CONVERSION OF RSGUP3 TO MODULAR LEGENDRE TRANSFORMS

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(1)

Aim

Convert the Legendre transforms in RSGUP3, and subsequently other versions of the "SGCM" to use the modular HEXP and HANAL routines in the "flux" program BGFLUX2.

This will enhance understanding and ease future modification of the model(s).

There is likely to be a cost penalty in CPU time on vector machines, since the transform vector length will generally be reduced from NFLD*NL*NHEM to NL*NHEM. This will be worst for low vertical resolution experiments. It may be important to arrange common blocks and transforms to perform operations on several arrays in the same pass, maximising NFLD.

A beneficial side-effect should be a reduction in memory requirement of the model, due to removal of the (NFLD*NL*NHEM) Legendre polynomial work space, since the modular transforms use only a single level polynomial array. The spectral array copies of ζ,D are also unnecessary.

The modular transforms will allow the black-box grid ⇔ spectral transform routines in BGFLUX2 to be included in the model(s) (GSTRAN, SGTRAN).

The general spectral filter routine, SPFILT, can also be included.

NB • Use the modified versions of HEXP and HANAL from BGFLUX2 (~July 1994). These contain the following changes from the original versions:

i) The ∇^{-2} transform types (4-8) are included in HANAL.

ii) The input GV Fourier array is not modified by ~~HEXP~~ HANAL.

LEGENDRE FUNCTIONS

The Legendre functions $P_n^m(\mu)$, $(1-\mu^2)\frac{\partial P_n^m}{\partial \mu}$ are defined and normalised identically in the two programs, RSGUP3, BGFLUX2.

The ∇^{-2} versions of the functions differ in the two programs:

RSGUP3:

$$
\begin{array}{lll|l}
ALP & \equiv & P_n^m(\mu) & A_n^m \leftrightarrow [\ f\ ]_m \\
DALP & \equiv & (1-\mu^2)\dfrac{dP_n^m}{d\mu} & A_n^m \leftrightarrow \left[(1-\mu^2)\dfrac{\partial f}{\partial \mu}\right]_m \\
DP & \equiv & +\dfrac{m}{n(n+1)} P_n^m(\mu) & A_m^m \leftrightarrow \left[-m\nabla^{-2} f\right]_m \\
DQ & \equiv & +\dfrac{(1-\mu^2)}{n(n+1)}\dfrac{dP_n^m}{d\mu} & A_n^m \leftrightarrow \left[-(1-\mu^2)\dfrac{\partial}{\partial \mu}\nabla^{-2} f\right]_m
\end{array}
$$

BGFLUX2:

$$
\begin{array}{lll|l}
ALP & \equiv & P_n^m(\mu) & A_n^m \leftrightarrow [\ f\ ]_m \\
DALP & \equiv & (1-\mu^2)\dfrac{dP_n^m}{d\mu} & A_n^m \leftrightarrow \left[(1-\mu^2)\dfrac{\partial f}{\partial \mu}\right]_m \\
RLP & \equiv & -\dfrac{1}{n(n+1)} P_n^m(\mu) & A_n^m \leftrightarrow \left[+\nabla^{-2} f\right]_m \\
DRLP & \equiv & -\dfrac{(1-\mu^2)}{n(n+1)}\dfrac{dP_n^m}{d\mu} & A_n^m \leftrightarrow \left[+(1-\mu^2)\dfrac{\partial}{\partial \mu}\nabla^{-2} f\right]_m
\end{array}
$$

Note that $-\frac{a^2}{n(n+1)}$ is the eigenvalue of the ∇^{-2} operator.

Note that the Fourier transform of $im[f]_m$ is $\frac{\partial f}{\partial \lambda}$, or, equivalently, $im[f]_m$ is the transform of $\frac{\partial f}{\partial \lambda}$.

- Thus only the ∇^{-2} functions need to be replaced in the model.

GAUSSIAN WEIGHTS.

In both RSGUP3 and BGFLUX2, the array AW in common LEGAU contains the modified Gaussian weights used in the latitude integrals of the direct Legendre transforms.

$$AW(J) \rightarrow w_j' = \frac{2}{1-\mu_j^2} w_j \quad , \text{ where } w_j \equiv (\Delta\mu)_j .$$

$$\text{Then } \int_{-1}^{+1} A_m(\mu).P_n^m(\mu) d\mu \equiv \sum_{j=1}^{2\times JG}{}' A_{mj} P_{nj}^m w_j = \sum_{j=1}^{JG}{}' \left[(1-\mu_j^2) \bar{A}_{mj} \right] P_{nj}^m w_j'$$

$$\text{where } \bar{A}_{mj} = \begin{cases} \frac{1}{2}(A_m(+\mu) + A_m(-\mu)), \text{ even field} \\ \frac{1}{2}(A_m(+\mu) - A_m(-\mu)), \text{ odd field.} \end{cases} \text{ for global data.}$$

The factor of 2 accounts for the hemispheric-only integration, over JG latitudes.

NB BGFLUX2 assumes that common LEGAU contains the index of the current latitude in variable JH: HEXP and HANAL use this. It is not currently included in RSGUP3.

INDIRECT LEGENDRE TRANSFORMS IN RSGUP3: NHEXP.

	Spectral Field.	Fourier Field.	Trans. Type	Method.	Arrays	Spectral Sym.	Fourier Sym.	
SP	$\ln p_*$	$(1-\mu^2)\frac{\partial \ln p_*}{\partial \mu}$	(4)	$A_m = \sum_n' A_n^m (1-\mu^2)\frac{\partial P_n^m}{\partial \mu}$	SP × DALP	E	O	PJT
SPDU	$\ln p_*$	$\ln p_*$	(2)	$A_m = \sum_n' A_n^m P_n^m$	SP × ALP	E	E	PLT
DDA	$-iD$	$U_\chi = \frac{\partial \chi}{\partial \lambda}$	–	$A_m = \sum_n' (-iD_n^m)\frac{m}{n(n+1)} P_n^m$	DDA × DP	E	E	UGT
ZDA	$-i\zeta_{ABS}$	$V_\psi = \frac{\partial \psi}{\partial \lambda}$	–	$A_m = \sum_n' (-i\zeta_n^m)\frac{m}{n(n+1)} P_n^m$	ZDA × DP	O	O	VGT
ZDB	ζ_{REL}	$U_\psi = -(1-\mu^2)\frac{\partial \psi}{\partial \mu}$	–	$A_m = \sum_n' \zeta_n^m \frac{(1-\mu^2)}{n(n+1)}\frac{\partial P_n^m}{\partial \mu}$	ZDB × DQ	O	E	UG
DDB	$-D$	$V_\chi = (1-\mu^2)\frac{\partial \chi}{\partial \mu}$	–	$A_m = \sum_n' (-D_n^m)\frac{(1-\mu^2)}{n(n+1)}\frac{\partial P_n^m}{\partial \mu}$	DDB × DQ	E	O	VG
Z	ζ_{ABS}	ζ_{ABS}	(1)		Z × ALP	O	O	ZG
D	D	D	(2)	$A_m = \sum_n' A_n^m P_n^m$	D × ALP	E	E	DG
T	T	T	(2)		T × ALP	E	E	TG

The table above describes the transforms in the main loop of NHEXP. The input spectral arrays are on the left of the table and the output Fourier arrays on the right.

The subsequent loops make copies:

$$PJG = PJT \qquad \left[(1-\mu^2)\frac{\partial \ln p_*}{\partial \mu}\right]_m$$
$$PLG = PLT \qquad \left[\ln p_*\right]_m$$
$$PMG = (im)\,PLT \qquad \left[\frac{\partial \ln p_*}{\partial \lambda}\right]_m$$

and add the rotational and divergent winds:

$$UG = UG + UGT \qquad \left[U_\psi + U_\chi\right]_m$$

$$VG = VG + VGT \qquad \left[V_\chi + V_\psi\right]_m$$

Subroutine EONS then computes Fourier coefficients at $\pm\mu$ (global).

INDIRECT LEGENDRE TRANSFORMS IN SGCM: MODULAR VERSION.

Subroutine LTI is called from the main program, once per latitude (pair-global). LTI calls HEXP to perform the transform for individual fields.

Spectral Field	Fourier Field.	Trans. Type	Method	Arrays.	Spectral Sym.	Fourier Sym.
ζ_A	ζ_A	1	$A_m=\sum_n' A_n^m P_n^m$	ZG = ALP × Z	O	O
D	D	2	$A_m=\sum_n' A_n^m P_n^m$	DG = ALP × D	E	E
T	T	2		TG = ALP × T	E	E
$\ln p_*$	$\ln p_*$	2		PLG = ALP × SP	E	E
$\ln p_*$	$(1-\mu^2)\frac{\partial \ln p_*}{\partial \mu}$	4	$A_m=\sum_n' A_n^m (1-\mu^2)\frac{dP_n^m}{d\mu}$	PJG = DALP × SP	E	O

Then remove planetary vorticity from $\zeta_A \rightarrow \zeta_R$.

ζ_R	$\psi = \nabla^{-2}\zeta_R$	5	$A_m=\sum_n' A_n^m\left(\frac{1}{n(n+1)}P_n^m\right)$	PSIG = RLP × Z	O	O
D	$\chi = \nabla^{-2}D$	6	$A_m=\sum_n' A_n^m\left(\frac{-1}{n(n+1)}P_n^m\right)$	CHIG = RLP × D	E	E
ζ_R	$-U_\psi = (1-\mu^2)\frac{\partial\psi}{\partial\mu}$	7	$A_m=\sum_n' A_n^m\left(\frac{-(1-\mu^2)}{n(n+1)}\frac{dP_n^m}{d\mu}\right)$	UG = RDLP × Z	O	E
D	$V_\chi = (1-\mu^2)\frac{\partial\chi}{\partial\mu}$	8	$A_m=\sum_n' A_n^m\left(-\frac{(1-\mu^2)}{n(n+1)}\frac{dP_n^m}{d\mu}\right)$	VG = RDLP × D	E	O

Then restore planetary vorticity in $\zeta_R \rightarrow \zeta_A$.

Then sum terms to obtain total winds:

$$U = U_\psi + U_\chi = -\left(-U_\psi + \frac{\partial\chi}{\partial\lambda}\right) \quad : \text{UG} = -\text{UG} + (im)\,\text{CHIG}$$

$$V = V_\psi + V_\chi = \left(\frac{\partial\psi}{\partial\lambda} + V_\chi\right) \quad : \text{VG} = (im)\,\text{PSIG} + \text{VG}$$

Zonal gradient of surface pressure:

$$\left[\frac{\partial \ln p_*}{\partial\lambda}\right]_m = (im)\left[\ln p_*\right]_m \quad : \text{PMG} = (im)\,\text{PLG}.$$

Notes:

- Spectral and grid arrays for D, T, $\ln p_*$ must be grouped together in common to group as a single call to HEXP.
- The required Fourier/grid arrays for input to FFT must be grouped together: UG, VG, ZG, DG, TG, PLG, PMG, PJG.

DIRECT LEGENDRE TRANSFORMS : REQUIRED FIELDS

The direct Legendre transforms create the non-linear contributions to the tendencies of the prognostic variables in spectral space. Linear tendencies are computed separately in spectral space.

Following Hoskins & Simmons (1975), the fields to be created are

$p*$	: surface pressure (<u>not</u> logarithm of $p*$)	
$-\mathcal{P}$	: (−) tendency of $\ln p*$.	(non-linear part)
$\mathcal{T}$	: " " temperature	(" " ")
$\mathcal{D}$	: " " divergence	(" " ")
$\mathcal{Z}$	: " " vorticity.	(" " ")

These are defined by:

$$-\mathcal{P} = +\int_0^1 (\underline{V}.\nabla \ln p_*)\, d\sigma$$

$$\mathcal{T} = -\frac{1}{1-\mu^2}\frac{\partial}{\partial\lambda}(UT') - \frac{\partial}{\partial\mu}(VT') + \mathcal{T}_{(1)},$$

where $\mathcal{T}_{(1)} = \left(-\dot{\sigma}\frac{\partial T}{\partial\sigma}\right)_{NL} + \left(\frac{\kappa\omega T}{p}\right)_{NL} + DT'$

$$\mathcal{D} = \frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_U}{\partial\lambda} + \frac{\partial\mathcal{F}_V}{\partial\mu} - \nabla^2\frac{1}{2}\left[\frac{U^2+V^2}{1-\mu^2}\right]$$

$$\mathcal{Z} = \frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_V}{\partial\lambda} - \frac{\partial\mathcal{F}_U}{\partial\mu}.$$

It will be useful to define:

$$\mathcal{T}_{(2)} = \mathcal{T}_{(1)} - \frac{1}{1-\mu^2}\frac{\partial(UT')}{\partial\lambda}$$

The terms involving zonal gradients will be: $\mathcal{T}_X, \mathcal{D}_X, \mathcal{Z}_X$

The terms involving meridional gradients will be $\mathcal{T}_Y, \mathcal{D}_Y, \mathcal{Z}_Y$

DIRECT LEGENDRE TRANSFORMS IN RSGUP3 (HANAL)

	Fourier Field	Spectral Field	Transform type	Method	Arrays	Fourier Sym.	Spec. Sym.	
PLG	P_*	P_*	(2)	$A_n^m = \sum_j{}' (1-\mu^2) A_m P_n^m w_j'$	SPA = PLG ×CSJ×ALP×AW	E	E	SPA
VPG	$-\Phi$	$-\Phi$	(2)	$A_n^m = \sum_j{}' (1-\mu^2) A_m P_n^m w_j'$	VP = VPG ×CSJ×ALP×AW	E	E	VP
EG	(U^2+V^2)	$-\nabla^2 \frac{1}{2}\left[\frac{U^2+V^2}{1-\mu^2}\right]$	—	$A_n^m = \sum_j{}' \left[\frac{n(n+1)}{2}\right] A_m P_n^m w_j'$	DT = EG ×SQH×ALP×AW	E	E	DT
VTG	(VT')	$-\frac{\partial}{\partial \mu}(VT')$	(4)	$A_n^m = \sum_j{}' A_m (1-\mu^2)\frac{dP_n^m}{d\mu} w_j'$	TT = VTG × DALP×AW	O	E	TT
UZG	$\mathcal{F}_v$	$+\frac{\partial \mathcal{F}_v}{\partial \mu}$	(4)	$A_n^m = -\sum_j{}' A_m (1-\mu^2)\frac{dP_n^m}{d\mu} w_j'$	DT = UZG ×DALP×AW	O	E	DT
VZG	$\mathcal{F}_u$	$-\frac{\partial \mathcal{F}_u}{\partial \mu}$	(3)	$A_n^m = +\sum_j{}' A_m (1-\mu^2)\frac{dP_n^m}{d\mu} w_j'$	ZT = VZG × DALP×AW	E	O	ZT
TNLG	$\mathcal{T}_{(2)}(1-\mu^2) = (1-\mu^2)\mathcal{T}_{(1)} - \frac{\partial (UT')}{\partial \lambda}$	$\mathcal{T}_{(2)} = \mathcal{T}_{(1)} - \frac{1}{1-\mu^2}\frac{\partial (UT')}{\partial \lambda}$	(2)	$A_n^m = \sum_j{}' A_m P_n^m w_j'$	TT = TNLG ×ALP×AW	E	E	TT
VZGT	$\frac{\partial \mathcal{F}_u}{\partial \lambda}$	$\frac{1}{1-\mu^2}\frac{\partial \mathcal{F}_u}{\partial \lambda}$	(2)	$A_n^m = \sum_j{}' A_m P_n^m w_j'$	DT = VZGT × ALP×AW	E	E	DT
UZGT	$\frac{\partial \mathcal{F}_v}{\partial \lambda}$	$\frac{1}{1-\mu^2}\frac{\partial \mathcal{F}_v}{\partial \lambda}$	(1)	$A_n^m = \sum_j{}' A_m P_n^m w_j'$	ZT = UZGT ×ALP×AW	O	O	ZT

The table above describes the transforms performed in the main loops of routine HANAL. The input Fourier arrays are on the left and output spectral arrays on the right.

The preceding loops prepare the Fourier fields as follows:

1. $$(1-\mu^2)\mathcal{T}_{(2)} = (1-\mu^2)\mathcal{T}_{(1)} - \frac{\partial}{\partial\lambda}(UT')$$
   TNLG ≡ CSJ × TNLG − CMPA × UTG.

2. Zonal derivatives:
   $$\left[\frac{\partial \mathcal{F}_u}{\partial \lambda}\right]_m = im\left[\mathcal{F}_u\right]_m$$, VZGT = CMPA × VZG
   $$\left[\frac{\partial \mathcal{F}_v}{\partial \lambda}\right]_m = im\left[\mathcal{F}_v\right]_m$$, UZGT = CMPA × UZG

Note that the single-level transforms for SPA, VP (P_*, $-\Phi$) are vectorised over total wavenumber.

POSSIBLE APPROACHES TO MODULAR DIRECT TRANSFORMS

There are different ways to implement modular direct Legendre transforms in the SGCM at a detailed level. Some involve modifying the existing black-box routine HANAL used in BGFLUX2. Each has different memory and probably CPU timings. An aim in defining the method to be used is to maximise efficiency of the transforms. This may be important, to offset the loss of vector length when the current (7×NL+2) levels are split into transforms generally involving only NL levels.

① Retain current HANAL from BGFLUX2.

- There is no $\nabla^2(\ )$ transform, so the energy term in $\mathcal{D}$ must be evaluated in a separate spectral array and the ∇^2 taken in spectral space.
- Generally, two Fourier terms contribute to each spectral tendency, often with the opposite sign to the Fourier array. If extra spectral arrays are to be avoided, the Fourier arrays must have the correct sign before transforming, involving extra computations.

② Include sign argument in HANAL

- If an extra argument giving the sign of the transformed field with respect to the input field were included in HANAL, the second problem in option (1) would be avoided. The sign could be applied to AWT in HANAL, requiring a single multiplication at each latitude.

③ ∇^2 transform option in HANAL

- Transform types (9,10) for $\nabla^2(\ )$ could be added to HANAL. This would allow the energy term to contribute directly to the divergence tendency, but it would separate this transform from all others, reducing vectorisation. Unless new Legendre functions were used, the ∇^{-2} and ∇^2 transforms would use different techniques.

DIRECT LEGENDRE TRANSFORMS IN SGCM: MODULAR VERSION (OPTION 1).

This explains the structure of routine LTD, called from the main program at each latitude (pair). LTD calls HANAL to perform the transform for individual fields. Note that HANAL <u>increments</u> the spectral field, so the same spectral array can be used in several transforms.

1. First prepare the Fourier arrays, including sign changes:

$-\frac{1}{2}\left[\frac{U^2+V^2}{1-\mu^2}\right]_m$	: EG = -0.5 * EG / CSSQ(JH)
$-(VT')_m$	: VTG = - VTG
$\mathcal{T}_{(2)} = \mathcal{T}_{(1)} - \frac{1}{1-\mu^2}\frac{\partial}{\partial\lambda}(UT')$	: TNLG = TNLG - CMPA* UTG / CSSQ
$\frac{1}{1-\mu^2}\partial\mathcal{F}_U/\partial\lambda$	: FUG = CMPA * FUG / CSSQ
$\frac{1}{1-\mu^2}\partial\mathcal{F}_V/\partial\lambda$	: FVG = CMPA * FVG / CSSQ
$-\mathcal{F}_U$	: FUG = - FUG

2.

	Fourier Field	Spectral Field	Trans. Type	Method.	Arrays.	Fourier Sym.	Spec. Sym.	
PLG	$P*$	$P*$	2	$A_n^m=\sum_j{}'(1-\mu^2)A_m P_n^m w_j$	SPA = PLG ×~~CSSQ~~×ALP×AW	E	E	SPA
VPG	$-\mathcal{P}$	$-\mathcal{P}$	2.	(")	VP = VPG ×CSSQ×ALP×AW	E	E	VP
EG	$-\frac{1}{2}\left[\frac{U^2+V^2}{1-\mu^2}\right]$	$-\frac{1}{2}\left[\frac{U^2+V^2}{1-\mu^2}\right]$	2	(")	DTE = EG ×CSSQ×ALP×AW	E	E	DTE
TNLG	$\mathcal{T}_{(2)}$	$\mathcal{T}_{(2)}$	2	(")	TT = TNLG ×CSSQ×ALP×AW	E	E	TT
FUG	$\frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_U}{\partial\lambda}$	$\frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_U}{\partial\lambda}$	2	(")	DT = FUG ×CSSQ×ALP×AW	E	E	DT
FVG	$\frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_V}{\partial\lambda}$	$\frac{1}{1-\mu^2}\frac{\partial\mathcal{F}_V}{\partial\lambda}$	1	(")	ZT = FVG ×CSSQ×ALP×AW	O	O	ZT
VTG	$-(VT')$	$-\frac{\partial}{\partial\mu}(VT')$	4	$A_n^m=\sum_j{}' A_m(1-\mu^2)\frac{dP_n^m}{d\mu}w_j$	TT = VTG × DALP×AW	O	E	TT
FVGT	$+\mathcal{F}_V$	$+\frac{\partial\mathcal{F}_V}{\partial\mu}$	4	(")	DT = FVGT ×DALP×AW	O	E	DT
FUGT	$-\mathcal{F}_U$	$-\frac{\partial\mathcal{F}_U}{\partial\mu}$	3	(")	ZT = FUGT ×DALP×AW	E	O	ZT

3. Finally take ∇^2 of the spectral array DTE, <u>at the last latitude only</u>, and add this to DT. A separate black-box routine could be used for this.

<u>N.B.</u> DTE must be preset to zero at the beginning of the latitude loop.