Theoretical Aspects of Variational Data Assimilation

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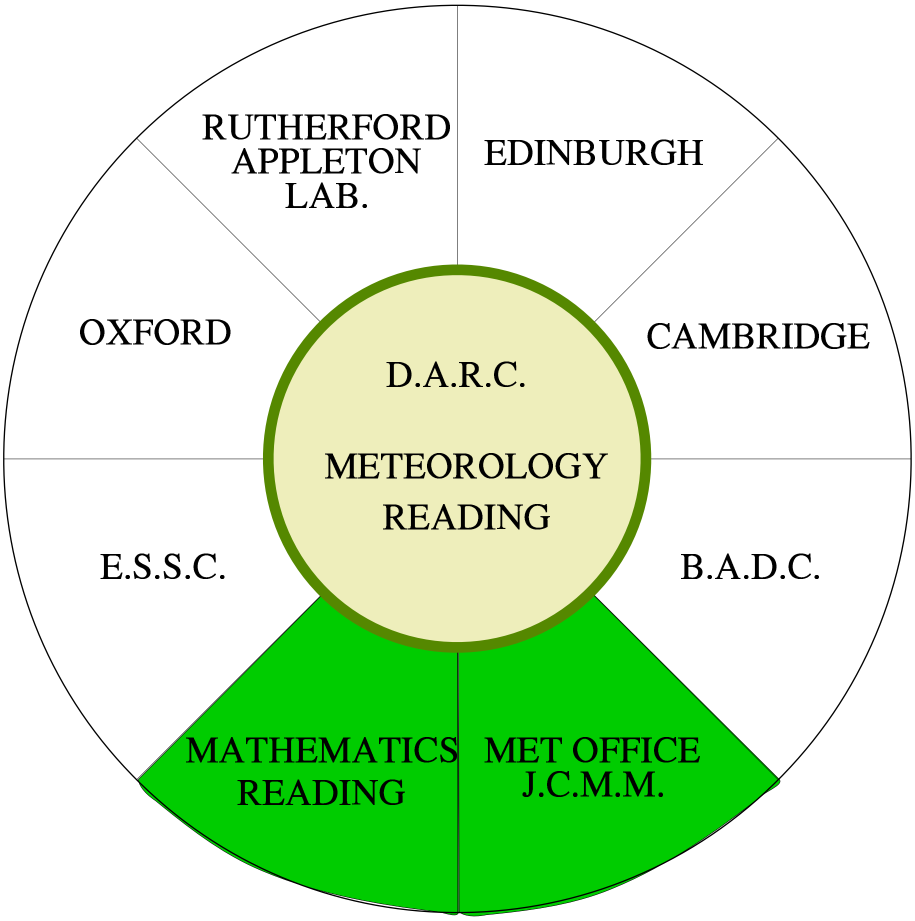

- Forward modelling of satellite radiances.
- Chemical data assimilation.
- Earth observation.
- Constructing research datasets.
- Theory.
- Training.
- &c &c &c

VARIABLE TRANSFORMS

The cost function,

$$J(\vec{x}) = J_b(\vec{x}) + J_o(\vec{x})$$

'background' term
(fit to previous forecast)

'observation' term
(fit to new observations)

$$J_b(\vec{x}) = \frac{1}{2}(\vec{x}_b - \vec{x})^T \mathbf{B}^{-1}(\vec{x}_b - \vec{x})$$

$$J_o(\vec{x}) = \frac{1}{2}(\vec{y}_o - \vec{H}(\vec{x}))^T (\mathbf{E} + \mathbf{F})^{-1}(\vec{y}_o - \vec{H}(\vec{x}))$$

Need the analysed state.
This is the $\vec{x}$ which minimizes J.

The background error correlation matrix, **B** is important:

- It specifies the spacial impact of observations.
- It is multivariate.
- It allows different quantities to be compared.
- It gives the accuracy of the background state.
- It provides the relative weight of background errors.

... but it is too large to construct, store and use (typically 10^{12} elements!). (We also don't know the 'truth', $\vec{x}_t$)

$$\mathbf{B} = (\vec{x} - \vec{x}_t)(\vec{x} - \vec{x}_t)^T$$

Use theory to help 'compress' the information using the *parameter transforms*, T_p, U_p, U_p^T.

Current Scheme

Met. vars. → Parameters

$$\begin{pmatrix} \vec{u} \\ \vec{v} \\ \vec{\theta} \\ \vec{p} \\ \vec{q} \end{pmatrix} \rightarrow \begin{pmatrix} \vec{\psi} \\ \vec{\chi} \\ {}^A p \\ \mu \end{pmatrix}$$

	ψ	χ	${}^A p$	μ
ψ	$\mathbf{B}_{\psi\psi}$			
χ		$\mathbf{B}_{\chi\chi}$		
${}^A p$			$\mathbf{B}_{{}^A p {}^A p}$	
μ				$\mathbf{B}_{\mu\mu}$

Proposed Scheme

Met. vars. → New parameters

$$\begin{pmatrix} \vec{u} \\ \vec{v} \\ \vec{\theta} \\ \vec{p} \\ \vec{q} \end{pmatrix} \rightarrow \begin{pmatrix} PV \\ \overline{PV} \\ \chi \\ \mu \end{pmatrix}$$

	PV	$\overline{PV}$	χ	μ
PV	$\mathbf{B}_{PVPV}$			
$\overline{PV}$		$\mathbf{B}_{\overline{PVPV}}$		
χ			$\mathbf{B}_{\chi\chi}$	
μ				$\mathbf{B}_{\mu\mu}$

$$\vec{v}_p = T_p \vec{x}$$

Another important benefit:
This transform separates the minimization increments to balanced and unbalanced states which can help constrain the minimization.

http://darc.nerc.ac.uk/
http://www.met.rdg.ac.uk/~ross/DataAssim/