The Implementation of Potential Vorticity as a Leading Control Variable in Var.

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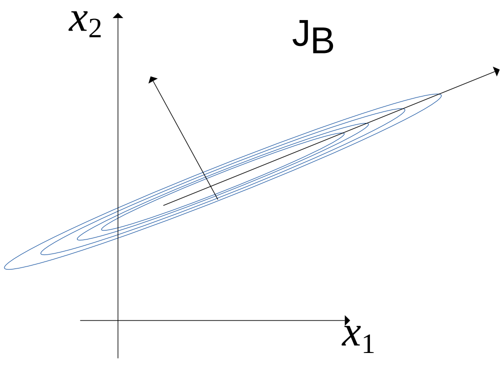
Why not use model variables as control parameters?

$$J[\vec{x}] = J_B + J_O$$

$$= \frac{1}{2}\vec{x}'^T \mathbf{B}^{-1}\vec{x}' + \frac{1}{2}(\vec{H}[\vec{x}' + \vec{x}_B] - \vec{y})^T \mathbf{R}^{-1}(\vec{H}[\vec{x} + \vec{x}_B] - \vec{y})$$

where $\vec{x}' = \vec{x} - \vec{x}_B$

- $\mathbf{B}$ (in $\vec{x}$-space) contains > 10^{14} elements and cannot be represented explicitly.
- $\mathbf{B}$ (in $\vec{x}$-space) is badly conditioned max e.v. / min e.v. $\sim 10^{10}$.



BADLY CONDITIONED
(model space)

Solution: for variational data assimilation vary weights of the eigenvectors of $\mathbf{B}$ (instead of components of $\vec{x}$).

$$\vec{v}' = \mathbf{U}^{-1}\vec{x}'$$
$$\mathbf{U}^{-1} = \Lambda^{-1/2}\mathbf{L}^{T}$$
$$\mathbf{B} = \mathbf{U}\mathbf{U}^{T}$$

Λ diagonal matrix of e.values, columns of $\mathbf{L}$ are e.functions.

$$J[\vec{v}'] = \frac{1}{2}\vec{v}'^{T}\vec{v}' + \frac{1}{2}(\vec{H}[\mathbf{U}\vec{v}' + \vec{x}_B] - \vec{y})^{T}\mathbf{R}^{-1}(\vec{H}[\mathbf{U}\vec{v}' + \vec{x}_B] - \vec{y})$$

This problem is much better conditioned.

But, this can't be done directly

$$\vec{v}' = \mathbf{U}^{-1}\vec{x}' = \mathbf{U}_h^{-1}\mathbf{U}_v^{-1}\mathbf{U}_p^{-1}\vec{x}'$$

↑ ★parameter transform★

↑ vertical transform

↑ horizontal transform

The role of $\mathbf{U}_p^{-1}$ is to 'block-diagonalize' the multi-variate correlations.

What parameters?

Existing scheme (pragmatic/engineering approach)

Subspace	Parameter	
$\vec{v}'_1$	ψ	Captures most of the flow …
$\vec{v}'_2$	χ	Captures most of the rest of the flow …
$\vec{v}'_3$	${}^{A}p$	Captures most of the rest of the flow …

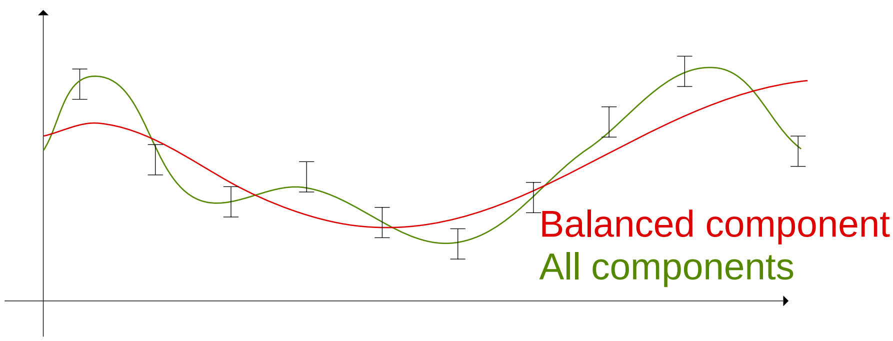
These are orthogonal but not uncorrelated

Proposed scheme (physics approach)

Subspace	Parameter	
$\vec{v}'_1$	s	"Balanced" / "slow manifold" (PV) …
$\vec{v}'_2$	${}^{U}p$	Captures most of the rest of the flow …
$\vec{v}'_3$	χ	Captures most of the rest of the flow …

- These are orthogonal, but are expected to be only weakly correlated.
- The new parameters are thought to evolve independently, each occupying a separate region in normal mode space.

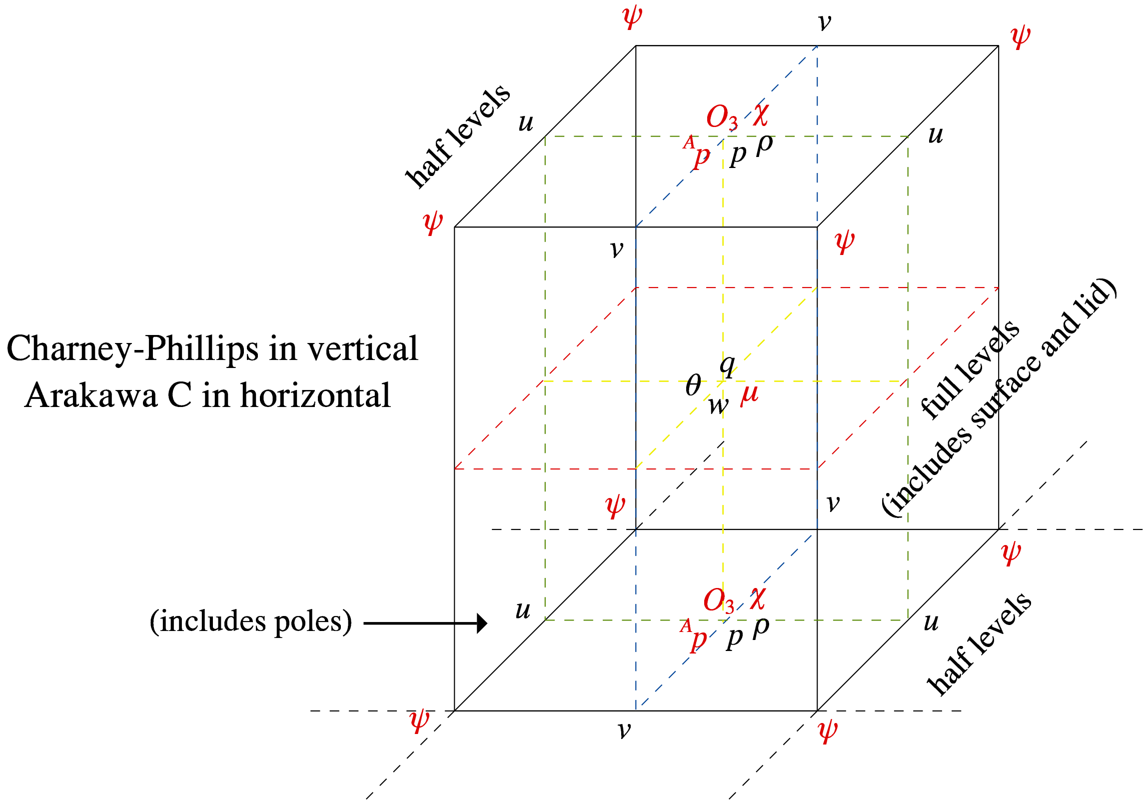
Why not assimilate using only the leading parameter?



What grid staggering for new parameters?

$$PV, \quad \bar{P}V, \quad \nabla \cdot \vec{v}, \quad s, \quad {}^{U}p, \quad \chi$$

Met Office Var. Grid Staggering
(black: model variables; red: existing Var. parameters)



There is one more full level than half levels