Geostrophic covariances

Assume pressure-pressure correlations are isotropic

$$\mu_{ij} = \exp -\frac{r_{ij}^2}{2L^2},$$

where μ_{ij} is the correlation of pressure at position i and pressure at position j, r_{ij} is the distance between positions i and j,

$$r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2,$$

and $(\sqrt{2})L$ is the length scale (approx. 750 km).

State geostrophic relations

$$-\frac{1}{\rho f}\frac{\partial p}{\partial x} = v,$$

$$\frac{1}{\rho f}\frac{\partial p}{\partial y} = u,$$

where p is pressure, ρ is density, f is Corriolis parameter $f = 2\Omega \sin\phi$, Ω is the Earth's angular velocity, ϕ is latitude, u is zonal wind, and v is meridional wind.

Need first and second spatial derivatives of p-p correlations

$$\frac{\partial \mu_{ij}}{\partial x_i} = -\frac{\mu_{ij}(x_i - x_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial x_j} = \frac{\mu_{ij}(x_i - x_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial y_i} = -\frac{\mu_{ij}(y_i - y_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial y_j} = \frac{\mu_{ij}(y_i - y_j)}{L^2}$$

$$\frac{\partial^2 \mu_{ij}}{\partial x_i \partial x_j} = \frac{\mu_{ij}}{L^2}\left(1 - \frac{(x_i - x_j)^2}{L^2}\right)$$

$$\frac{\partial^2 \mu_{ij}}{\partial y_i \partial y_j} = \frac{\mu_{ij}}{L^2}\left(1 - \frac{(y_i - y_j)^2}{L^2}\right)$$

$$\frac{\partial^2 \mu_{ij}}{\partial y_i \partial x_j} = -\frac{\mu_{ij}(x_i - x_j)(y_i - y_j)}{L^4}$$

$$\frac{\partial^2 \mu_{ij}}{\partial x_i \partial y_j} = -\frac{\mu_{ij}(x_i - x_j)(y_i - y_j)}{L^4}$$

Derive covariances for winds and pressure

Pressure - pressure covariances

$$\langle \delta p_i \delta p_j \rangle = \sigma^2 \mu_{ij}$$

σ is the pressure standard deviation (assume homogeneous)

Pressure - zonal wind covariances

$$\langle \delta p_i \delta u_j \rangle = \frac{1}{\rho f_j}\langle \delta p_i \frac{\partial}{\partial y_j}\delta p_j \rangle$$

$$= \frac{1}{\rho f_j}\frac{\partial}{\partial y_j}\langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho f_j}\frac{\partial \mu_{ij}}{\partial y_j}$$

Pressure - meridional wind covariances

$$\langle \delta p_i \delta v_j \rangle = -\frac{1}{\rho f_j}\langle \delta p_i \frac{\partial}{\partial x_j}\delta p_j \rangle$$

$$= -\frac{1}{\rho f_j}\frac{\partial}{\partial x_j}\langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho f_j}\frac{\partial \mu_{ij}}{\partial x_j}$$

Zonal wind - pressure covariances

$$\langle \delta u_i \delta p_j \rangle = \frac{1}{\rho f_i}\langle \frac{\partial}{\partial y_i}\delta p_i \delta p_j \rangle$$

$$= \frac{1}{\rho f_i}\frac{\partial}{\partial y_i}\langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho f_i}\frac{\partial \mu_{ij}}{\partial y_i}$$

Zonal - zonal wind covariances

$$\langle \delta u_i \delta u_j \rangle = \frac{1}{\rho^2 f_i f_j}\langle \frac{\partial}{\partial y_i}\delta p_i \frac{\partial}{\partial y_j}\delta p_j \rangle$$

$$= \frac{1}{\rho^2 f_i f_j}\frac{\partial^2}{\partial y_i \partial y_j}\langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho^2 f_i f_j}\frac{\partial^2 \mu_{ij}}{\partial y_i \partial y_j}$$

Zonal - meridional wind covariances

$$\langle \delta u_i \delta v_j \rangle = -\frac{1}{\rho^2 f_i f_j}\langle \frac{\partial}{\partial y_i}\delta p_i \frac{\partial}{\partial x_j}\delta p_j \rangle$$

$$= -\frac{1}{\rho^2 f_i f_j}\frac{\partial^2}{\partial y_i \partial x_j}\langle \delta p_i \delta p_j \rangle = -\frac{1}{\rho^2 f_i f_j}\frac{\partial^2 \mu_{ij}}{\partial y_i \partial x_j}$$

Meridional wind - pressure covariances

$$\langle \delta v_i \delta p_j \rangle = -\frac{1}{\rho f_i}\langle \frac{\partial}{\partial x_i}\delta p_i \delta p_j \rangle$$

$$= -\frac{1}{\rho f_i}\frac{\partial}{\partial x_i}\langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho f_i}\frac{\partial \mu_{ij}}{\partial x_i}$$

Meridional - zonal wind covariances

$$\langle \delta v_i \delta u_j \rangle = -\frac{1}{\rho^2 f_i f_j}\langle \frac{\partial}{\partial x_i}\delta p_i \frac{\partial}{\partial y_j}\delta p_j \rangle$$

$$= -\frac{1}{\rho^2 f_i f_j}\frac{\partial^2}{\partial x_i \partial y_j}\langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho^2 f_i f_j}\frac{\partial^2 \mu_{ij}}{\partial x_i \partial y_j}$$

Meridional - meridional wind covariances

$$\langle \delta v_i \delta v_j \rangle = \frac{1}{\rho^2 f_i f_j}\langle \frac{\partial}{\partial x_i}\delta p_i \frac{\partial}{\partial x_j}\delta p_j \rangle$$

$$= \frac{1}{\rho^2 f_i f_j}\frac{\partial^2}{\partial x_i \partial x_j}\langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho^2 f_i f_j}\frac{\partial^2 \mu_{ij}}{\partial x_i \partial x_j}$$

Covariance maps

The above covariances are plotted with the colour scheme: red (positive) and blue (negative) covariances.