General relationships

$$\zeta' = \vec{k} \cdot \nabla \times \vec{u}' \qquad (\nabla \text{ is 3-D nabla operator})$$

$$\zeta' = \nabla_z^2 \psi' \qquad (\nabla_z \text{ is 2-D (horizontal) nabla operator})$$

$$\vec{u}' = \vec{k} \times \nabla \psi'$$

$$\psi' = \psi'_B + \psi'_U$$

$$p' = p'_B + p'_U$$

System 1: The balanced equations

$$q(\psi', p') = \frac{\theta_{0z}}{\rho_0}\zeta' - \frac{f\theta_{0z}(1-\kappa)}{\rho_0^2 R \Pi_0 \bar{\theta}_0} p' - \frac{f\theta_{0z}}{\rho_0 R \Pi_0 \bar{\theta}_0} \overline{\frac{\theta_0}{\Pi_{0z}} \frac{\partial}{\partial z}\left(\frac{\kappa \Pi_0}{p_0} p'\right)} + \frac{fg}{\rho_0 c_p \bar{\Pi}_{0z}^2} \frac{\partial^2}{\partial z^2}\left(\frac{\kappa \Pi_0}{p_0} p'\right) - \frac{2fg\Pi_{0zz}}{\rho_0 c_p \bar{\Pi}_{0z}^3} \overline{\frac{\partial}{\partial z}\left(\frac{\kappa \Pi_0}{p_0} p'\right)}$$

$$\Pi_0 = \left(\frac{p_0}{100000}\right)^{\kappa}$$

$$q(\psi', p') = q(\psi'_B, p'_B)$$

$$\nabla_z^2 p'_B = \nabla_z \cdot (f \rho_0 \nabla_z \psi'_B)$$

System 2: The unbalanced equations

$$\bar{q}(\psi', p') = \nabla_z \cdot (f \rho_0 \nabla_z \psi') - \nabla_z^2 p'$$

$$\bar{q}(\psi', p') = \bar{q}(\psi'_U, p'_U)$$

$$\frac{\theta_{0z}}{\rho_0}\zeta'_U = \frac{f\theta_{0z}(1-\kappa)}{\rho_0^2 R \Pi_0 \bar{\theta}_0} p'_U + \frac{f\theta_{0z}}{\rho_0 R \Pi_0 \bar{\theta}_0} \overline{\frac{\theta_0}{\Pi_{0z}} \frac{\partial}{\partial z}\left(\frac{\kappa \Pi_0}{p_0} p'_U\right)} - \frac{fg}{\rho_0 c_p \bar{\Pi}_{0z}^2} \frac{\partial^2}{\partial z^2}\left(\frac{\kappa \Pi_0}{p_0} p'_U\right) + \frac{2fg\Pi_{0zz}}{\rho_0 c_p \bar{\Pi}_{0z}^3} \overline{\frac{\partial}{\partial z}\left(\frac{\kappa \Pi_0}{p_0} p'_U\right)}$$