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Model errors for MetUM winds – can we estimate these well with dense observations?

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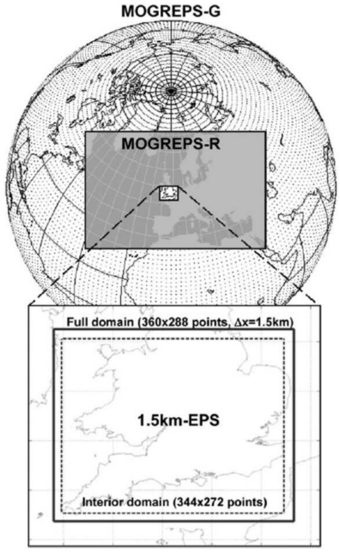
- The Met Office has a high resolution MetUM over the UK.
- Little is known about the statistics of its errors.
- Diagnosing model error statistics is difficult!

There are different aspects to errors in general:

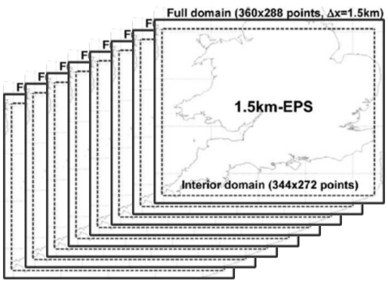
- They can have random and systematic components.
- They may be additive or multiplicative
- They may be Gaussian or non-Gaussian.
- They may be homogeneous or inhomogeneous.
- They may be isotropic or anisotropic.
- They may be correlated or uncorrelated.

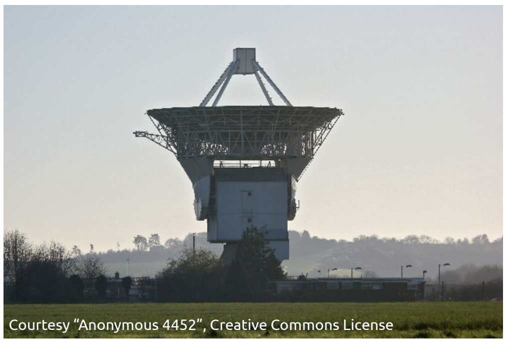


Courtesy Caron (2013), ©Crown Copyright 2013, Met Office

We have for one day (20/09/11):

- 1.8 million Doppler radar observations (Chilbolton Observatory).
- High-resolution forecasts (1.5km grid-length) over the Southern UK.
- A 93-member ensemble of forecasts.





Courtesy "Anonymous 4452", Creative Commons License

Assumptions:

	Random only?	Additive?	Gaussian?	Homo-geneous?	Isotropic?	Mutually uncorrel-ated?	Cross correl-ated?
Model errors	✓	✓	✓	✓	✓	✗	✗
Predictability errors	✓	✓	✓	✓	✓	✗	✗
Observation errors	✓	✓	✓	✓	✓	✓	✗

… plus that the ensemble is correctly spread, and that observations can be treated as 'point-like'. These assumptions are not always justified.

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General definitions

Forecast, analysis, or observation = 'Truth' + Error realisation

Innovation = Observation - Forecast 'observation'

Covariance = < (Quantity 1 – mean of Q1) × (Quantity 2 – mean of Q2) >

Specific definitions

Forecast (at time t)

$$\mathbf{x}_t^{\mathrm{f}} = \mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\epsilon}_t^{\mathrm{f}}$$

Analysis (at $t = 0$)

$$\mathbf{x}_0^{\mathrm{a}} = \mathbf{x}_0^{\mathrm{t}} + \boldsymbol{\epsilon}_0^{\mathrm{a}}$$

Observations

$$\mathbf{y}_t = \mathcal{H}_t(\mathbf{x}_t^{\mathrm{t}}) + \boldsymbol{\epsilon}_t^{\mathrm{ob}}$$

Observation operator applied to forecast

$$\begin{aligned} \mathcal{H}_t(\mathbf{x}_t^{\mathrm{f}}) &= \mathcal{H}_t(\mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\epsilon}_t^{\mathrm{f}}) \\ &\approx \mathcal{H}_t(\mathbf{x}_t^{\mathrm{t}}) + \mathbf{H}_t\boldsymbol{\epsilon}_t^{\mathrm{f}} \end{aligned}$$

Forecast model (no stochastic scheme)

$$\mathcal{M}(\mathbf{x}_{t-1}^{\mathrm{t}}) = \mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\eta}_t$$

Using the definitions

Applying the forecast model

$$
\begin{aligned}
\mathbf{x}_t^{\mathrm{f}} &= \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{a}}) \\
&= \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{t}} + \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}) \\
&\approx \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{t}}) + \mathbf{M}_t\boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \\
\mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\epsilon}_t^{\mathrm{f}} &= \mathbf{x}_t^{\mathrm{t}} + \underbrace{\boldsymbol{\eta}_t}_{\text{model error}} + \underbrace{\mathbf{M}_t\boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}}_{\text{predictability error}}
\end{aligned}
$$

Innovations

$$
\begin{aligned}
\mathbf{d}_t^{\mathrm{o,b}} &= \mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^{\mathrm{f}}) \\
&\approx \mathcal{H}_t(\mathbf{x}_t^{\mathrm{t}}) + \boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathcal{H}_t(\mathbf{x}_t^{\mathrm{t}}) - \mathbf{H}_t\boldsymbol{\epsilon}_t^{\mathrm{f}} \\
&= \boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathbf{H}_t\boldsymbol{\epsilon}_t^{\mathrm{f}}
\end{aligned}
$$

Ensemble perturbations

$$
\begin{aligned}
\boldsymbol{\delta}_t^{\mathrm{f}} &\approx \mathbf{x}_t^{\mathrm{f}} - \langle \mathbf{x}_t^{\mathrm{f}} \rangle \\
&= \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{a}}) - \langle \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{a}}) \rangle \\
&= \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{t}} + \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}) - \langle \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{t}} + \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}) \rangle \\
&\approx \mathcal{M}_t(\mathbf{x}_{t-1}^{\mathrm{t}}) + \mathbf{M}_t\boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} - \langle \mathcal{M}_{t-1\to t}(\mathbf{x}_{t-1}^{\mathrm{t}}) \rangle \\
&= \mathbf{M}_t\boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}.
\end{aligned}
$$

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Procedure – interpretation of Daley (1992)

Daley, Roger (1992) The effect of serially correlated observation and model error on atmospheric data assimilation, Monthly weather review 120 (1), 164 – 177.

1. Analysis of innovations

$$
\begin{aligned}
\left\langle \mathbf{d}_t^{\mathrm{o,b}} \mathbf{d}_t^{\mathrm{o,b}\,\mathrm{T}} \right\rangle &= \left\langle \left(\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^{\mathrm{f}})\right) \left(\mathbf{y}_t - \mathcal{H}_t(\mathbf{x}_t^{\mathrm{f}})\right)^{\mathrm{T}} \right\rangle \\
&= \left\langle \left(\boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathbf{H}_t \boldsymbol{\epsilon}_t^{\mathrm{f}}\right) \left(\boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathbf{H}_t \boldsymbol{\epsilon}_t^{\mathrm{f}}\right)^{\mathrm{T}} \right\rangle \\
&= \left\langle \left(\boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathbf{H}_t \boldsymbol{\eta}_t - \mathbf{H}_t \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}\right) \left(\boldsymbol{\epsilon}_t^{\mathrm{ob}} - \mathbf{H}_t \boldsymbol{\eta}_t - \mathbf{H}_t \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}\right)^{\mathrm{T}} \right\rangle \\
&\approx \underbrace{\underbrace{\left\langle \boldsymbol{\epsilon}_t^{\mathrm{ob}} \boldsymbol{\epsilon}_t^{\mathrm{ob}\,\mathrm{T}} \right\rangle}_{\mathbf{R}_t}}_{\text{observation error part}} + \underbrace{\mathbf{H}_t \underbrace{\left\langle \boldsymbol{\eta}_t \boldsymbol{\eta}_t^{\mathrm{T}} \right\rangle}_{\mathbf{Q}_t} \mathbf{H}_t^{\mathrm{T}}}_{\text{model error part}} + \underbrace{\mathbf{H}_t \mathbf{M}_t \underbrace{\left\langle \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}\,\mathrm{T}} \right\rangle}_{\mathbf{A}_{t-1}} \mathbf{M}_t^{\mathrm{T}} \mathbf{H}_t^{\mathrm{T}}}_{\text{predictability error part}}
\end{aligned}
$$

Warning
Analysis of innovations will be susceptible to biases in obs and in model

2. Ensemble analysis (no localisation)

$$
\begin{aligned}
\left\langle \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{f}} - \langle \mathbf{x}_t^{\mathrm{f}} \rangle\right]\right) \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{f}} - \langle \mathbf{x}_t^{\mathrm{f}} \rangle\right]\right)^{\mathrm{T}} \right\rangle &\approx \Big\langle \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\eta}_t + \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} - \langle \mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\eta}_t + \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \rangle\right]\right) \times \\
&\qquad \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\eta}_t + \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} - \langle \mathbf{x}_t^{\mathrm{t}} + \boldsymbol{\eta}_t + \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \rangle\right]\right)^{\mathrm{T}} \Big\rangle \\
&= \Big\langle \left(\mathbf{H}_t \left[\mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} - \langle \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \rangle\right]\right) \times \\
&\qquad \left(\mathbf{H}_t \left[\mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} - \langle \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}} \rangle\right]\right)^{\mathrm{T}} \Big\rangle \\
&\approx \left\langle \left(\mathbf{H}_t \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}\right) \left(\mathbf{H}_t \mathbf{M}_t \boldsymbol{\epsilon}_{t-1}^{\mathrm{a}}\right)^{\mathrm{T}} \right\rangle \\
&= \underbrace{\mathbf{H}_t \mathbf{M}_t \mathbf{A}_{t-1} \mathbf{M}_t^{\mathrm{T}} \mathbf{H}_t^{\mathrm{T}}}_{\text{predictability error part}}
\end{aligned}
$$

3. Estimate the model error contribution

$$
\mathbf{H}_t \left\langle \boldsymbol{\eta}_t \boldsymbol{\eta}_t^{\mathrm{T}} \right\rangle \mathbf{H}_t^{\mathrm{T}} \approx \left\langle \mathbf{d}_t^{\mathrm{o,b}} \mathbf{d}_t^{\mathrm{o,b}\,\mathrm{T}} \right\rangle - \left\langle \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{f}} - \langle \mathbf{x}_t^{\mathrm{f}} \rangle\right]\right) \left(\mathbf{H}_t \left[\mathbf{x}_t^{\mathrm{f}} - \langle \mathbf{x}_t^{\mathrm{f}} \rangle\right]\right)^{\mathrm{T}} \right\rangle - \mathbf{R}_t
$$

Warning
Model error is estimated as the residual of imperfectly known contributions

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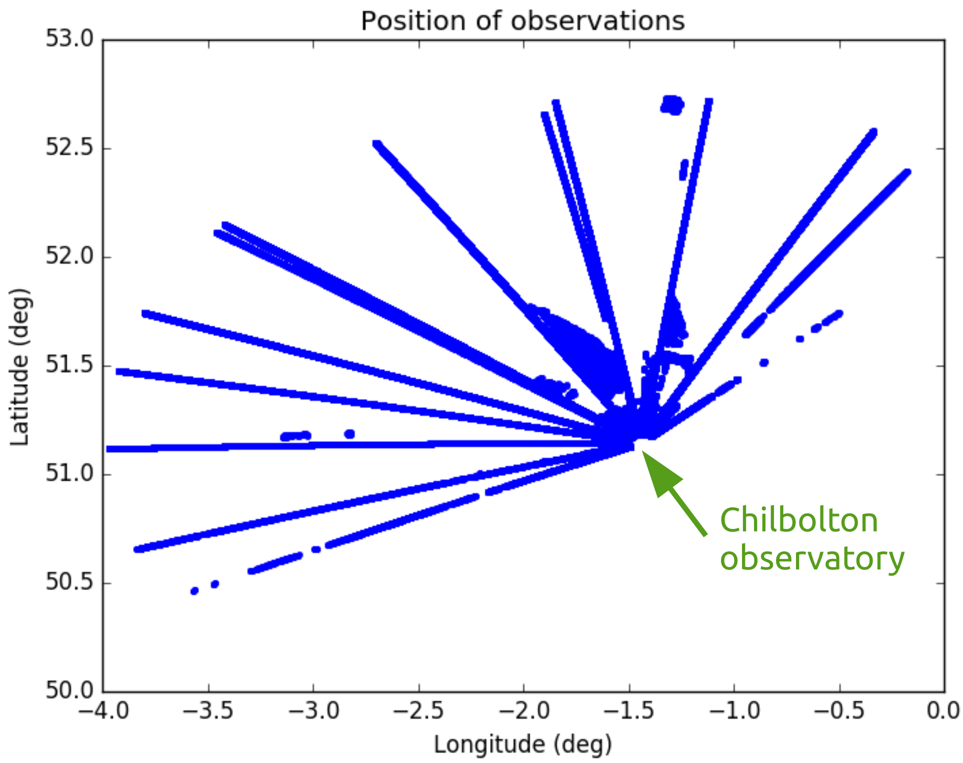
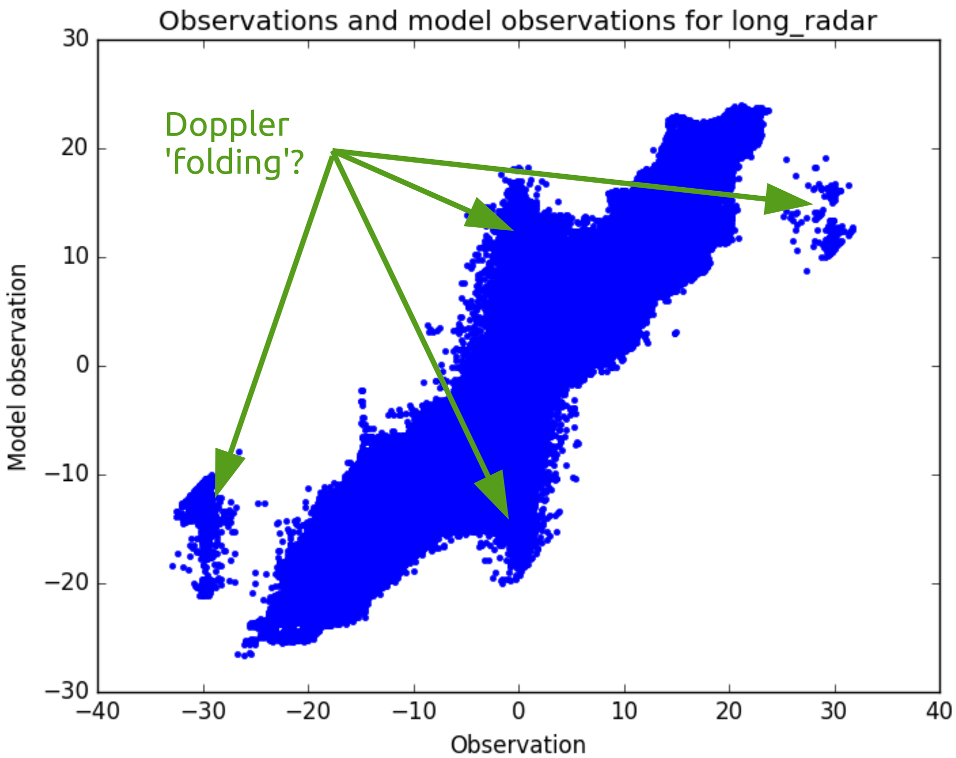
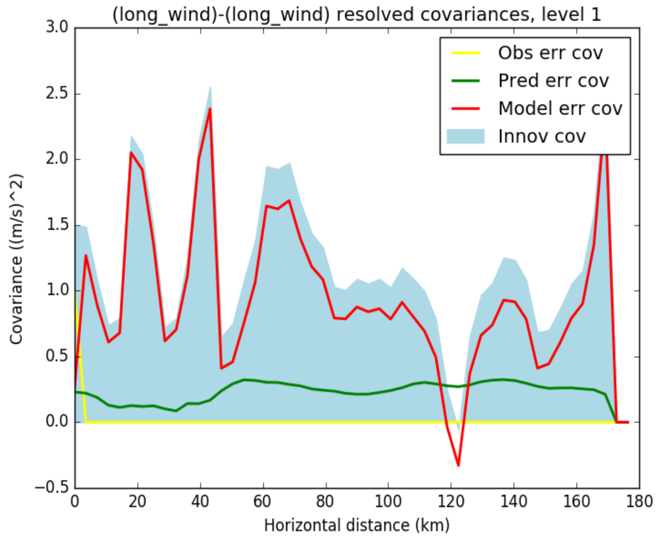
The observations and the model's equivalents



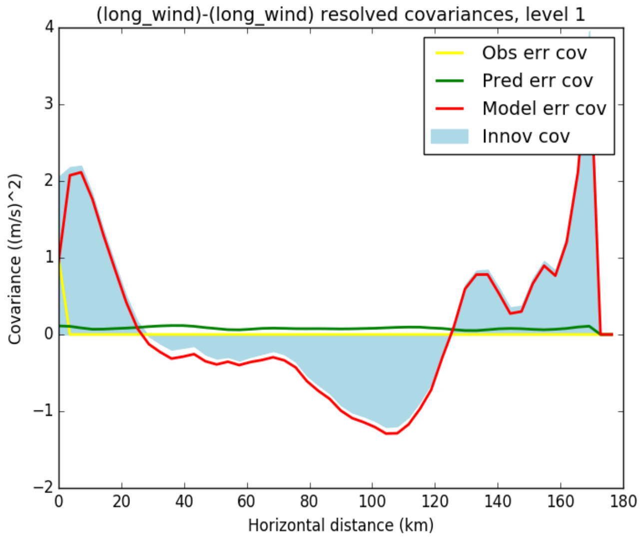


Provisional computations

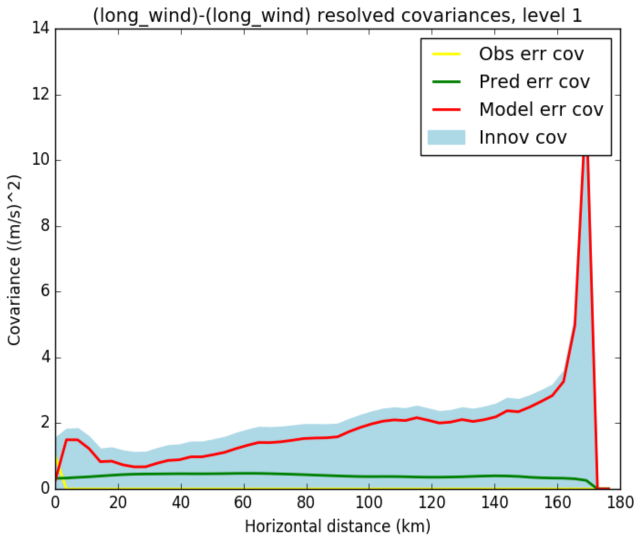
Forecast lead time 1hr ± 1hr



Forecast lead time 2hr ± 1hr



Forecast lead time 3hr ± 1hr



Comments

- No. of samples ~$O(num\ obs)^2$.
- Sampling errors are deemed negligible.
- Have assumed contribution from (i) *w* (vertical wind), and (ii) effect of atmospheric refraction, to 'model obs' are negligible.
- The above results show systematic problems with the procedure on P. 4.
- Some covariances look noisy and increase with distance.
  - Is this an artefact?
  - If not, why does this happen?
- What is the source of the large innovation covariance at ~170 km?
- Need to do quality control on observations (e.g. Doppler 'folding')?
- Need to check for systematic errors, and remove where possible.
- How can these results be used?