

# Geostrophic covariances

Corrected version, January 2011

**Assume pressure-pressure correlations are isotropic**

$$\mu_{ij} = \exp -\frac{r_{ij}^2}{2L^2},$$

where  $\mu_{ij}$  is the correlation of pressure at position  $i$  and pressure at position  $j$ ,  $r_{ij}$  is the distance between positions  $i$  and  $j$ ,

$$r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2,$$

and  $(\sqrt{2})L$  is the length scale (approx. 750 km).

## State geostrophic relations

$$\frac{1}{\rho f} \frac{\partial p}{\partial x} = v,$$

$$-\frac{1}{\rho f} \frac{\partial p}{\partial y} = u,$$

where  $p$  is pressure,  $\rho$  is density,  $f$  is Coriolis parameter  $f = 2\Omega \sin \phi$ ,  $\Omega$  is the Earth's angular velocity,  $\phi$  is latitude,  $u$  is zonal wind, and  $v$  is meridional wind.

## Need first and second spatial derivatives of p-p correlations

$$\frac{\partial \mu_{ij}}{\partial x_i} = -\frac{\mu_{ij}(x_i - x_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial x_j} = \frac{\mu_{ij}(x_i - x_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial y_i} = -\frac{\mu_{ij}(y_i - y_j)}{L^2}$$

$$\frac{\partial \mu_{ij}}{\partial y_j} = \frac{\mu_{ij}(y_i - y_j)}{L^2}$$

$$\frac{\partial^2 \mu_{ij}}{\partial x_i \partial x_j} = \frac{\mu_{ij}}{L^2} \left( 1 - \frac{(x_i - x_j)^2}{L^2} \right)$$

$$\frac{\partial^2 \mu_{ij}}{\partial y_i \partial y_j} = \frac{\mu_{ij}}{L^2} \left( 1 - \frac{(y_i - y_j)^2}{L^2} \right)$$

$$\frac{\partial^2 \mu_{ij}}{\partial y_i \partial x_j} = -\frac{\mu_{ij}(x_i - x_j)(y_i - y_j)}{L^4}$$

$$\frac{\partial^2 \mu_{ij}}{\partial x_i \partial y_j} = -\frac{\mu_{ij}(x_i - x_j)(y_i - y_j)}{L^4}$$

## Derive covariances for winds and pressure

### Pressure - pressure covariances

$$\langle \delta p_i \delta p_j \rangle = \sigma^2 \mu_{ij}$$

$\sigma$  is the pressure standard deviation (assume homogeneous)

### Pressure - zonal wind covariances

$$\begin{aligned} \langle \delta p_i \delta u_j \rangle &= -\frac{1}{\rho f} \langle \delta p_i \frac{\partial}{\partial y_j} \delta p_j \rangle \\ &= -\frac{1}{\rho f} \frac{\partial}{\partial y_j} \langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho f} \frac{\partial \mu_{ij}}{\partial y_j} \end{aligned}$$

### Pressure - meridional wind covariances

$$\langle \delta p_i \delta v_j \rangle = \frac{1}{\rho f} \langle \delta p_i \frac{\partial}{\partial x_j} \delta p_j \rangle$$

$$= \frac{1}{\rho f} \frac{\partial}{\partial x_j} \langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho f} \frac{\partial \mu_{ij}}{\partial x_j}$$

### Zonal wind - pressure covariances

$$\begin{aligned} \langle \delta u_i \delta p_j \rangle &= -\frac{1}{\rho f} \langle \frac{\partial}{\partial y_i} \delta p_i \delta p_j \rangle \\ &= -\frac{1}{\rho f} \frac{\partial}{\partial y_i} \langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho f} \frac{\partial \mu_{ij}}{\partial y_i} \end{aligned}$$

### Zonal - zonal wind covariances

$$\begin{aligned} \langle \delta u_i \delta u_j \rangle &= \frac{1}{\rho^2 f^2} \langle \frac{\partial}{\partial y_i} \delta p_i \frac{\partial}{\partial y_j} \delta p_j \rangle \\ &= \frac{1}{\rho^2 f^2} \frac{\partial^2}{\partial y_i \partial y_j} \langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho^2 f^2} \frac{\partial^2 \mu_{ij}}{\partial y_i \partial y_j} \end{aligned}$$

### Zonal - meridional wind covariances

$$\begin{aligned}\langle \delta u_i \delta v_j \rangle &= -\frac{1}{\rho^2 f^2} \left\langle \frac{\partial}{\partial y_i} \delta p_i \frac{\partial}{\partial x_j} \delta p_j \right\rangle \\ &= -\frac{1}{\rho^2 f^2} \frac{\partial^2}{\partial y_i \partial x_j} \langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho^2 f^2} \frac{\partial^2 \mu_{ij}}{\partial y_i \partial x_j}\end{aligned}$$

### Meridional wind - pressure covariances

$$\begin{aligned}\langle \delta v_i \delta p_j \rangle &= \frac{1}{\rho f} \left\langle \frac{\partial}{\partial x_i} \delta p_i \delta p_j \right\rangle \\ &= \frac{1}{\rho f} \frac{\partial}{\partial x_i} \langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho f} \frac{\partial \mu_{ij}}{\partial x_i}\end{aligned}$$

### Meridional - zonal wind covariances

$$\begin{aligned}\langle \delta v_i \delta u_j \rangle &= -\frac{1}{\rho^2 f^2} \left\langle \frac{\partial}{\partial x_i} \delta p_i \frac{\partial}{\partial y_j} \delta p_j \right\rangle \\ &= -\frac{1}{\rho^2 f^2} \frac{\partial^2}{\partial x_i \partial y_j} \langle \delta p_i \delta p_j \rangle = -\frac{\sigma^2}{\rho^2 f^2} \frac{\partial^2 \mu_{ij}}{\partial x_i \partial y_j}\end{aligned}$$

### Meridional - meridional wind covariances

$$\begin{aligned}\langle \delta v_i \delta v_j \rangle &= \frac{1}{\rho^2 f^2} \left\langle \frac{\partial}{\partial x_i} \delta p_i \frac{\partial}{\partial x_j} \delta p_j \right\rangle \\ &= \frac{1}{\rho^2 f^2} \frac{\partial^2}{\partial x_i \partial x_j} \langle \delta p_i \delta p_j \rangle = \frac{\sigma^2}{\rho^2 f^2} \frac{\partial^2 \mu_{ij}}{\partial x_i \partial x_j}\end{aligned}$$

## Covariance maps

The above covariances are plotted with the colour scheme: red (positive) and blue (negative) covariances.

