The Data Assimilation Research Centre

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A “Potential Vorticity” (PV)-based control parameter for background error covariance modelling

Uncertainty in prior knowledge (B)

$$J=\tfrac{1}{2}(\mathbf{x}-\mathbf{x_B})^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x_B})+J_0[\mathbf{y},\mathbf{x}]$$

We assume that forecast errors are:

- Unbiased
- Normally distributed
- Mulitvariately and auto-correlated

B has many roles in DA:

- Defines ‘closeness to forecast’ (regularization)
- Gives larger weight to parts of forecast better known
- Allows observations to be used synergistically
- Ensures analysis is smooth
- Imposes appropriate balance

A rotational wind constrained scheme

A rotational wind constrained scheme for the leading variable is common in Var.

$$\mathbf{v}=(\psi',{}^A p',\chi')^T$$

total streamfunction — unbal. pressure — velocity potential

$$\begin{pmatrix}u'\\v'\\p'\end{pmatrix}=\begin{pmatrix}-\partial/\partial y & 0 & \partial/\partial x\\ \partial/\partial x & 0 & \partial/\partial y\\ \mathrm{LBE} & \mathbf{I} & 0\end{pmatrix}\begin{pmatrix}\psi'\\ {}^A p'\\ \chi'\end{pmatrix}$$

LBE = Linear Bal. Eq.

ψ', which is taken to be the ‘balanced’ variable is not universally balanced

How are forecast errors treated?

Control variable transform:

$$\mathbf{x}'=\mathbf{x}-\mathbf{x}_B=\mathbf{B}_0^{1/2}\mathbf{v}$$

$$J[\mathbf{v}]=\tfrac{1}{2}\mathbf{v}^T\mathbf{v}+J_0[\mathbf{y},\mathbf{x}_B+\mathbf{B}_0^{1/2}\mathbf{v}]$$

Elements of **v** are uncorrelated

Implied **B**: $\mathbf{B}_0^{1/2}\mathbf{B}_0^{T/2}$

If $\mathbf{B}_0$ is a good model then $\mathbf{B}\approx\mathbf{B}_0$

A PV constrained scheme

A PV constrained scheme is more valid for the leading control variable than the rotational wind scheme.

$$\mathbf{v}=(s',{}^U p',\chi')^T$$

bal. streamfunction — unbal. pressure — velocity potential

$$\begin{pmatrix}u'\\v'\\p'\end{pmatrix}=\begin{pmatrix}\mathbf{U}_s & \mathbf{U}_{up} & \mathbf{U}_\chi\end{pmatrix}\begin{pmatrix}s'\\ {}^U p'\\ \chi'\end{pmatrix}$$

s’, is universally ‘balanced’ in synoptic and large-scale data assimilation

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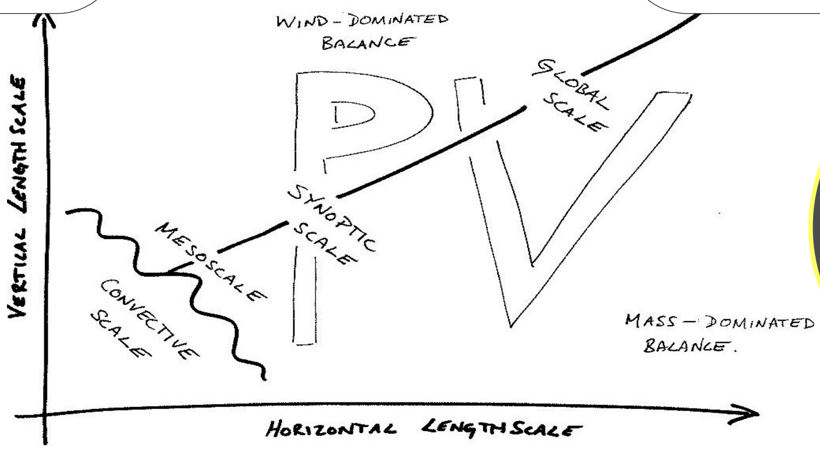

Copies of this poster are available from

www.met.rdg.ac.uk/~ross/DataAssim.html

The balanced transform

$$\begin{pmatrix}u'_s\\v'_s\\p'_s\end{pmatrix}=\mathbf{U}_s s'=\begin{pmatrix}-\partial s'/\partial y\\ \partial s'/\partial x\\ \nabla^{-2}\nabla\cdot(f\rho_0\nabla s')\end{pmatrix}$$

LBE

The ‘balanced streamfunction’ is associated with PV’.

$$\overline{PV}'=0,\ \nabla\cdot\vec{u}'=0$$

The rotational unbalanced transform

$$\begin{pmatrix}u'_{up}\\v'_{up}\\p'_{up}\end{pmatrix}=\mathbf{U}_{up}{}^U p'=\begin{pmatrix}\mathbf{R}\,{}^U p'\\ \mathbf{S}\,{}^U p'\\ {}^U p'\end{pmatrix}$$

R and S: operators giving winds that have no linear PV’

The ‘unbalanced pressure’ is associated with anti-PV (residual of LBE).

$$\overline{PV}'=\nabla\cdot(f\rho_0\nabla\psi')-\nabla^2 p'$$

$$PV'=0,\ \nabla\cdot\vec{u}'=0$$

Unbalanced divergent transform

$$\begin{pmatrix}u'_\chi\\v'_\chi\\p'_\chi\end{pmatrix}=\mathbf{U}_\chi\chi=\begin{pmatrix}\partial\chi/\partial x\\ \partial\chi/\partial y\\ 0\end{pmatrix}$$

The ‘velocity potential’ is associated with the divergence.

$$PV'=0,\ \overline{PV}'=0$$

The inverse transform

The inverse transform is needed to calibrate the statistics.

Let $\mathbf{A}x'=\begin{pmatrix}PV'\\ \overline{PV}'\\ \nabla\cdot\vec{u}\end{pmatrix}=\begin{pmatrix}\mathbf{A}^*_{PV}\\ \mathbf{A}^*_{\overline{PV}}\\ \mathbf{A}^*_{\nabla\cdot u}\end{pmatrix}x'$. Find $\mathbf{AU}x'$:

$$\begin{pmatrix}PV'\\ \overline{PV}'\\ \nabla\cdot\vec{u}'\end{pmatrix}=\begin{pmatrix}\mathbf{A}^*_{PV}\mathbf{U}_s & & \\ & \mathbf{A}^*_{\overline{PV}}\mathbf{U}_{up} & \\ & & \mathbf{A}^*_{\nabla\cdot u}\mathbf{U}_\chi\end{pmatrix}\begin{pmatrix}s'\\ {}^U p'\\ \chi'\end{pmatrix}$$

Use GCR method to solve three uncoupled equations.

Example transform for shallow water equations (f-plane)

Linear balance equation: $\phi-f\psi=0$ $\qquad u'=-\partial(\nabla^{-2}\zeta)/\partial y$

Linearized potential vorticity: $PV=\phi_0\zeta-f\phi$ $\qquad v'=\partial(\nabla^{-2}\zeta)/\partial x$

$$\begin{pmatrix}u'\\v'\\\phi'\end{pmatrix}=\begin{pmatrix}-\partial/\partial y & -\frac{f}{\phi_0}\partial\nabla^{-2}/\partial y & \partial/\partial x\\ \partial/\partial x & \frac{f}{\phi_0}\partial\nabla^{-2}/\partial x & \partial/\partial y\\ f & \mathbf{I} & 0\end{pmatrix}\begin{pmatrix}\mathbf{B}_s^{1/2}s'\\ \mathbf{B}_{u\phi}^{1/2}v'\\ \mathbf{B}_{\nabla\cdot u}^{1/2}\phi'\end{pmatrix}$$