Summary of Equations - Met Office Var Scheme

1. $\vec{w}$-space formulation

$$J(\vec{w}') = \frac{1}{2}(\vec{w}'^b - \vec{w}')^T \mathbf{B}^{-1}(\vec{w}'^b - \vec{w}') + \frac{1}{2}(\vec{y}^o - H(\vec{w}', \vec{w}^g))^T (\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\vec{w}', \vec{w}^g))$$

$$\frac{\mathrm{d}J}{\mathrm{d}\vec{w}'} = -\mathbf{B}^{-1}(\vec{w}'^b - \vec{w}') - \mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\vec{w}', \vec{w}^g))$$

$$\frac{\mathrm{d}^2 J_b}{\mathrm{d}\vec{w}'^2} = \mathbf{B}^{-1} + \mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}\mathbf{H}$$

$$\vec{w} = \text{model state } (' \text{ pertbtn, } b \text{ backgrnd, } g \text{ guess})$$

$$H(\vec{w}', \vec{w}^g) \approx H(\vec{w}^g) + \mathbf{H}\vec{w}'$$

2. $\vec{v}$-space formulation

$$\vec{v} = \mathbf{T}\vec{w}' \qquad \vec{w}' = \mathbf{U}\vec{v} \qquad \mathbf{U}^T\mathbf{B}^{-1}\mathbf{U} = \mathbf{I}$$

$$J(\vec{v}) = \frac{1}{2}(\vec{v}^b - \vec{v})^T \mathbf{U}^T\mathbf{B}^{-1}\mathbf{U}(\vec{v}^b - \vec{v}) + \frac{1}{2}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))^T (\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))$$

$$= \frac{1}{2}(\vec{v}^b - \vec{v})^T(\vec{v}^b - \vec{v}) + \frac{1}{2}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))^T (\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))$$

$$\frac{\mathrm{d}J}{\mathrm{d}\vec{v}} = -\mathbf{U}^T\mathbf{B}^{-1}\mathbf{U}(\vec{v}^b - \vec{v}) - \mathbf{U}^T\mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))$$

$$= (\vec{v}^b - \vec{v}) - \mathbf{U}^T\mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}(\vec{y}^o - H(\mathbf{U}\vec{v}, \vec{w}^g))$$

$$\frac{\mathrm{d}^2 J_b}{\mathrm{d}\vec{v}^2} = \mathbf{U}^T\mathbf{B}^{-1}\mathbf{U} + \mathbf{U}^T\mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}\mathbf{H}\mathbf{U}$$

$$= \mathbf{I} + \mathbf{U}^T\mathbf{H}^T(\mathbf{E} + \mathbf{F})^{-1}\mathbf{H}\mathbf{U}$$