Summary of Phase Function from scatter90 and for RFM-DISORT

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18th March 2011

1 Definition of the phase function

The phase function is the angular distribution of light intensity scattered by a particle at a given wavelength. It is given at an angle θ which is relative to the incident beam. The phase function is the intensity (radiance) at θ relative to the normalized integral of the scattered intensity at all angles (Seinfeld & Pandis, p693). It is defined by Seinfeld & Pandis (eq 15.9) as:

$$P(\theta) = \frac{F(\theta)}{\int_0^\pi F(\theta) sin\theta d\theta} \tag{1}$$

where F is intensity (radiance). $P(\theta)$ can be thought of as a probability density function, showing the chances of a photon of light being scattered in a particular direction, θ. This also means that when integrated over the sphere, the phase function must equal zero, which is known as the 'normalisation condition,' and shown by:

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} p(\cos\theta) \sin\theta d\theta d\phi = 1 \tag{2}$$

Note that the azimuthal dependence ϕ of P is often removed, which is possible under the assumption of a spherical particle.

However, the phase fucntion can also be defined as an infinite series of orthogonal basis functions, such as the Legendre polylnomials, $P_l(cos\theta)$. Expressing the phase function in this way allows the shape of $p(cos\theta)$ to be expressed to whatever level of accuracy is required by changing N. From Petty, p427 Eq A1,

$$P(\cos\theta) = \sum_{l=0}^{N-1} \beta_l P_l(\cos\theta) \tag{3}$$

Here β_l are the moments of the phase function, and $P_l(\cos\theta)$ are the Legendre polynomials.

The first few Legendre polynomials are given by the following, where $\mu = \cos(\theta)$:

$$P_0(\mu) = 1 \tag{4}$$

$$P_1(\mu) = \mu \tag{5}$$

$$P_2(\mu) = \frac{1}{2}(3\mu^2 - 1) \tag{6}$$

$$P_3(\mu) = \frac{1}{2}(5\mu^3 - 3\mu) \tag{7}$$

$$P_4(\mu) = \frac{1}{8}(35\mu^4 - 30\mu^2 + 3) \tag{8}$$

$$P_5(\mu) = \frac{1}{8}(65\mu^5 - 70\mu^3 + 15\mu) \tag{9}$$

β_l, the moments of the phase function, can be defined as followed, and are also output from Mie scattering codes:

$$\beta_l = \frac{2l+1}{2}\int_{-1}^{1} P_l(\cos\theta)p(\cos\theta)d\cos\theta \tag{10}$$

β_0 will always be 1, due to the normalization condition. This means that the zeroth moment of the phase function is always 1.

Additionally it turns out that $g = \beta_1/3$, where g is the asymmetry parameter. This means that if you use only g as input to a radiation code, then you are essentially using only two terms (β_0 $(= 1)$ and β_1 $(= 3g)$) to represent the phase function.

2 Phase Function from Scatter90

When using the Mie scattering code in Edwards & Slingo, you produce phase function output, and have the option to choose how many moments of the phase function (χ) you require.

Firstly it is important to note that the moments of the phase function output by scatter90 are different to those described above, e.g. as in Petty, and also in many other text books. Scatter90 uses a phase function described by:

$$P(\cos\theta) = \sum_{l=0}^{2N-1}(2l+1)\chi_l P_l(\cos\theta) \tag{11}$$

where

$$\chi_l = \frac{1}{2}\int_{-1}^{1} P_l(\cos\theta)p(\cos\theta)d\cos\theta \tag{12}$$

Therefore in this format,

$$\beta_l = (2l+1)\chi_l \tag{13}$$

Note that when the phase function is expressed using χ, the case is still that $\chi_0 = 1$, so that the zeroth moment of the phase function is 1. Note that scatter90 will not produce output for the zeroth moment.

Secondly, when using the χ format of the phase function equation, the relation of g to the first moment changes. We can see that:

$$\beta_l = (2l+1)\chi_l \tag{14}$$

Since when $l = 1$, $g = \beta_1/3$, then

$$\beta_1 = 3\chi_1 \tag{15}$$

$$g = \chi_1 \tag{16}$$

Therefore, in the out put from Scatter90, g is indeed the first moment of the phase function.

2.1 Number of Moments Requested in Scatter90

It is interesting to run scatter90 with different numbers of moments of the phase function requested. You will find as you change the number of moments requested, the value of the moments themselves change, including the first one. For example, for a case of Saharan dust, the first moment changes from 0.7 for 220 moments, to 0.6 when 4 moments are requested. These differences are quite significant. This is not surprising when you understand what the code is doing - the more moments requested mean that the overall representation of the phase function will be more accurate, and the first N moments of the phase function will be more accurate. Therefore it is always best to run the code requesting as many moments of the phase function as possible (over 200 recommended, perhaps more for complex phase functions). If applications of the output are then required which use fewer moments, they can then just be lifted from this more detailed output.

The above is particularly relevant if you are using the first moment (g) from the output, and not running with many moments of the phase function. E.g. you ask for 4 moments of the phase function, and take the first as information on g. If you do this, then your value of g is likely to be very inaccurate. Instead, ask for say 220 moments, an then lift the first for g.

Conversely, if you ask scatter90 for only 1 moment of the phase function, it will give you just that. HOWEVER, the code does something different here, and outputs a highly accurate value of g. I have not looked at Fortran code in detail, but I suspect it does detailed scattering calculations (e.g. uses many moments of the phase function), and only outputs the first moment of the phase function. This can be seen and tested by doing different runs of scatter90.

Various calculations need to be done to convert moments of the phase function (either β_l or χ_l) into the scattering phase function, $p(\cos\theta)$. There are some Fortran codes kicking around which can do this.

2.2 Phase Function for RFM-DISORT

An interesting question is whether other scattering and radiation codes output or require input of the phase function of β_l or χ_l. If there is a mismatch between what is output/input and required, then the results could be quite weird!

For RFM-DISORT, the format of the phase function required is χ_l (see equation 5a in the DISORT documentation), so scatter90 and ESRAD data can be seamlessly used in RFM-DISORT. Note that although it is always 1, RFMD does require input of χ_0 values, which is dealt with nicely in Nazim's IDL code.

3 Further Reading

A lot of the background material here came from Petty, 'A First Course in Atmospheric Radiation' and some also from Seinfeld & Pandis' 'Atmospheric Chemistry and Physics.'

4 Acknowledgements

Thanks to various people who have done some digging and explaining to help, including Christine Chiu, Ellie Highwood, Cristina Charlton-Perez & Jean-Claude Thelen.