STATISTICAL CONSIDERATIONS IN ATMOSPHERIC DATA ASSIMILATION

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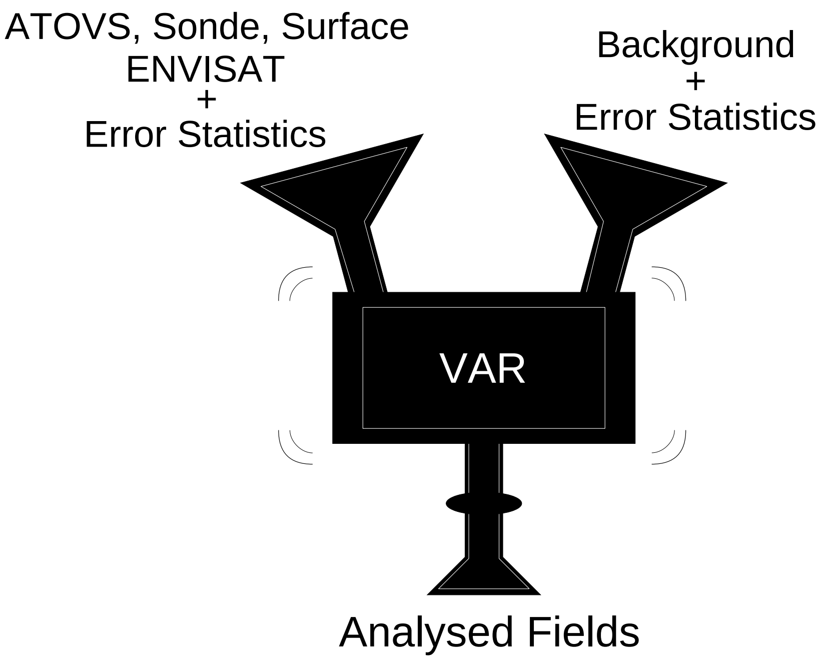


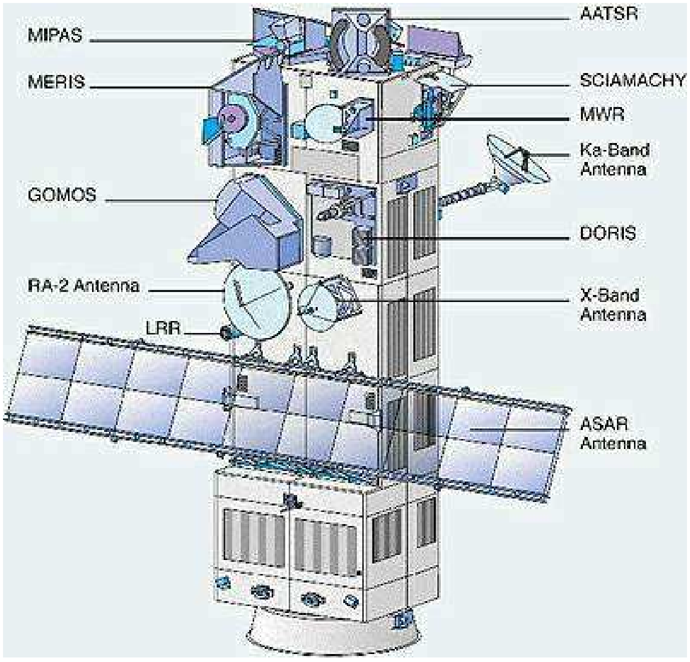
In data assimilation we must:

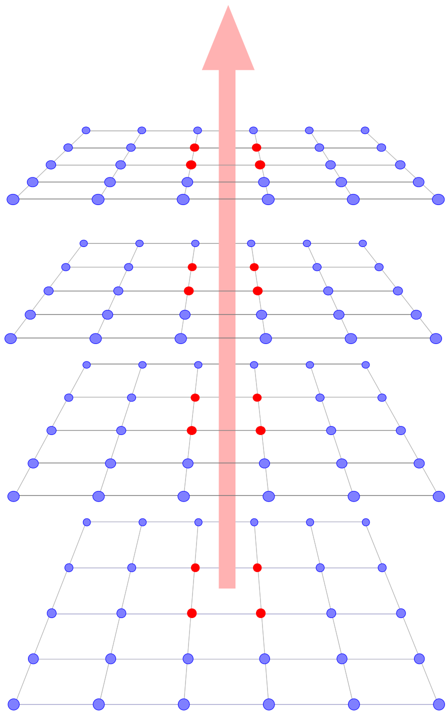
- Reproduce accurately observations via the observation operators.
- Represent realistically the error statistics of all information.

Some of DARC's activities:

1. How can we best assimilate data from ENVISAT?
2. How can we use physics to better estimate uncertainties in the background fields?

1. How can we best assimilate data from ENVISAT?





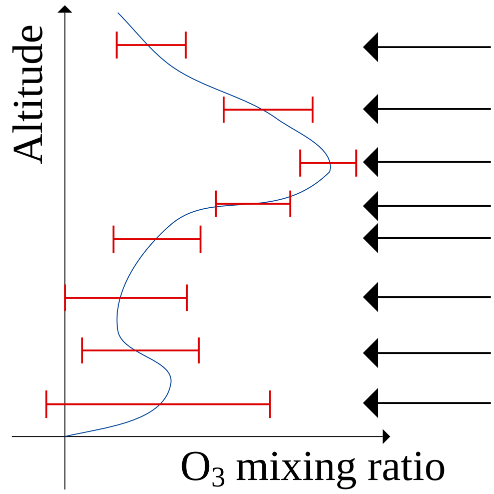
Observational data:

$\vec{y}$: vector of retrieved profiles from satellite group,

$\mathbf{S}_y$: error covariance matrix of $\vec{y}$.

Existing approach at Met Office (obsolete)

Assimilate $\vec{y}$ by interpolation
and with fixed and diagonal $\mathbf{S}_y$.



PROBLEMS:

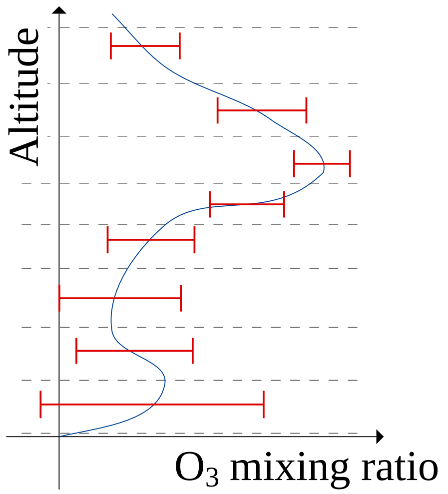
- Retrieved profile does not represent point values of O_3.
- Errors are correlated and profile dependent.

DARC approach #1 (implemented)

Assimilate $\vec{y}$ by layer averaging
and with fixed and diagonal $\mathbf{S}_y$.

PROBLEMS:

- Retrieved profile does not represent simple layer averaged O_3.
- Errors are correlated and profile dependent.
- Difficult to implement for humidity (R.H.).



DARC approach #2 (planned) - S.Migliorini, C.Rodgers

Assimilate $\vec{y}$ by averaging kernels and with full error statistics.

Retrieved state and 'truth'

$$\vec{y} = \mathbf{A}\vec{x}_t + (\mathbf{I} - \mathbf{A})\vec{x}_a + \overrightarrow{\text{error}},$$

$$\vec{x}_t : \text{'truth'}, \quad \vec{x}_a : \text{a-priori}, \quad \mathbf{A} : \text{averaging kernel} = \frac{\partial \vec{y}}{\partial \vec{x}_t}.$$

Modification #1, assimilate:

$$\vec{y}_1 = \vec{y} - (\mathbf{I} - \mathbf{A})\vec{x}_a \qquad (= \mathbf{A}\vec{x}_t),$$

with error covariance:

$$\mathbf{E} = \mathbf{G}\mathbf{R}\mathbf{G}^T$$

$$\text{where} \quad \mathbf{G} = \mathbf{S}\mathbf{K}^T(\mathbf{K}\mathbf{S}\mathbf{K}^T + \mathbf{R})^{-1}$$

$$\text{and} \quad \mathbf{A} = \mathbf{G}\mathbf{K}$$

$$\mathbf{K} : \text{weighting function in retrieval,}$$

$$\mathbf{S} : \text{error cov. of } \vec{x}_a, \quad \mathbf{R} : \text{error cov. of radiances.}$$

Modification #2, assimilate prewhitened profiles:

$$\vec{y}_2 = (\Lambda^{-1/2}\mathbf{L})(\vec{y} - (\mathbf{I} - \mathbf{A})\vec{x}_a)$$

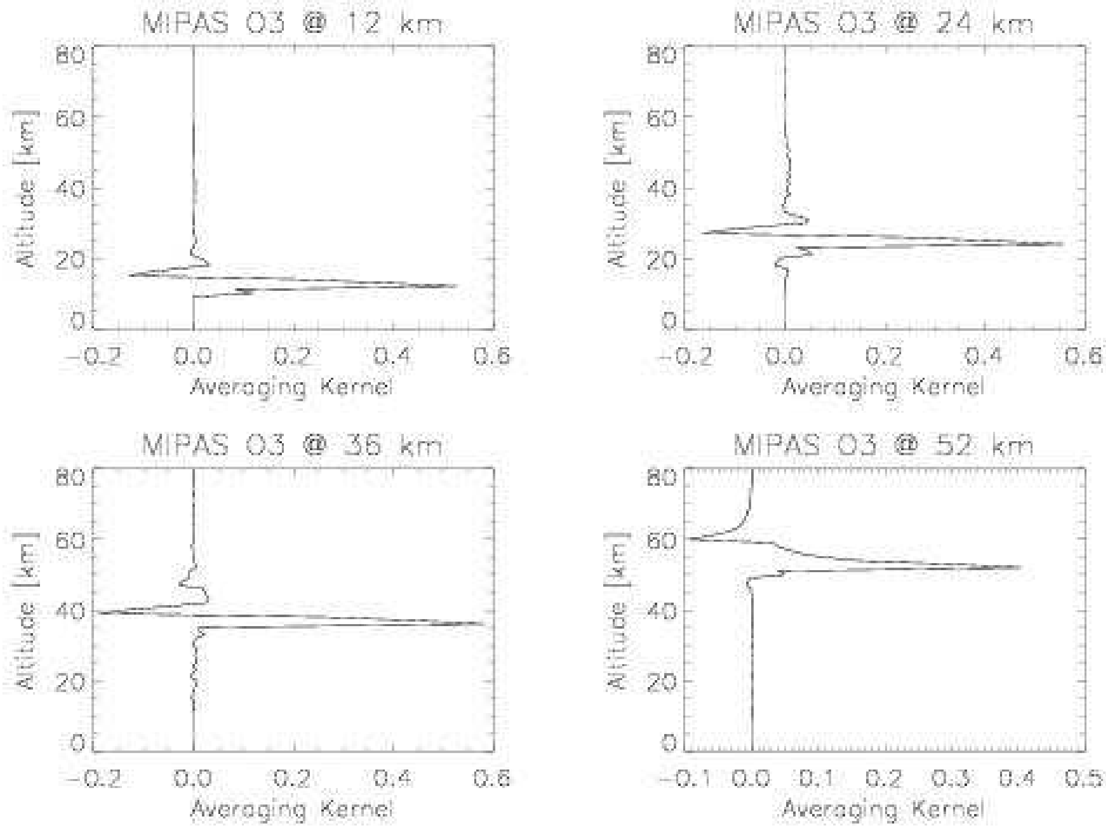
$$\text{prewhitening transformation} \quad \Lambda^{-1/2}\mathbf{L} = \mathbf{E}^{-1/2}$$

ENVISAT contribution to J_O:

$$J_O = \frac{1}{2}\left(\vec{y}_2 - \underbrace{(\Lambda^{-1/2}\mathbf{L}\mathbf{A})}\vec{x}\right)^T \mathbf{I} \left(\vec{y}_2 - \underbrace{(\Lambda^{-1/2}\mathbf{L}\mathbf{A})}\vec{x}\right)$$

observation operator in Var. (forward model) — unit error covariance matrix

Example Averaging Kernels for MIPAS Ozone



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2. How can we use physics to better estimate uncertainties in the background fields?

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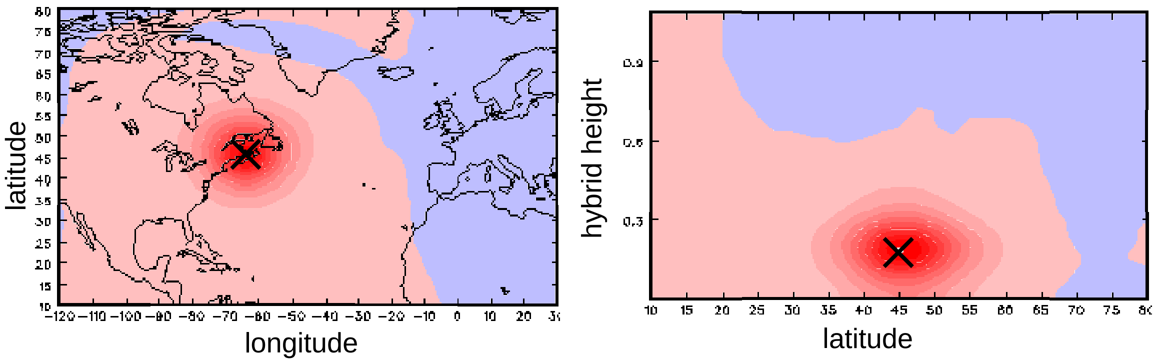
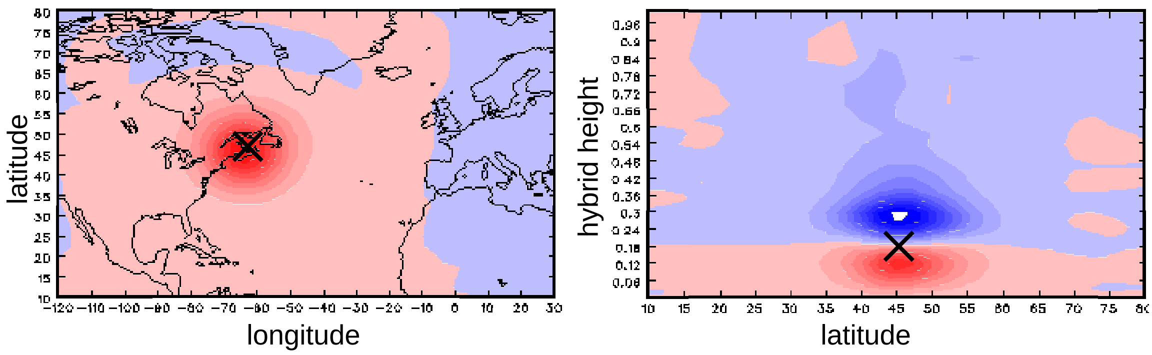
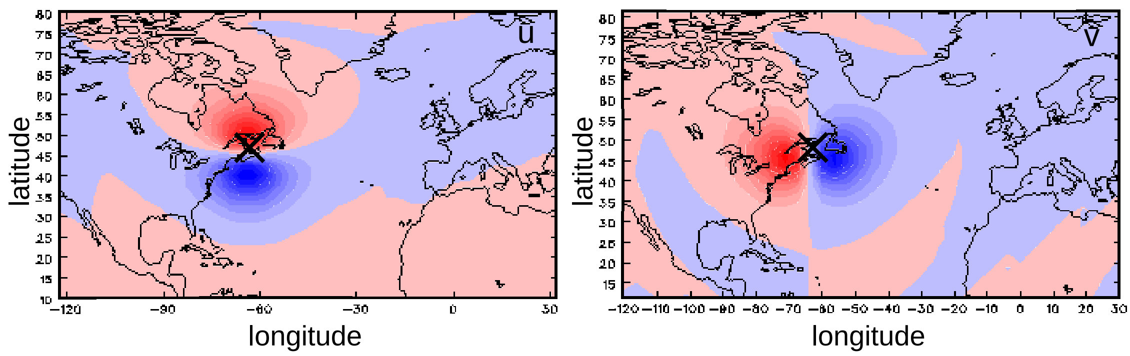
Pressure-Pressure & Pressure-Theta



Theta-Pressure Covariances



Horizontal wind-Pressure Covariances

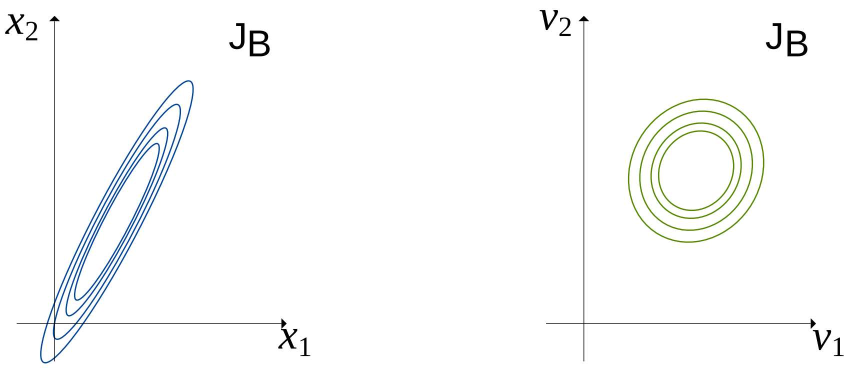


Var. uses control variables, $\vec{v}'$, prewhitened according to B.

$$\vec{v}' = \mathbf{T}\vec{x}'$$

$$\mathbf{T} = \Lambda^{-1/2}\mathbf{L}$$

$$\mathbf{L} \; : \; \text{eigenvectors of } \mathbf{B}, \quad \Lambda \; : \; \text{eigenvalues.}$$



BADLY CONDITIONED (model space)

WELL CONDITIONED (control variable space)

Part of the transformation between $\vec{x}$ and $\vec{v}$ spaces is to choose alternative parameters.

<u>Pragmatic approach (*engineering*)</u>

- $\vec{v}_1$ - capture most of flow - (ψ).
- $\vec{v}_2$ - capture most of remaining part of flow - (χ).
- $\vec{v}_3$ - capture most of remaining part of flow - (${}^{A}p$).
- etc.

<u>Theoretical approach (*physics*)</u>

Choose parameters that are uncorrelated, spanned by mutually exclusive normal modes.

- $\vec{v}_1$ - 'balanced' streamfunction - slow manifold - (s).
- $\vec{v}_2$ - unbalanced part of vortical flow (${}^{U}p$).
- $\vec{v}_3$ - divergent part of flow (χ).

Statistics are accumulated for each parameter