THEORETICAL ASPECTS OF VARIATIONAL DATA ASSIMILATION

Ross Bannister
Data Assimilation Research Centre

Collaborators: Theory Applications Group, Met Office (JCMM)
Mathematics Dept., Reading Univ.

The Cost Function

$$J(\vec{w}') = J_b(\vec{w}') + J_o(\vec{w}')$$

'background' term (fit to previous forecast)

'observation' term (fit to new observations)

$$J_b(\vec{w}') = \frac{1}{2}(\vec{w}'^b - \vec{w}')^T \mathsf{B}^{-1}(\vec{w}'^b - \vec{w}')$$

$$J_o(\vec{w}') = \frac{1}{2}(\vec{y}^o - H(\vec{w}', \vec{w}^g))^T(\mathsf{E} + \mathsf{F})^{-1}(\vec{y}^o - H(\vec{w}', \vec{w}^g))$$

Need the analysed state.
This is the $\vec{w}'$ which minimizes J.

The background error covariance matrix

It is crucial to know B (and its inverse).
Most basically ...

B =

	$\vec{u}$	$\vec{v}$	$\vec{\theta}$	$\vec{p}$	$\vec{q}$
$\vec{u}$	B_{uu}	B_{uv}	$B_{u\theta}$	B_{up}	B_{uq}
$\vec{v}$	B_{uv}	B_{vv}	$B_{v\theta}$	B_{vp}	B_{vq}
$\vec{\theta}$	$B_{u\theta}$	$B_{v\theta}$	$B_{\theta\theta}$	$B_{\theta p}$	$B_{\theta q}$
$\vec{p}$	B_{up}	B_{vp}	$B_{\theta p}$	B_{pp}	B_{pq}
$\vec{q}$	B_{uq}	B_{vq}	$B_{\theta q}$	B_{pq}	B_{qq}

Too big to store
Too big to calculate
Too big to use ...
... (and we need to know its inverse!)

Problem: **B** is too large to work with (even at half resolution).

# fields	# long. points	# lat. points	# levels	# elements **B**
5	216	163	30	$> 10^{13}$
5	48	37	42	$> 10^{11}$

Presently ...

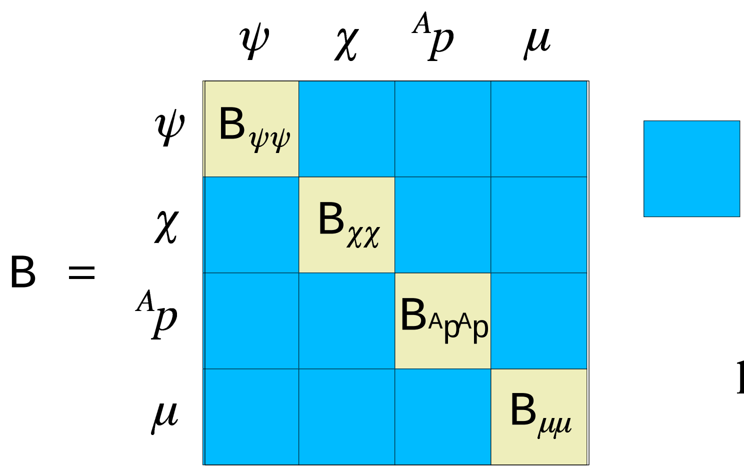
Make assumptions about the nature of the error correlations (ie compact the information needed to approximate B and B^{-1}).

One important stage in completing this process is the *parameter transform,* U_p.

Meteorological variables → Parameters

$$\begin{pmatrix} \vec{u} \\ \vec{v} \\ \vec{\theta} \\ \vec{p} \\ \vec{q} \end{pmatrix} \rightarrow \begin{pmatrix} \vec{\psi} \\ \vec{\chi} \\ {}^{A}p \\ \mu \end{pmatrix}$$

Parameters are assumed to be uncorrelated ...



sub-matrix assumed zero (but non-zero)

$\mathbf{B}^{-1}$ has a similar structure

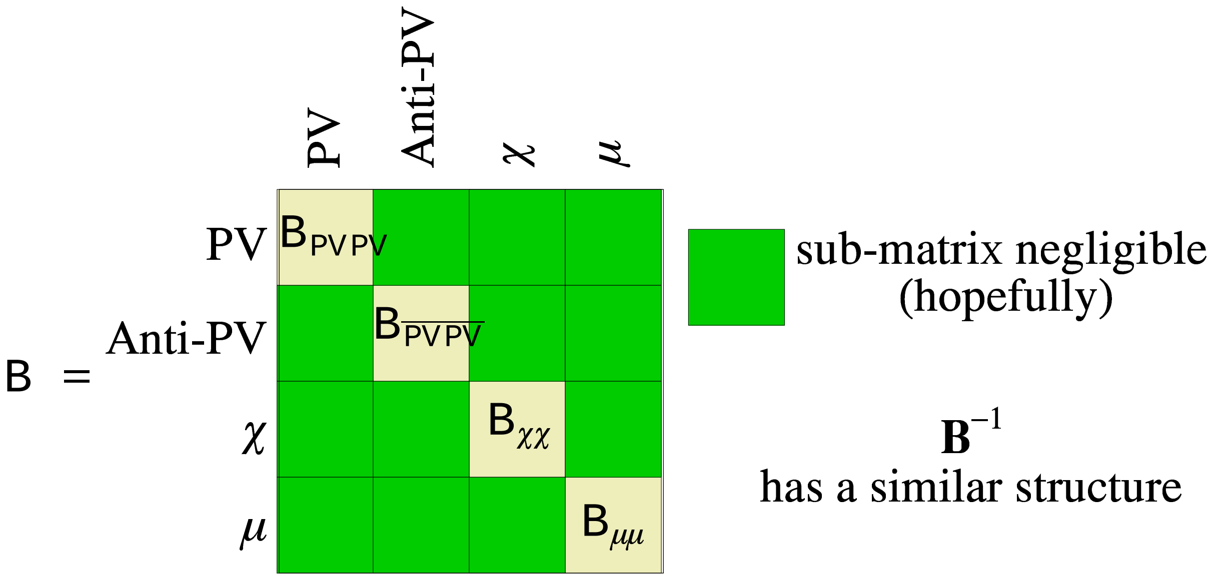
There are no good theoretical reasons to suppose that these variables are uncorrelated (and they are not uncorrelated).

New parameters ...

There are better reasons to suppose that a new set of (PV based) parameters are more uncorrelated.

Meteorological variables → New parameters

$$\begin{pmatrix} \vec{u} \\ \vec{v} \\ \vec{\theta} \\ \vec{p} \\ \vec{q} \end{pmatrix} \rightarrow \begin{pmatrix} \text{PV} \\ \text{"Anti-PV"} \\ \chi \\ \mu \end{pmatrix}$$



$\mathbf{B}^{-1}$

has a similar structure

To implement this new scheme in the Met Office 3d Var system, we need to know:

the transformation, U_p (new parameters to met. variables),

the inverse transformation, U_p^{-1},

the adjoint transformation, U_p^{T},

and vertical statistics for each parameter.