THE DISPLACEMENT OF THE POLAR POINTS IN SPHERICAL CALCULATIONS

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The formula for the grid box area about a grid point centred on latitude ϕ is,

$$A_1(\phi) = R^2 \cos\phi\delta\lambda\delta\phi, \tag{1}$$

where R is the Earth's radius and $\delta\lambda$ and $\delta\phi$ is the angular longitudinal and latitudinal grid box size (see Fig. 1).

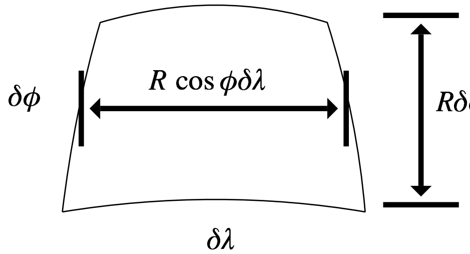

Fig. 1: A typical grid box on the sphere. The point in question is at the centre and the boundaries of the box are half way to the neighbouring points of the same staggering.

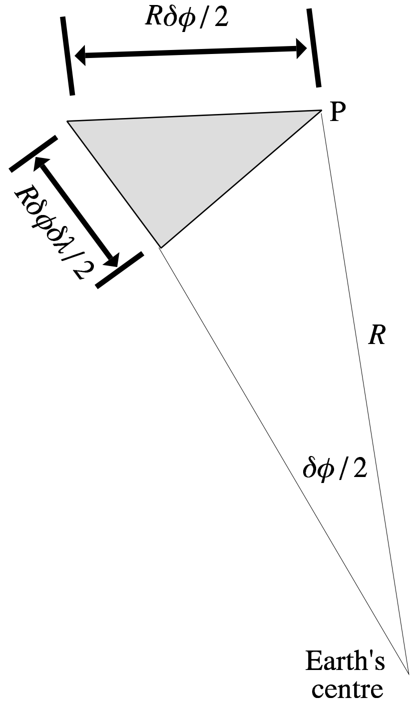

Fig. 2: The grid box near the pole (shaded). P denotes the pole itself

The geometry of the special grid box at the pole is different to the general case (see Fig. 2), which leads to a different formula for the area,

$$A_2(\phi = \pi/2) = \frac{R^2\delta\phi^2\delta\lambda}{8}. \tag{2}$$

We would like Eq. 1 to be used at the pole, but in order for it to give the same result as Eq. 2, we must imagine displacing the polar points slightly away from the pole (to be done in the A_1 area formula only). We let the effective latitude at which the polar points are imagined to be situated be ϕ_{eff}. This has the definition,

$$\phi_{eff} = \frac{\pi}{2} - \Delta, \tag{3}$$

where Δ is the displacement angle. Equating Eqs. 1 and 2 under these circumstances and then cancelling gives,

$$\begin{aligned} R^2 \cos \phi_{eff} \delta\lambda \delta\phi &= \frac{R^2 \delta\phi^2 \delta\lambda}{8}, \\ \cos \phi_{eff} &= \frac{\delta\phi}{8}. \end{aligned} \tag{4}$$

Substituting Eq. 3 into 4, and noting that $\cos(\pi/2 - \Delta) = \sin \Delta$, at small Δ we have,

$$\Delta \approx \frac{\delta\phi}{8}. \tag{5}$$

Thus, if we consider the polar points to be situated $\delta\phi/8$ away from the pole then we can use the general formula (Eq. 1) when evaluating areas.