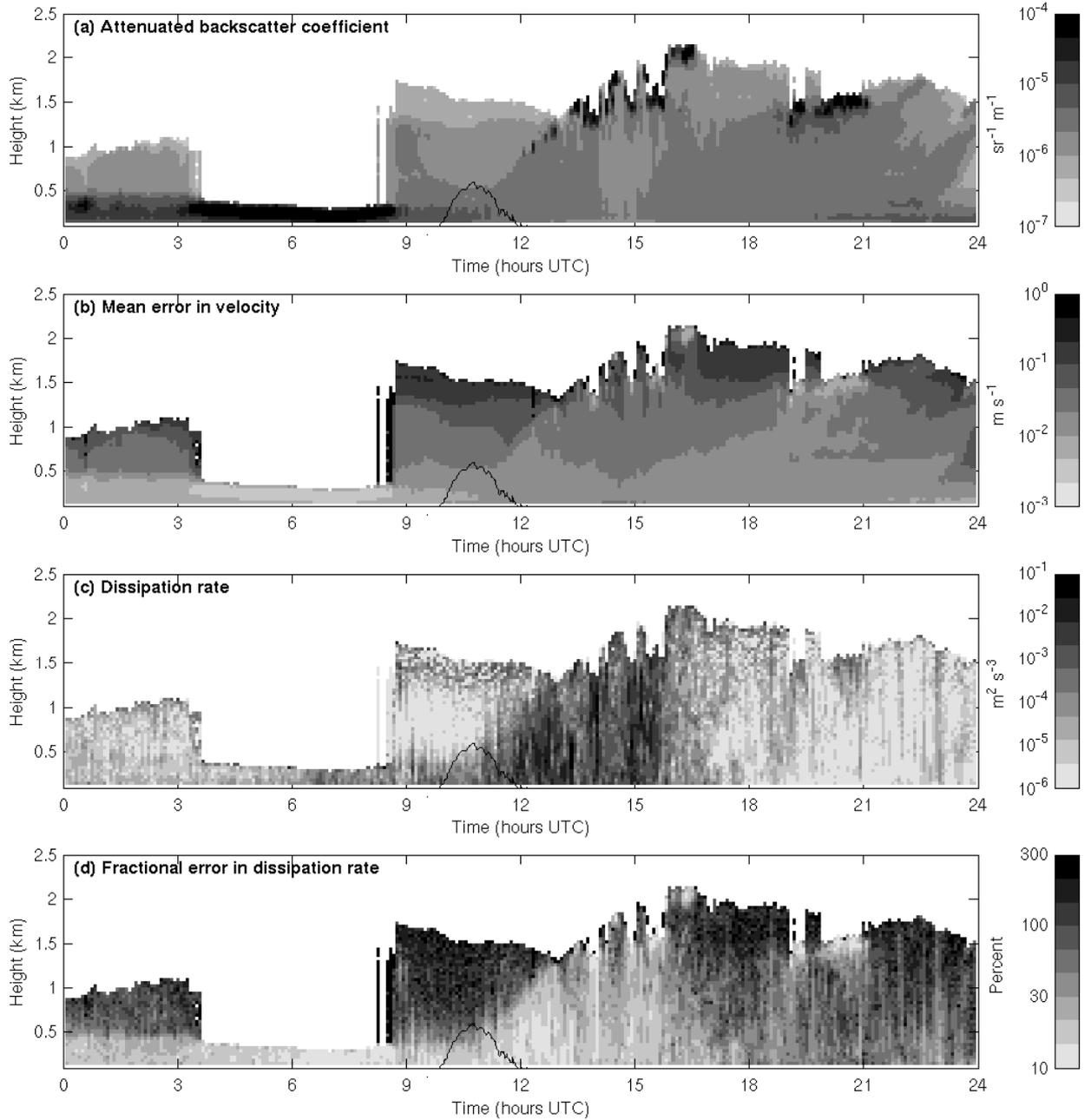


FIG. 7: Top panel (a); attenuated backscatter coefficient from the Doppler lidar at Chilbolton for 22 April 2008. Panel (b) displays the ensemble mean of the theoretical uncertainty in observed velocities for each sample used to derive (c) the dissipation rate. The estimated fractional error in dissipation rate is given in panel (d). The black line in each panel (from about 1000 to 1200 UTC) denotes the balloon flight track.

to estimate the Doppler lidar ϵ values, whereas the balloon values were calculated from in-situ measurements of the horizontal wind, so that we could examine how the lidar technique would perform in an operational context.

A comparison of the in-situ measurements of ϵ with those inferred from the Doppler lidar shows good general agreement both in time, Fig. 8, and in height,

Fig. 9. The balloon observes a significant decrease in ϵ with height of over 3 orders of magnitude; the lidar method captures this decrease, and is also able to cover the wide variation in magnitude. For values close to the ground, between 100 and 250 m, better agreement is found between the balloon and lidar estimates derived from 4 consecutive samples and 3 adjacent gates. Concerns noted earlier about

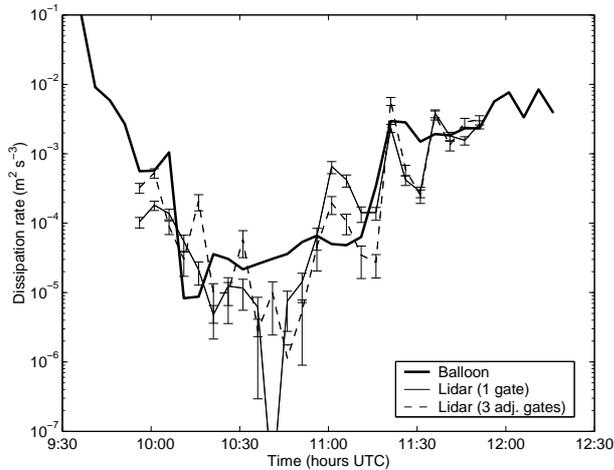


FIG. 8: Observed rate of dissipation of turbulent kinetic energy, ϵ , in the boundary layer from the in-situ balloon measurements, and the lidar on 22 April 2008.

the length scales involved near the surface being too large when 10 consecutive samples at one gate are used to derive the lidar estimates are most likely responsible for this discrepancy. The values of lidar ϵ below $10^{-6} \text{ m}^2 \text{ s}^{-3}$ are those which display the greatest disagreement with the balloon measurements and are again due to the limitations in using 10 consecutive samples at one gate. As shown in Fig. 9, this occurs at a height of almost 600 m, which, at 10.40 UTC, is not yet encompassed by the growing convective boundary layer. Two factors limit the ability of the lidar to estimate ϵ in this particular case; not only is the lidar SNR low above the convective boundary layer, the length scales over which the velocity variance is calculated are again unlikely to be wholly contained within the inertial sub-range. Using 4 consecutive samples and 3 adjacent gates to estimate ϵ at this height does show considerable improvement but it is still not certain that the shorter length scales involved will remain within the inertial sub-range outside the convective boundary layer.

One method of diagnosing whether the length scales probed by the lidar only encompass the inertial sub-range is to vary the number of samples, N , used to calculate ϵ . If the derived values of ϵ are no longer consistent with each other then the probability is high that the observed variance, $\sigma_{\bar{v}}^2$, contains contributions from the outer scales of turbulence.

5. Conclusion

We have demonstrated the potential for estimating ϵ from a Doppler lidar by using the standard deviation of successive samples of the mean Doppler velocity. We have shown that the noise contribution to the velocity variance can be estimated reliably and

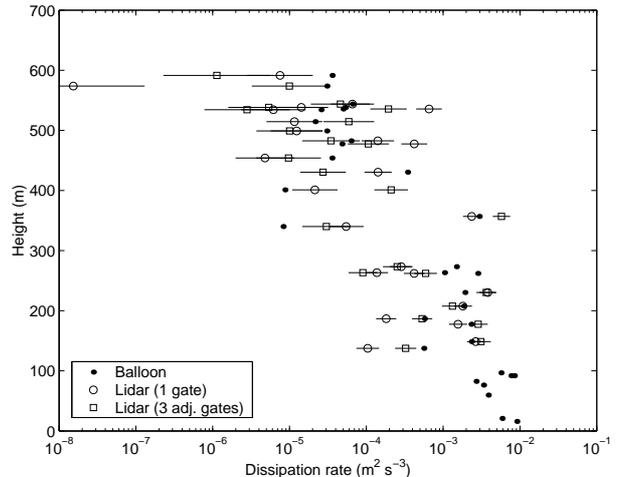


FIG. 9: Observed rate of dissipation of turbulent kinetic energy, ϵ , in the boundary layer from the in-situ balloon measurements, and the lidar on 22 April 2008.

that there is sufficient SNR throughout most of the well-mixed boundary layer for good Doppler observations. The range of values found for ϵ agree well with the wide range of dissipation rates measured by Chen (1975); Siebert *et al.* (2005). The agreement with the in-situ balloon-borne measurements is very encouraging; however, it should be noted that this comparison was mainly performed within the well-mixed boundary-layer, where the lidar signals are generally strong and the scales of motion contained within the inertial sub-range are large enough to encompass the entire N sampling times required to derive $\sigma_{\bar{v}}$.

In principle the retrieval of ϵ in low SNR conditions can be improved by discarding the individual Doppler velocity measurements with large errors before computing $\sigma_{\bar{v}}$, as discussed by Frehlich (2001). For low-power Doppler lidars, which require averaging of many pulses to achieve a reasonable sensitivity, there are a limited number of individual samples available within the required timeframe for keeping length scales within the inertial sub-range and it is highly likely that removal of the noisy samples will bias the calculation of ϵ . A threshold on the relative frequency of noisy samples within a variance measurement provides a simple quality flag for the ϵ estimates.

It is clear that a shorter integration time is preferable for ensuring that the length scales probed are always within the inertial sub-range to ensure that (1) is applicable. The boundary between the outer scale and the inertial sub-range may well lie at much smaller scales in some regions of the atmosphere (and in some types of boundary layers). However, for the particular instruments considered here, there is a tradeoff between the sensitivity of the instrument

and the applicability of the method. This method can still be applied where longer integration times have been used to improve the instrument sensitivity but there will be more situations when the length scales are no longer within the inertial sub range. From the measurements discussed here it seems that an instrument temporal resolution of 4 s, resulting in a $\sigma_{\bar{v}}$ estimate over < 60 s, is sufficient to remain within the inertial sub-range throughout the boundary layer (providing the signal strength is high), whereas an instrument with a temporal resolution of 30 s may be limited to convective boundary layers. With 30-second samples there is potential for reducing length scales by taking samples from adjacent gates so that the number of consecutive samples in time can be reduced accordingly.

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