How is balance of a forecast ensemble affected by adaptive and non-adaptive localization schemes?

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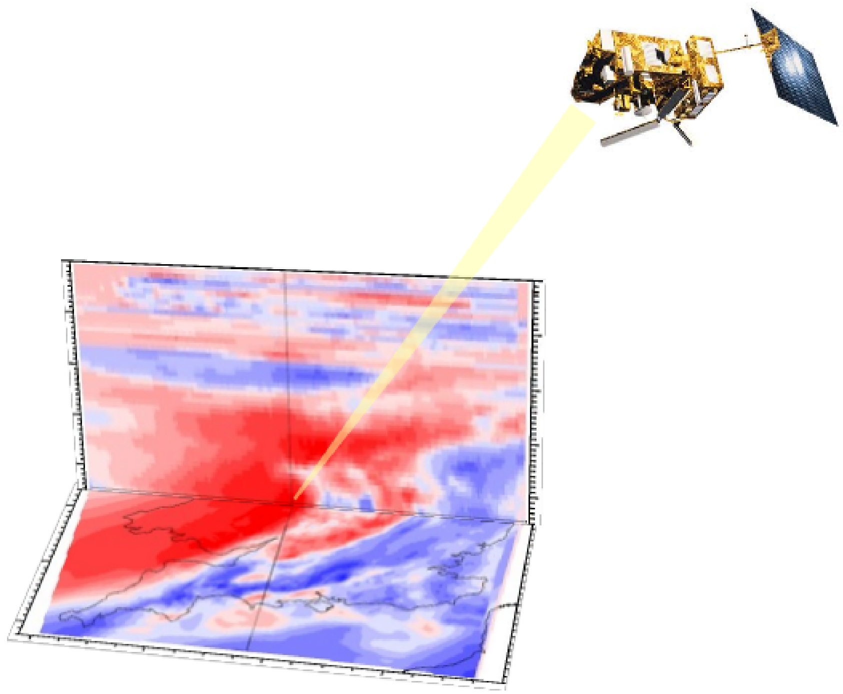


Outline

1 Meteorological balance

1(a) What is balance and why is it important to worry about?

- Initial conditions of meteorological models need to be appropriately balanced.

Momentum equations

$$
\begin{aligned}
\frac{Du}{Dt} &= fv - \frac{1}{\rho}\frac{\partial p}{\partial x} - D_x, \\
\frac{Dv}{Dt} &= -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} - D_y, \\
\frac{Dw}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - D_x,
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\Omega\sin(y/a), \\
\Omega &= 7.29\times 10^{-5}\mathrm{rads}^{-1}, \\
a &= 6.371\times 10^{6}\mathrm{m}, \\
g &= 9.806\mathrm{ms}^{-1}.
\end{aligned}
$$

- Geostrophic balance

$$
fv - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0.
$$

Dimensionless variables

$$
u = U\tilde{u},\ v = U\tilde{v},\ w = W\tilde{w},\ p = P\tilde{p},
$$

$$
x = L\tilde{x},\ z = H\tilde{z},\ t = L/U\tilde{t},
$$

$$
\begin{aligned}
\cancel{\mathrm{Ro}\frac{D\tilde{u}}{D\tilde{t}}} &= v - \frac{P}{f\rho U L}\frac{\partial\tilde{p}}{\partial\tilde{x}} - \cancel{\tilde{D}_x}, \\
\cancel{\mathrm{Ro}\frac{D\tilde{v}}{D\tilde{t}}} &= -\tilde{u} - \frac{P}{f\rho U L}\frac{\partial\tilde{p}}{\partial\tilde{y}} - \cancel{\tilde{D}_y}, \\
\cancel{\mathrm{Ro}\frac{W}{U}\frac{D\tilde{w}}{D\tilde{t}}} &= -\frac{P}{f\rho U H}\frac{\partial\tilde{p}}{\partial\tilde{z}} - \frac{g}{fU} - \cancel{\tilde{D}_x},
\end{aligned}
$$

$$
\mathrm{Ro} = \frac{U}{fL} = \mathcal{O}(10^{-1}),\ \frac{W}{U} = \mathcal{O}(10^{-2}).
$$

- Hydrostatic balance

$$
-\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0.
$$

- Geostrophic balance

$$fv - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0.$$

- Geostrophic balance is characteristic of mid-latitude flow (small Ro wind follows the isobars).

  ▷ Some ageostrophic flow is needed in initial conditions to match the atmosphere.

  ▷ Too much ageostrophic flow can damage a forecast.

  ▷ Unbalanced motion relaxes to near balanced motion by geostrophic adjustment (gravity waves).

  ▷ All modern meteorological models are capable of supporting gravity waves so excessive gravity waves will cause a problem.

- Hydrostatic balance

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0.$$

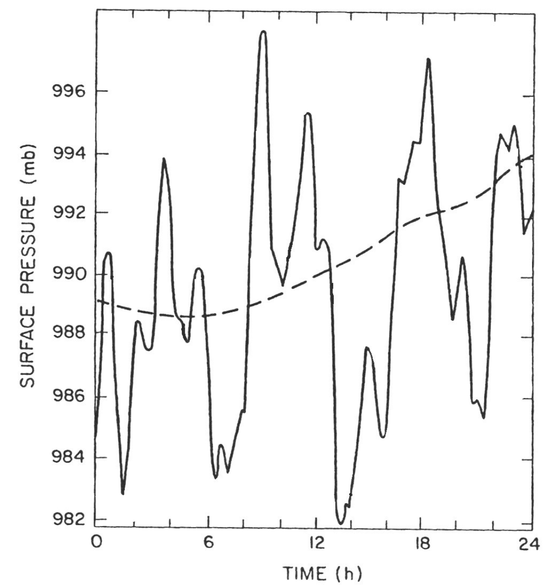


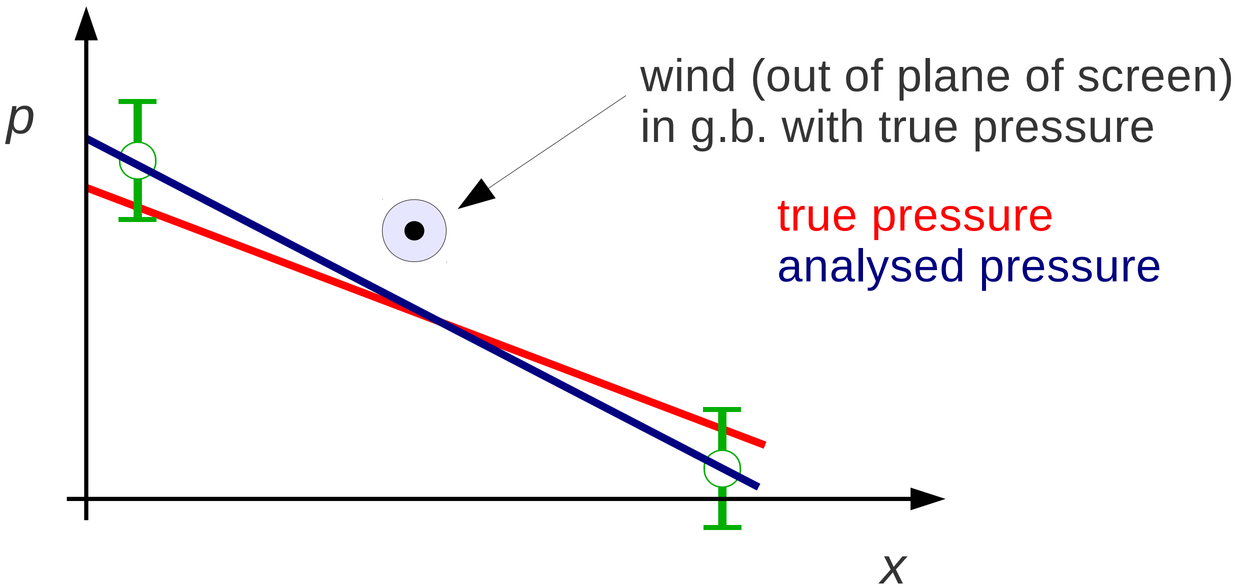
Figure 6.1 Surface pressure as a function of time during the integration of a primitive equations model. Uninitialized (solid), initialized (dashed). (After Williamson and Temperton, *Mon. Wea. Rev.* **109**: 745, 1981. The American Meteorological Society.)

- Hydrostatic balance is characteristic of non-convective flow (small Ro and W/U).

  ▷ Meteorological models that permit convection explicitly must allow non-hydrostatic flow.

1(b) How can assimilation of observations lead to imbalance?

- Observations sample from the truth.
  - ▷ The 'true manifold' $\neq$ the model manifold.
- Observations are not perfect.



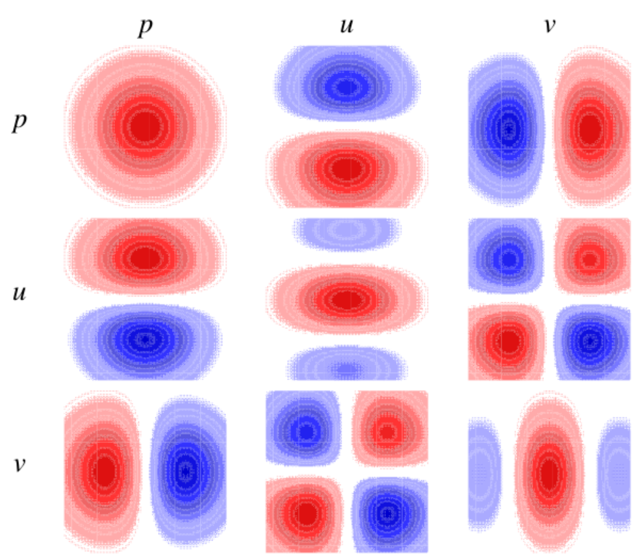
- Essentially discovered by L.F. Richardson in the 1920s.

1(c) How to respect balance in data assimilation?

- Initialization
  - ▷ Post-posteriori filtering of imbalance according to a set of rules.
  - ▷ Moves away from observations just assimilated.
- Forecast error covariance matrix

$$\mathbf{x}^{\mathrm{a}} = \mathbf{x}^{\mathrm{f}} + \mathbf{P}^{\mathrm{f}}\mathbf{H}^{\mathrm{T}} \left(\mathbf{R} + \mathbf{H}\mathbf{P}^{\mathrm{f}}\mathbf{H}^{\mathrm{T}}\right)^{-1} \left[\mathbf{y} - \mathbf{H}\mathbf{x}^{\mathrm{f}}\right].$$

  - ▷ $\mathbf{P}^{\mathrm{f}}$ is used not just for regularization.



▷ $\mathbf{P}^{\mathrm{f}}$ contains the 'statistics of balance' (e.g. for strong hydrostatic balance: in $\mathbf{\Delta} = \mathbf{x}^{\mathrm{a}} - \mathbf{x}^{\mathrm{f}}$ (analysis increments), the correlation between $\Delta[\partial p/\partial z]$ and $\Delta[\rho g]$ should be -1:

$$\frac{\partial p}{\partial z} + \rho g = 0.$$

▷ $\mathbf{P}^{\mathrm{f}}$ in VAR is modelled using explicit balance conditions (called B).

← Example structure functions giving the output field (p, u or v down the side) associated with a point in the centre of the domain (either of p, u or v along the top). Red is positive, blue is negative.

1(d) How much balance should be in an analysis increment?

- Practically - we don't know.
- Have climatological ideas (e.g. winter average for mid-latitudes), but this could change from day-to-day (e.g. high-pressure vs. low pressure systems, fronts, convection, boundary layer characteristics, etc.)
- In the Kalman Filter this information will be wrapped-up in $\mathbf{P}^{\mathrm{f}}$.
- In the Ensemble Kalman Filter this information is wrapped-up in the ensemble (see later).
- Want the analysis increments to be balanced in the way described by the Kalman update equation

$$\mathbf{x}^{\mathrm{a}} = \mathbf{x}^{\mathrm{f}} + \mathbf{P}^{\mathrm{f}}\mathbf{H}^{\mathrm{T}}\left(\mathbf{R} + \mathbf{H}\mathbf{P}^{\mathrm{f}}\mathbf{H}^{\mathrm{T}}\right)^{-1}\left[\mathbf{y} - \mathbf{H}\mathbf{x}^{\mathrm{f}}\right].$$

2 Ensemble data assimilation

2(a) Sampling error

Dynamical sample covariance

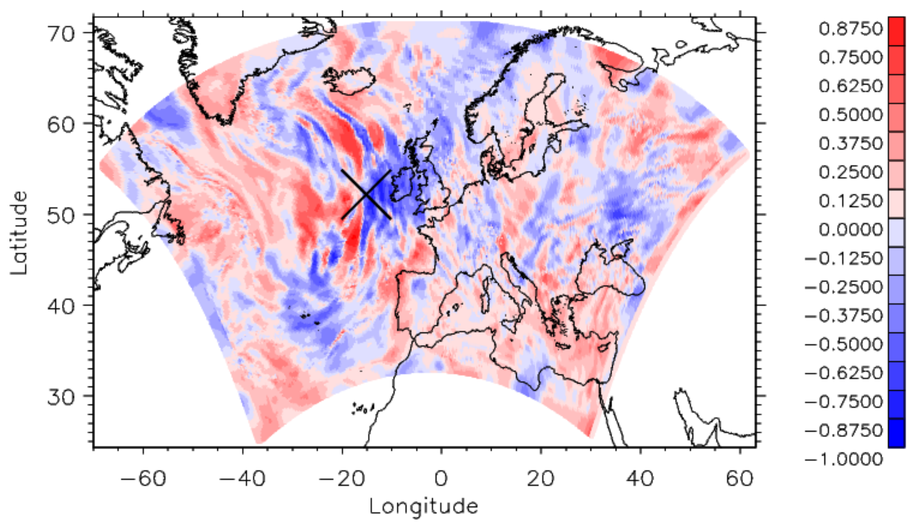
$$\mathbf{P}^{\mathrm{D}}_{(N)} = \frac{1}{N-1}\sum_{l=1}^{N}\delta\mathbf{x}_l\delta\mathbf{x}_l^{\mathrm{T}} = \frac{1}{N-1}\mathbf{X}\mathbf{X}^{\mathrm{T}},$$
$$\mathbf{P}^{\mathrm{D}}_{(N)} \in \mathbb{R}^{n\times n}$$

Dynamical forecast ensemble

$$\mathbf{X} = \{\delta\mathbf{x}_1, \delta\mathbf{x}_2, \ldots \delta\mathbf{x}_N\},$$
$$\delta\mathbf{x}_l \in \mathbb{R}^n$$
$$\mathbf{X} \in \mathbb{R}^{n\times N}$$

Sample covariance error

$$[\mathcal{E}(\delta\mathbf{P}^{\mathrm{D}}_{(N)})]_{ij} \sim \frac{\sigma_i\sigma_j}{\sqrt{N-1}}\left(1-\left(\frac{[\mathbf{P}]_{ij}}{\sigma_i\sigma_j}\right)^2\right).$$



← v-p correlation function from ensemble with $N = 24$.

2(b) Localization

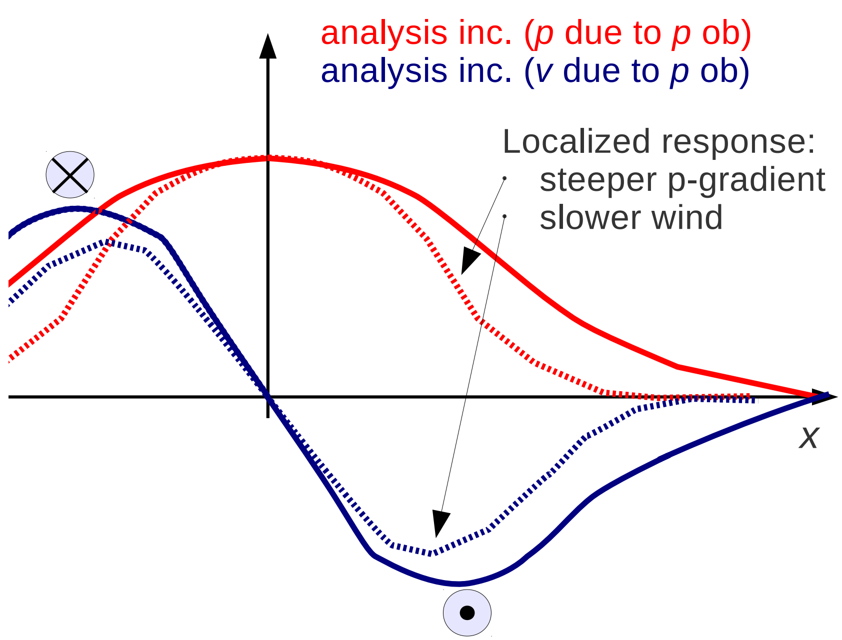
$$\mathbf{P}^{\mathrm{L}}_{(N,K)} = \mathbf{P}^{\mathrm{D}}_{(N)} \circ \mathbf{\Omega}_{(K)}$$

$$\mathbf{P}^{\mathrm{D}}_{(N)} \in \mathbb{R}^{n \times n}$$
$$\mathbf{\Omega}_{(K)} \in \mathbb{R}^{n \times n}$$

- Notation:
  - ▷ $\mathbf{P}^{\mathrm{D}}_{(N)}$ forecast error covariance matrix from N ensemble members (rank $\sim N$).
  - ▷ $\mathbf{\Omega}_{(K)}$ localization / regularization matrix (rank K).
    - ▷ $\mathbf{\Omega}_{(K)}$ is a correlation matrix.

2(c) Balance (non-)preservation

- Balance is defined by the null-space (or near null space) of $\mathbf{P}^{\mathrm{f}}$.
  - ▷ Localization changes the null-space.
- Localization assumes that structures become less important with distance from an observation.
  - ▷ Some structures grow with distance.
  - ▷ Example - can affect position of peaks in geostrophic lobes.
- Localization changes the values and gradients of fields[1].



[1] Lorenc A.C., The potential of the ensemble Kalman Filter for NWP - a comparison with 4D-VAR, Quart. J. Roy. Meteor. Soc. 129, 3183-3203 (2003).

2(d) Possible mitigation strategies

- Avoid doing localization with highly anisotropic fields like u and v - apply to ψ and χ instead[2].
  - ▷ OK for geostrophic balance, unclear what to do for other balances.
- Perform localization on 'control variables' and introduce a balance operator.
  - ▷ Like the Met Office's hybrid data assimilation system[3].
  - ▷ OK when balances are known and appropriate.
- → **Adaptive localization schemes?** ←
  - ▷ **SENCORP (Smoothed ENsemble COrrelations Raised to a Power)[4].**
  - ▷ **ECO-RAP (Ensemble COrrelations Raised to A Power)[5] [6].**

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[2]Kepert J.S., Covariance localization and balance in an ensemble Kalman filter, Quart. J. Roy. Meteor. Soc. 135, 1157-1176, DOI:10.1002/qj.443 (2009).

[3]Clayton A.M., Lorenc A.C. and Barker D.M., Operational implementation of a hybrid ensemble/4D-Var global data assimilation system at the Met Office, Q.J.R. Meteorol. Soc., DOI:10.1002/qj.2054 (2012).

[4]Bishop C.H. and Hodyss D., Flow adaptive moderation of spurious ensemble correlations and its used in ensemble-based data assimilation, Quart. J. Roy. Met. Soc. 133, 2029-2044 (2007), DOI:10.1002/qj.169.

[5]Bishop C.H. and Hodyss D., Ensemble covariances adaptively localized with ECO-RAP, Part 1: Tests on simple error models, Tellus A 61, 84-96 (2009).

[6]Bishop C.H. and Hodyss D., Ensemble covariances adaptively localized with ECO-RAP, Part 2: A strategy for the atmosphere, Tellus A 61, 97-111 (2009).

3 Balance diagnostics

3(a) Given an ensemble, what is the balance?

General: equation of motion and two-term balance equation

$$\frac{\partial Q(\mathbf{x})}{\partial t} = \mathcal{A}(\mathbf{x}) + \mathcal{B}(\mathbf{x}) + \mathcal{C}(\mathbf{x}), \qquad 0 = \mathcal{A}(\mathbf{x}) + \mathcal{B}(\mathbf{x}).$$

$\mathcal{A}(\mathbf{x})$ will be exactly anti-correlated with $\mathcal{B}(\mathbf{x})$ if this balance condition is obeyed exactly.

Geostrophic balance (actually linear balance): the divergence equation

$$\frac{D\delta'}{Dt} = \mathcal{M}' + \mathcal{W}' + \text{horiz. Coriolis} + \text{metric} + \text{forcing} + \text{other},$$

$$\mathcal{M}' = c_p\left(\theta_{\mathrm{v}0}\nabla_z^2\Pi' + \nabla_z^2\Pi_0\,\theta'_{\mathrm{v}}\right), \qquad \mathcal{W}' = -\mathbf{k}\cdot\left(\nabla\times\mathbf{u}' + (\nabla f)\times\mathbf{u}\right).$$

Hydrostatic balance: the vertical momentum equation

$$\frac{Dw'}{Dt} = \mathcal{P}' + \mathcal{T}' + \text{vert. Coriolis} + \text{metric} + \text{forcing} + \text{other},$$

$$\mathcal{P}' = \theta_{\mathrm{v}0}\frac{\partial\Pi'}{\partial z}, \qquad \mathcal{T}' = \frac{\partial\Pi_0}{\partial z}\theta'_{\mathrm{v}}.$$

3(b) What is the balance correlation in a localized ensemble?

Dynamical sample cov $\quad \mathbf{P}^{\mathrm{D}}_{(N)} = \frac{1}{N-1}\sum_{l=1}^{N} \delta\mathbf{x}_l \delta\mathbf{x}_l^{\mathrm{T}} = \frac{1}{N-1}\mathbf{X}\mathbf{X}^{\mathrm{T}} \qquad \mathbf{P}^{\mathrm{D}}_{(N)} \in \mathbb{R}^{n\times n}$

Dynamical forecast ensemble $\quad \mathbf{X} = \{\delta\mathbf{x}_1, \delta\mathbf{x}_2, \ldots \delta\mathbf{x}_N\} \qquad \mathbf{X} \in \mathbb{R}^{n\times N}$

Localized sample cov $\quad \mathbf{P}^{\mathrm{L}}_{(N,K)} = \mathbf{P}^{\mathrm{D}}_{(N)} \circ \mathbf{\Omega}_{(K)}$

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Localized sample cov $\quad \mathbf{P}^{\mathrm{L}}_{(N,K)} = \mathbf{P}^{\mathrm{D}}_{(N)} \circ \mathbf{\Omega}_{(K)}$

Correlation $\quad \mathbf{\Omega}_{(K)} = \frac{1}{K-1}\sum_{k=1}^{K} \boldsymbol{\omega}_k \boldsymbol{\omega}_k^{\mathrm{T}} = \frac{1}{K-1}\mathbf{K}\mathbf{K}^{\mathrm{T}} \quad \mathbf{\Omega}_{(K)} \in \mathbb{R}^{n\times n}$

Correlation ensemble $\quad \mathbf{K} = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots \boldsymbol{\omega}_K\} \quad \mathbf{K} \in \mathbb{R}^{n\times K}$

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Correlation ensemble $\quad \mathbf{K} = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots \boldsymbol{\omega}_K\} \qquad \mathbf{K} \in \mathbb{R}^{n\times K}$

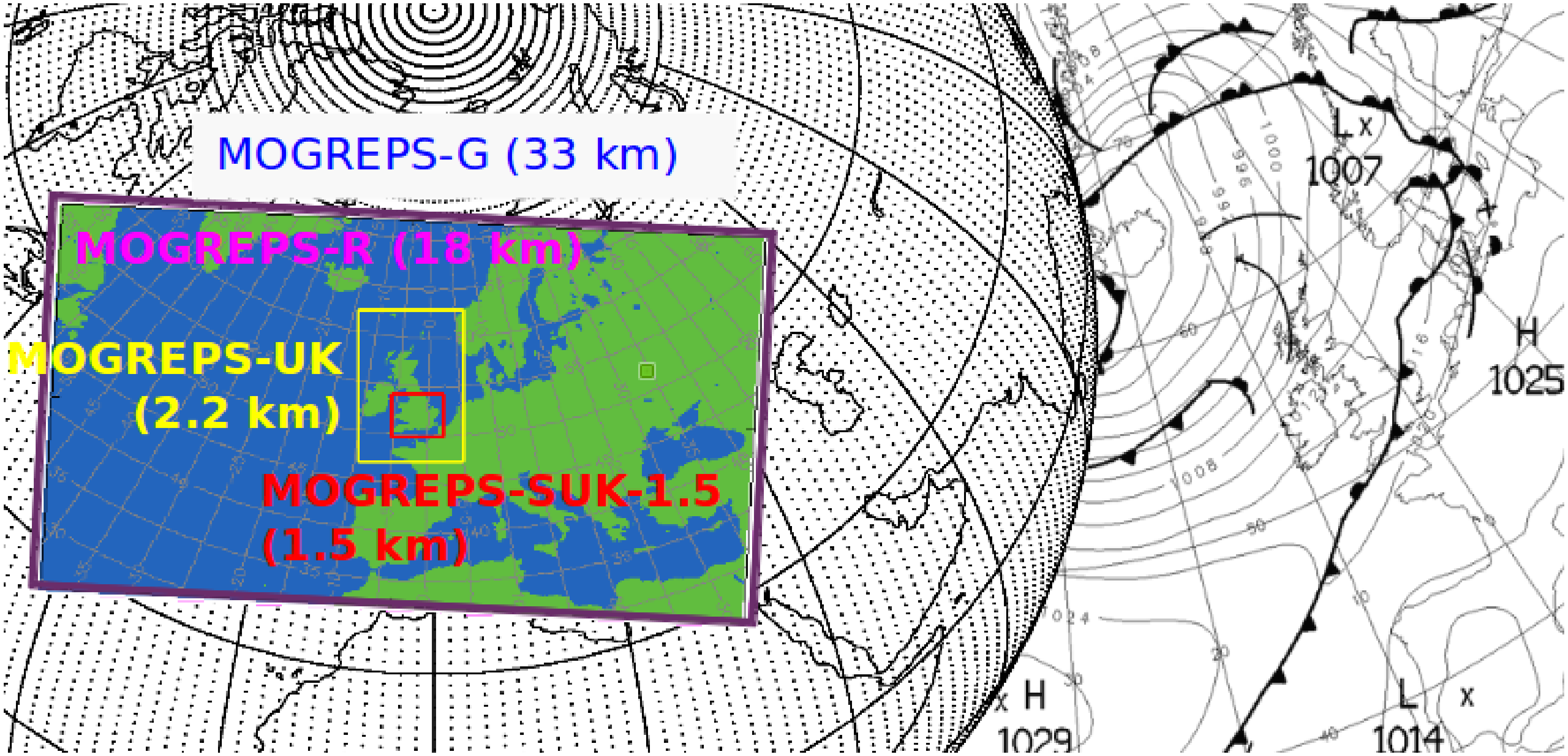
Localized sample cov revisited $\quad \mathbf{P}^{\mathrm{L}}_{(N,K)} = \frac{1}{M-1}\sum_{m=1}^{M} \delta\tilde{\mathbf{x}}_m \delta\tilde{\mathbf{x}}_m^{\mathrm{T}} = \frac{1}{M-1}\tilde{\mathbf{X}}\tilde{\mathbf{X}}^{\mathrm{T}} \qquad (M = NK)$

Localized ensemble $\quad \tilde{\mathbf{X}} = \{\delta\tilde{\mathbf{x}}_1, \delta\tilde{\mathbf{x}}_2, \ldots \delta\tilde{\mathbf{x}}_M\} \qquad \tilde{\mathbf{X}} \in \mathbb{R}^{n\times M}$

$$\delta\tilde{\mathbf{x}}_m = \delta\mathbf{x}_l \circ \boldsymbol{\omega}_k$$

$$\tilde{\mathbf{X}} = \sqrt{\frac{M-1}{(N-1)(K-1)}}\,\mathbf{X} \,\triangle\, \mathbf{K}$$

3(c) The meteorological case



- 20/09/2011
- MOGREPS: Met Office Global and Regional Ensemble Precipitation System

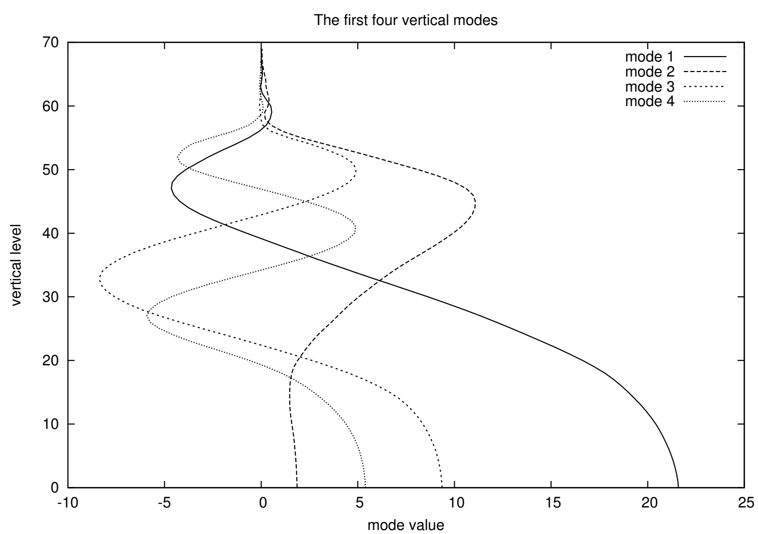
4 Adaptive and non-adaptive localization

4(a) Spectral representation (univariate non-adaptive localization for variable s)

$$\mathbf{K}_s = \overline{\mathbf{F}_s \mathbf{\Lambda}_s^{1/2}}$$

$$[\mathbf{K}_s]_{\mathbf{rk}} = \overline{\underbrace{\cos(k_x r_x + \delta_s^x)\cos(k_y r_y + \delta_s^y)\nu(r_z, k_z)}_{\mathbf{F}_s}\underbrace{\lambda_s^{\mathrm{H}}(k_x^2 + k_y^2)\lambda_s^{\mathrm{V}}(k_z)}_{\mathbf{\Lambda}_s^{1/2}}}$$

- Choose the # horiz. wns, $K_{x\backslash y}$, and # vert. modes, K_z: $K = K_{x\backslash y} K_z$.
- Over-bar means normalize - make sum of squares of each row of matrix unity.
- In practice $K \ll n$.



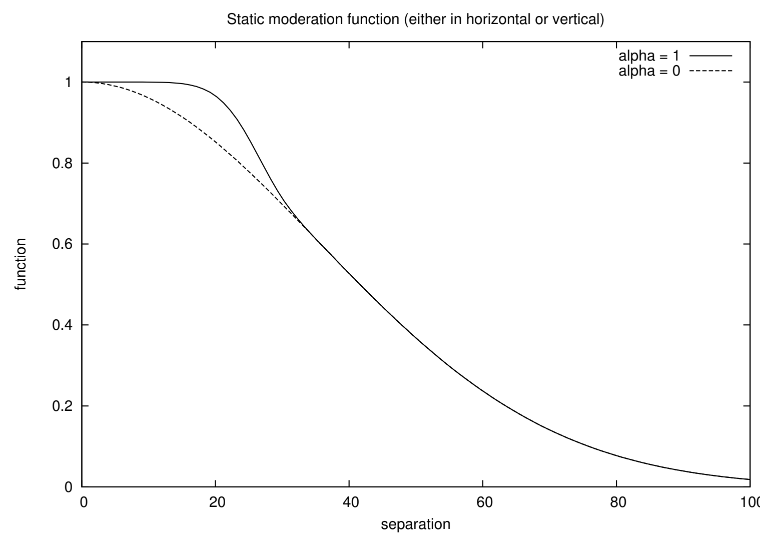


Figure 1: Left: First four vert. modes ($\nu(r_z, k_z)$). Right: Moderation fns, ${\lambda_s^{\mathrm{H}}}^2(r_{x\backslash y})$, ${\lambda_s^{\mathrm{V}}}^2(r_z)$.

4(b) Static localization scheme 1 (S1)

$$\mathbf{K}^{\mathrm{S1}} = \overline{\begin{pmatrix} \mathbf{F}_{\delta u}\boldsymbol{\Lambda}_{\delta u}^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\delta v}\boldsymbol{\Lambda}_{\delta v}^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta w}\boldsymbol{\Lambda}_{\delta w}^{1/2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta \Pi}\boldsymbol{\Lambda}_{\delta \Pi}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta \theta}\boldsymbol{\Lambda}_{\delta \theta}^{1/2} \end{pmatrix}} \in \mathbb{R}^{n \times 5K}$$

4(c) Static localization scheme 2 (S2)

$$\mathbf{K}^{\mathrm{S2}} = \overline{\begin{pmatrix} \mathbf{F}_{\delta u}\boldsymbol{\Lambda}_{\delta u}^{1/2} \\ \mathbf{F}_{\delta v}\boldsymbol{\Lambda}_{\delta v}^{1/2} \\ \mathbf{F}_{\delta w}\boldsymbol{\Lambda}_{\delta w}^{1/2} \\ \mathbf{F}_{\delta \Pi}\boldsymbol{\Lambda}_{\delta \Pi}^{1/2} \\ \mathbf{F}_{\delta \theta}\boldsymbol{\Lambda}_{\delta \theta}^{1/2} \end{pmatrix}} \in \mathbb{R}^{n \times K}$$

4(d) SENCORP (Smoothed ENsemble COrrelations Raised to a Power) localization

$$\mathbf{\Omega} \;=\; \mathbf{C}^{\circ Q}$$

1. From the ensemble members, $\delta \mathrm{x}_l$, create smoothed members, $\delta \mathbf{w}_l$.
2. Normalize.
3. Calculate correlation matrix
$$\mathbf{C} = \frac{1}{N-1} \sum_{l=1}^{N} \delta \mathbf{w}_l \delta \mathbf{w}_l^{\mathrm{T}}.$$
4. $\mathbf{C}^{\circ Q}$ is the Schur power of $\mathbf{C}$ with itself Q times.

$$Q/2 \in \mathbb{Z} \text{ and } Q > 0$$

$$\mathbf{K} = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots \boldsymbol{\omega}_K\} = \sqrt{\frac{K-1}{(N-1)^Q}} \mathbf{W} \bigtriangleup \mathbf{W} \bigtriangleup \mathbf{W} \bigtriangleup \cdots \qquad \mathbf{K} \in \mathbb{R}^{n \times K}$$

$$\boldsymbol{\omega}_k = \sqrt{\frac{K-1}{(N-1)^Q}} \delta \mathbf{w}_{l_1} \circ \ldots \circ \delta \mathbf{w}_{l_Q}$$

$$K = N^Q, \text{ but } \mathrm{rank}(\mathbf{C}) < K$$

4(e) ECO-RAP (Ensemble COrrelations Raised to A Power) localization scheme 1 (E1)

A combination of SENCORP and S1

$$\mathbf{K}^{\mathrm{E1}} = \overline{\mathbf{C}^{\circ Q} \begin{pmatrix} \mathbf{F}_{\delta u}\boldsymbol{\Lambda}_{\delta u}^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\delta v}\boldsymbol{\Lambda}_{\delta v}^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta w}\boldsymbol{\Lambda}_{\delta w}^{1/2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta \Pi}\boldsymbol{\Lambda}_{\delta \Pi}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{\delta \theta}\boldsymbol{\Lambda}_{\delta \theta}^{1/2} \end{pmatrix}} \in \mathbb{R}^{n \times 5K}$$

4(f) ECO-RAP (Ensemble COrrelations Raised to A Power) localization scheme 2 (E2)

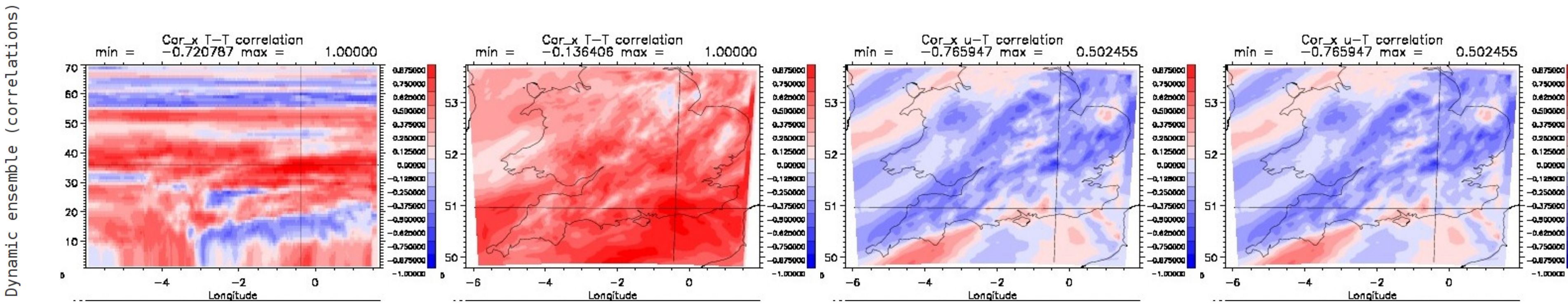
A combination of SENCORP and S2

$$\mathbf{K}^{\mathrm{E2}} = \overline{\mathbf{C}^{\circ Q} \begin{pmatrix} \mathbf{F}_{\delta u}\boldsymbol{\Lambda}_{\delta u}^{1/2} \\ \mathbf{F}_{\delta v}\boldsymbol{\Lambda}_{\delta v}^{1/2} \\ \mathbf{F}_{\delta w}\boldsymbol{\Lambda}_{\delta w}^{1/2} \\ \mathbf{F}_{\delta \Pi}\boldsymbol{\Lambda}_{\delta \Pi}^{1/2} \\ \mathbf{F}_{\delta \theta}\boldsymbol{\Lambda}_{\delta \theta}^{1/2} \end{pmatrix}} \in \mathbb{R}^{n \times K}$$

Practicality: expensive so have to restrict influence of $\mathbf{C}^{\circ Q}$.

5 Implied structure functions and balance diagnostics (dynamic ensemble)

(a) T-T (long/ht), (b) T-T (long/lat), (c) u-T (long/lat), (d) v-T long lat



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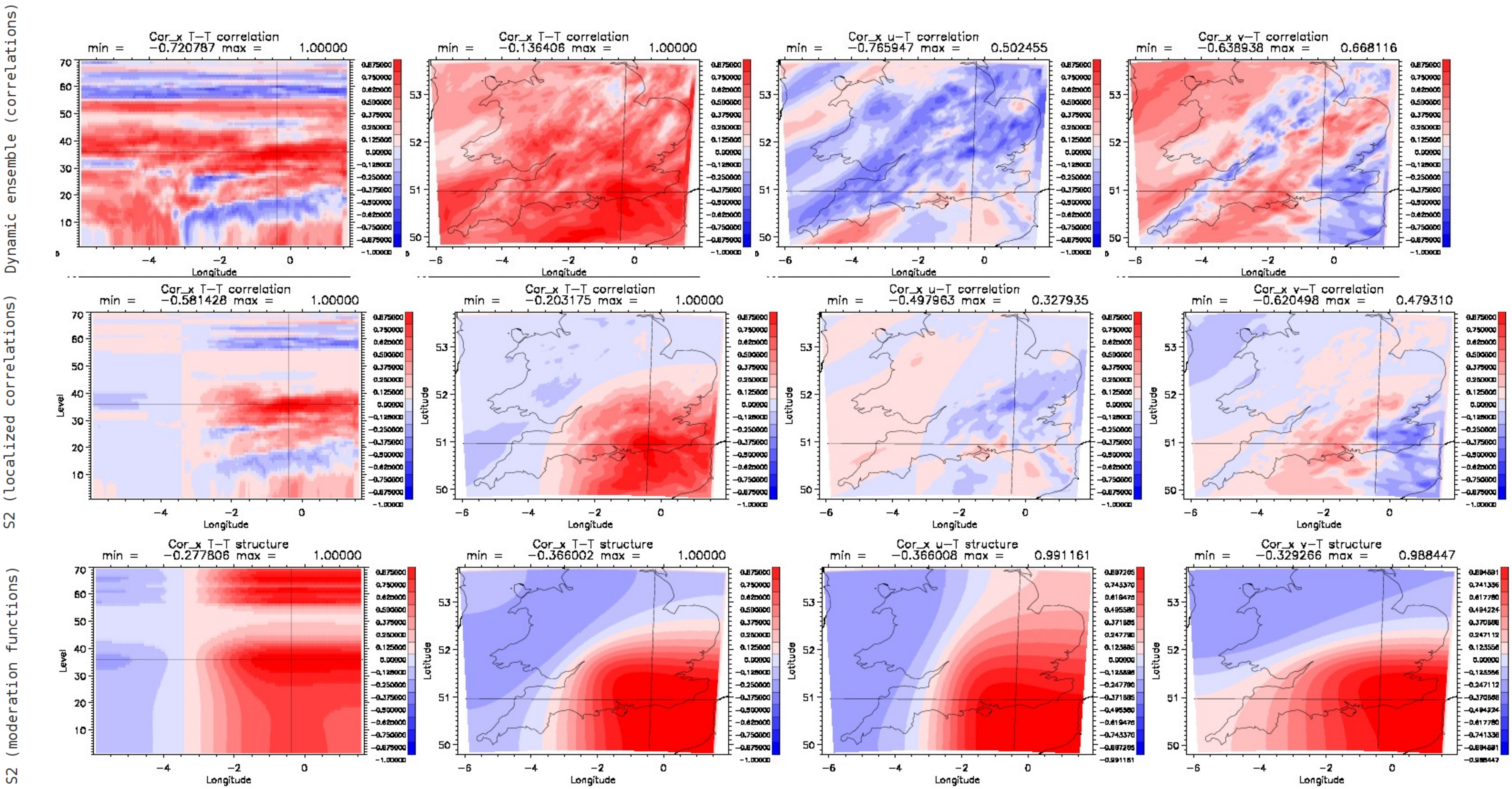
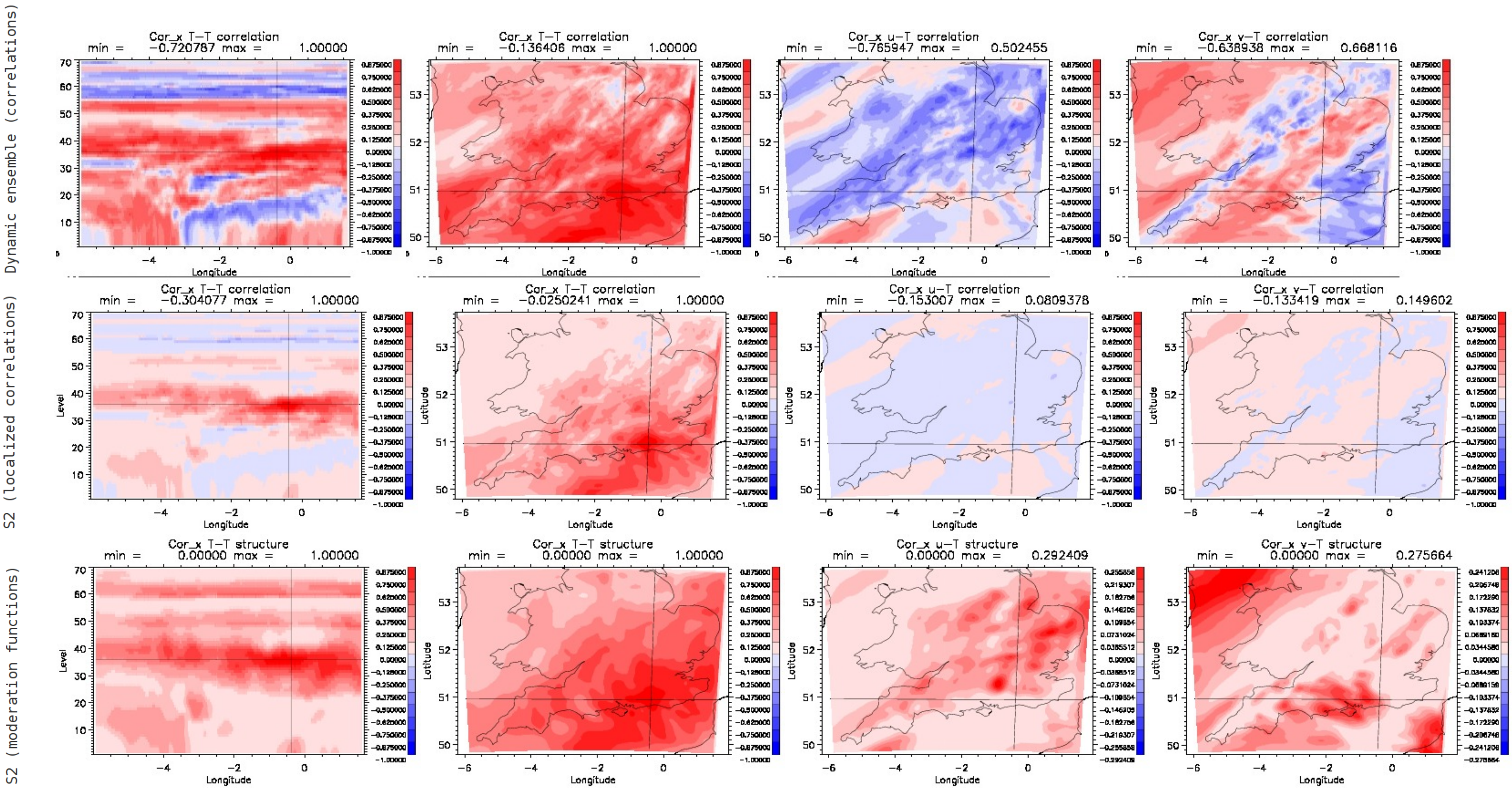
(a) T-T (long/ht), (b) T-T (long/lat), (c) u-T (long/lat), (d) v-T long lat



5 Implied structure functions and balance diagnostics (adaptive localization, SENCORP)

(a) T-T (long/ht), (b) T-T (long/lat), (c) u-T (long/lat), (d) v-T long lat



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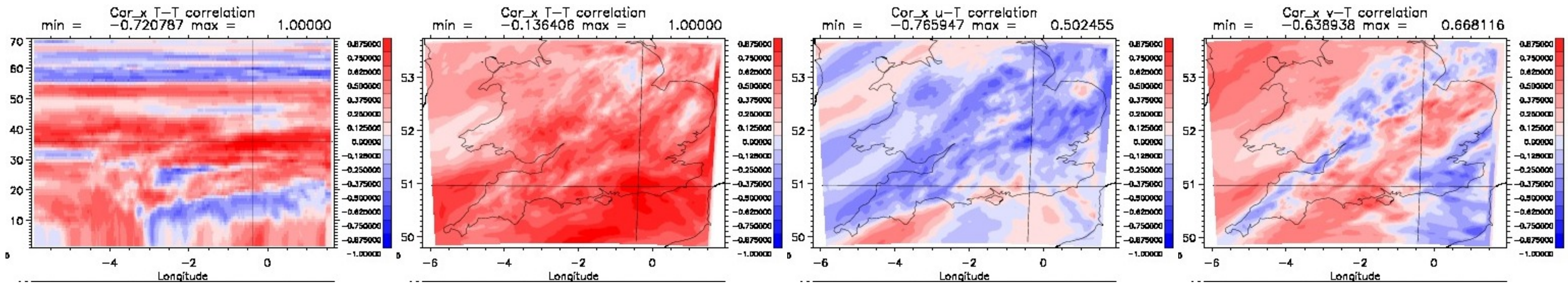
Dynamic ensemble (correlations)



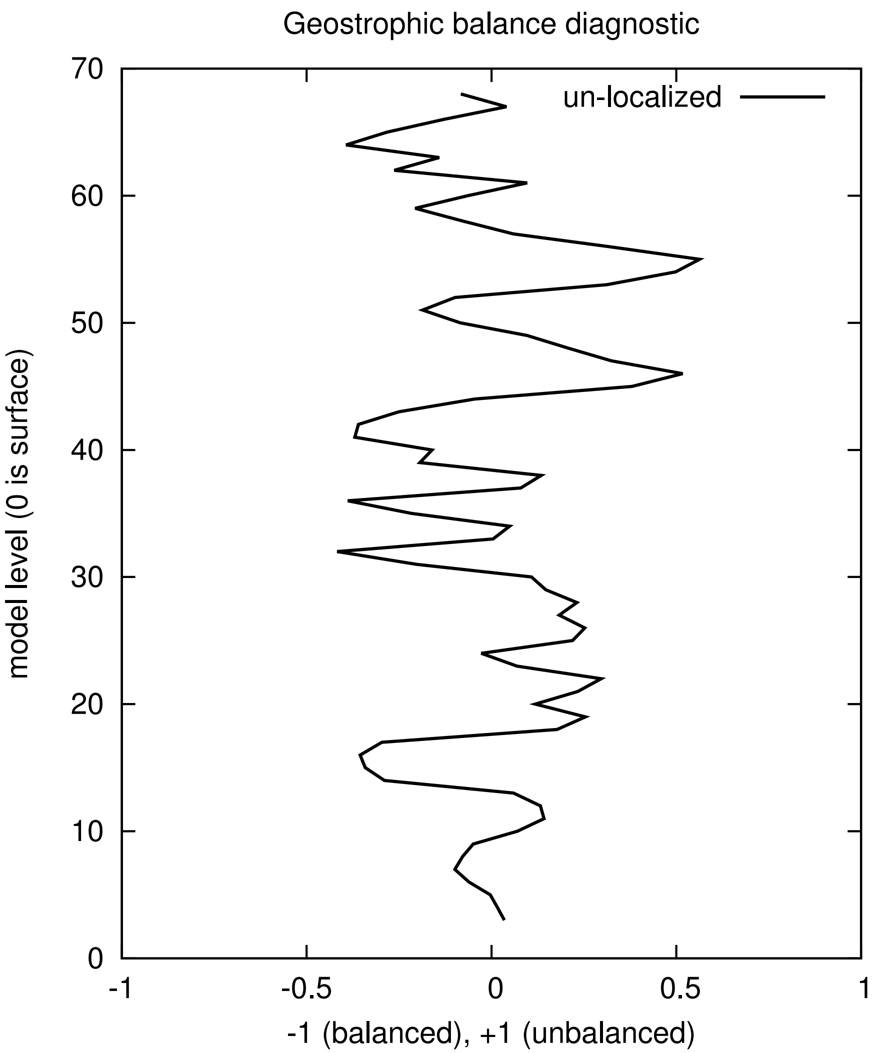
E2 (localized correlations)

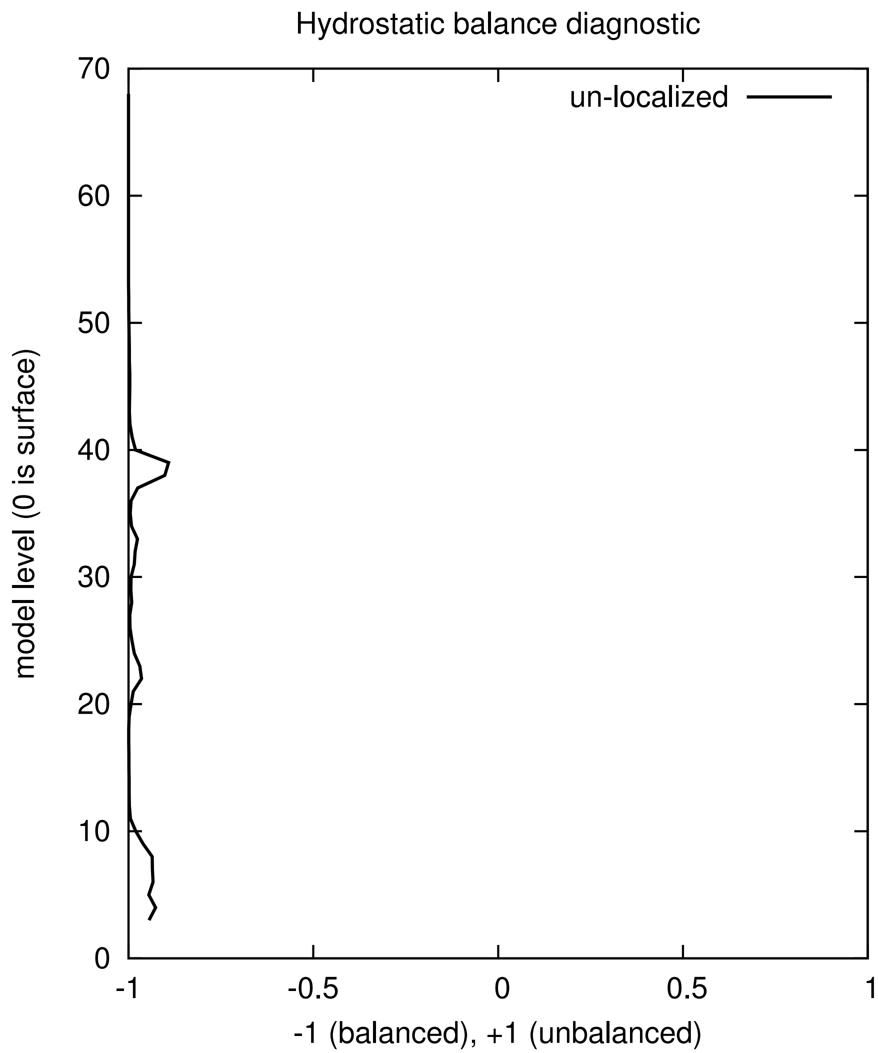
Fields not computed

E2 (moderation functions)

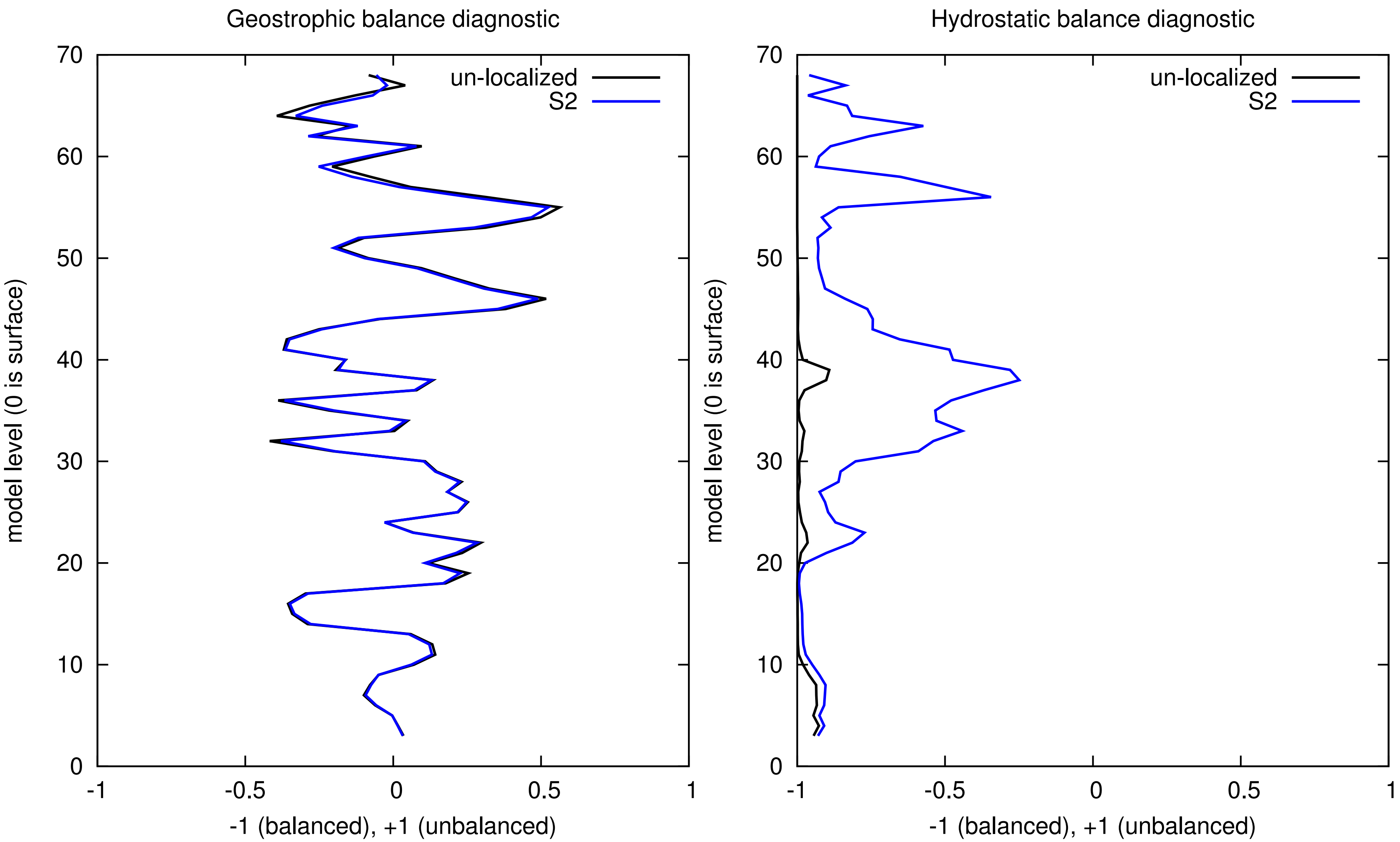
Fields not computed

5 Implied structure functions and balance diagnostics (dynamic ensemble)

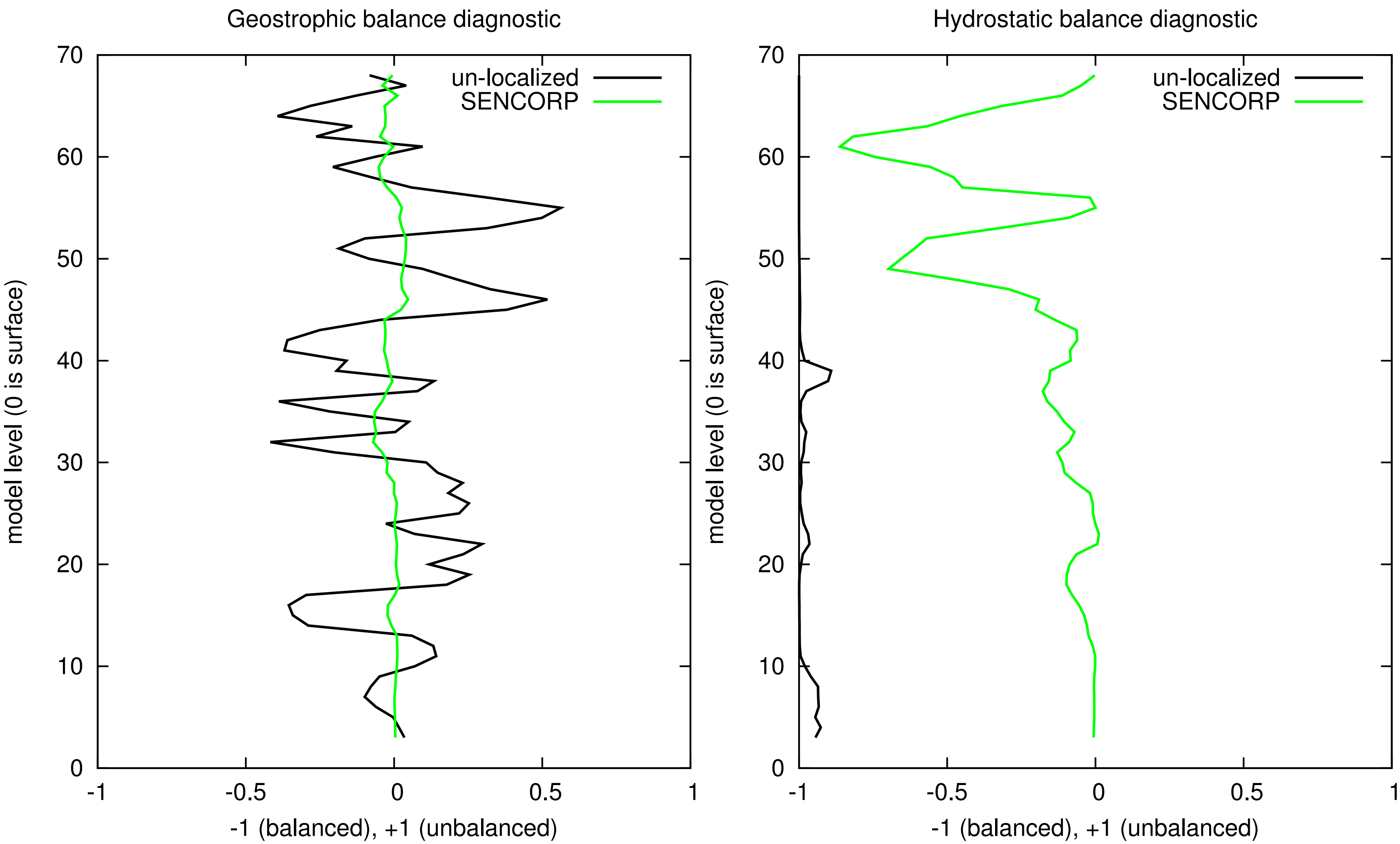




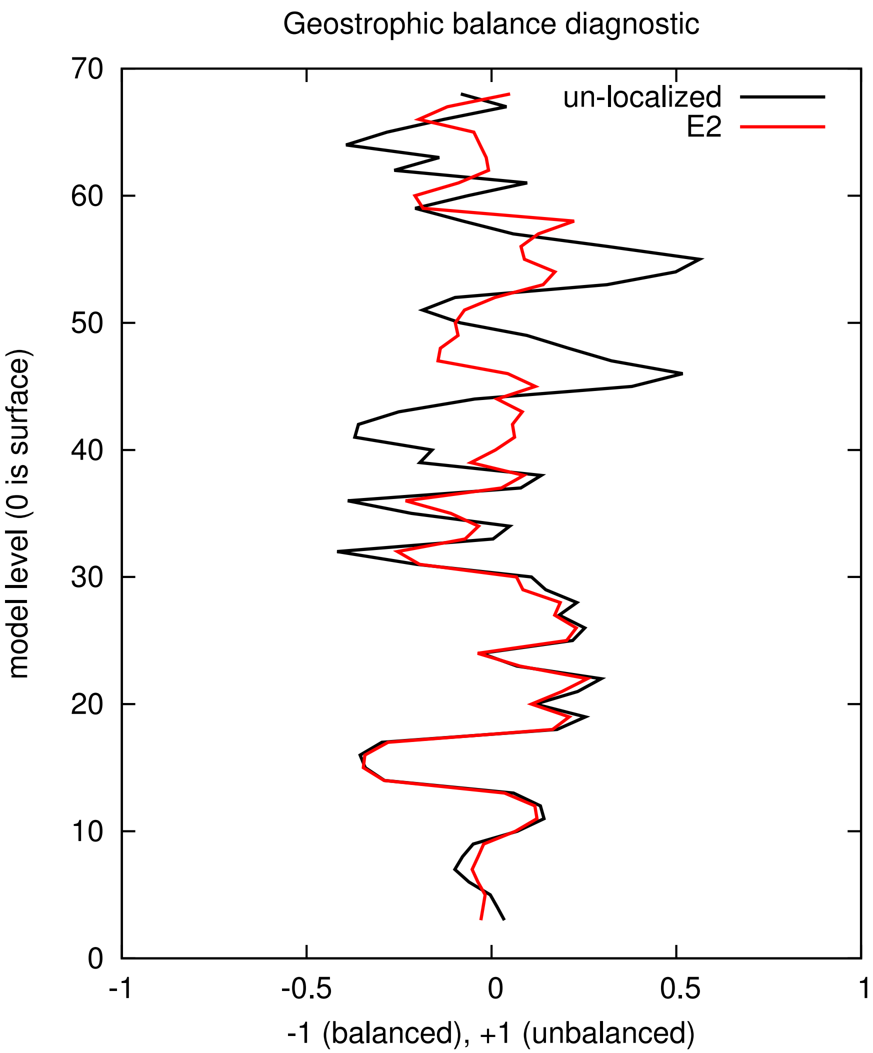
5 Implied structure functions and balance diagnostics (static localization, S2)

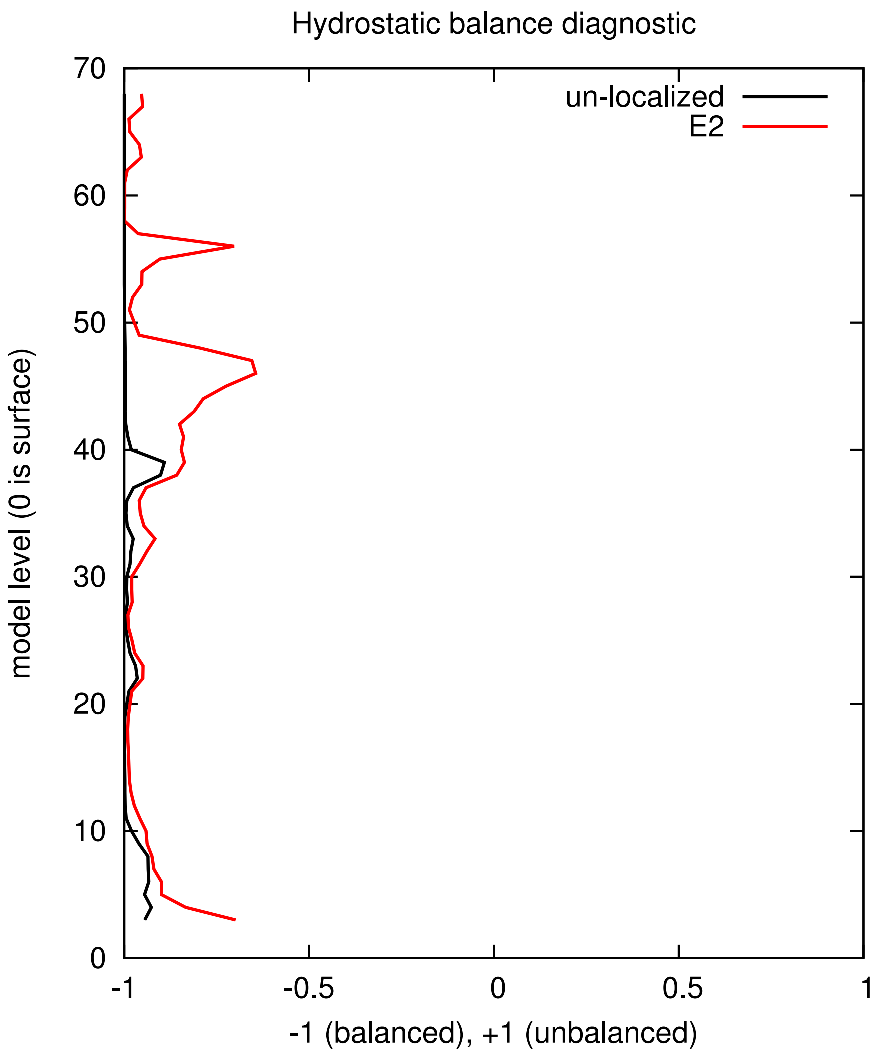


5 Implied structure functions and balance diagnostics (adaptive localization, SENCORP)



5 Implied structure functions and balance diagnostics (adaptive localization, E2)





6 Conclusions

- Balanced analyses for NWP.
  - ▷ Crude DA does not respect balance.
  - ▷ Not imposing balance when it should be, . . ., and imposing balance when it shouldn't be.
  - ▷ Adding localization to EnKF disturbs balance.
- Balance diagnostics.
  - ▷ Given an ensemble, find the correlation between leading terms.
  - ▷ Apply to dynamical (raw) ensemble.
  - ▷ Apply to localized ensemble (combining of dynamical and correlation ensemble).
    - ▷ Correlation ensemble is 'square-root' of localization matrix.
- Localization schemes.
  - ▷ Static (spectral) scheme (S2).
  - ▷ SENCORP scheme (localization defined from smoothed ensemble).
  - ▷ ECORAP (combination of S2 and SENCORP) scheme (S2).
- Findings
  - ▷ SENCORP doesn't perform well.
  - ▷ S2 performs well for geos. balance.
  - ▷ E2 performs will for hydro. balance.
- Notes
  - ▷ Many parameters (truncation, length-scales, order Q, other things).
  - ▷ Look at other profiles and cases.
  - ▷ Is it worth the computational effort?
  - ▷ Other balances (anelastic, moisture, . . .).