

# 'Toy' models for Data Assimilation

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# Toy models

- We will use two ‘simple’ models to test our data assimilation techniques.
- Both were designed by Edward Lorenz. Under certain choices of parameters, they exhibit chaotic behavior and present a challenge for data assimilation and predictability.
- Their size allows us to visualize results in a simple manner and explore different settings: observation operators, magnitude of background and observational error, time frequency of observations, etc.

# 1. Lorenz 1963 3-variable model

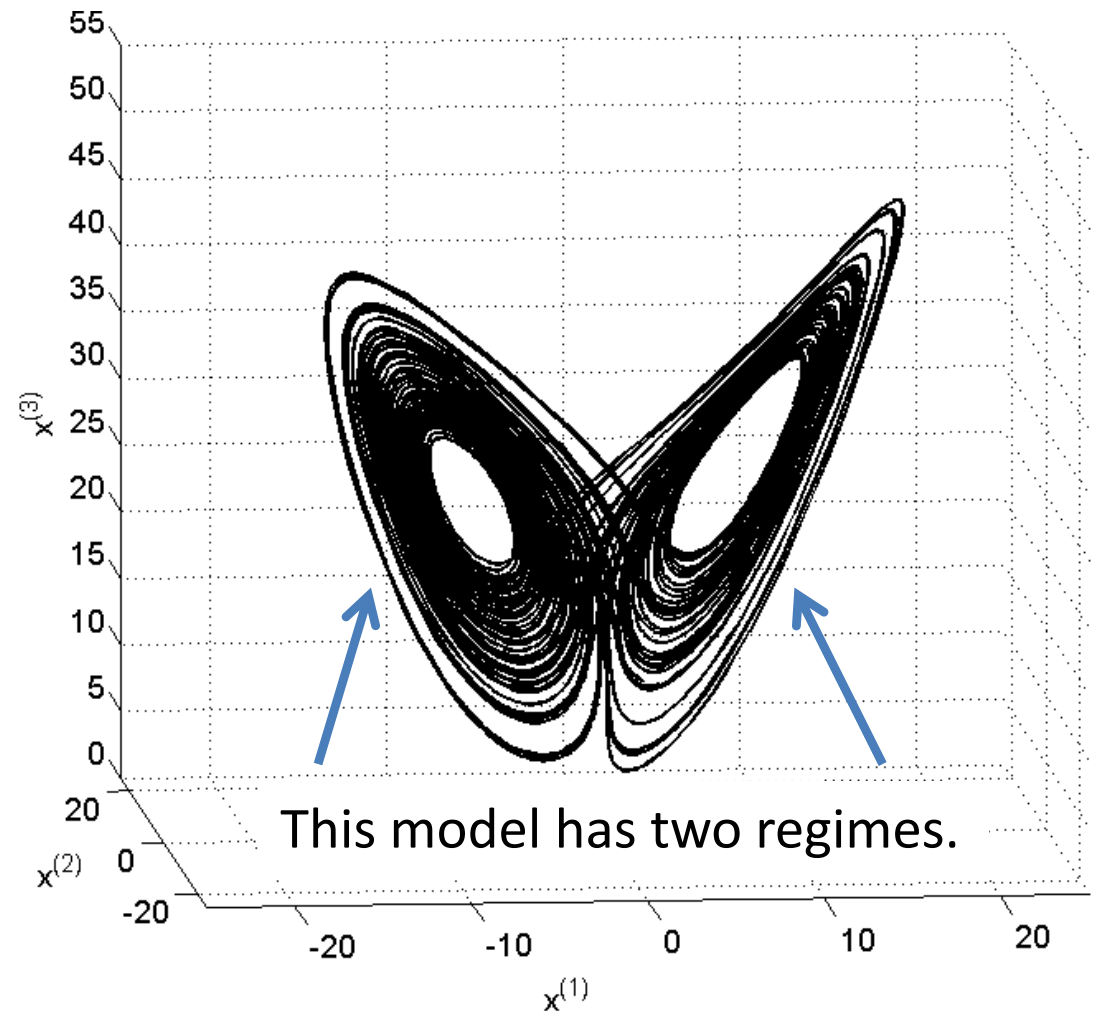
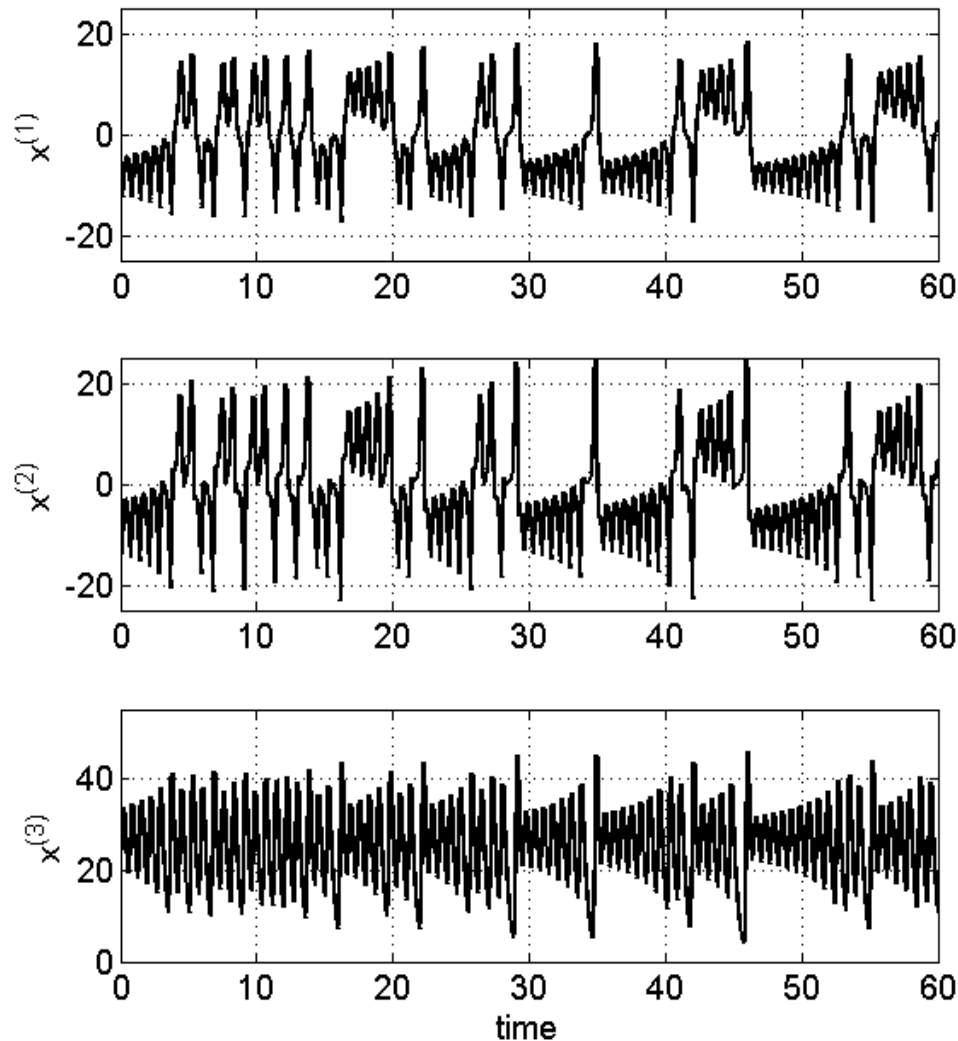
This model comes from a simplified description of the Rayleigh-Benard convection. It has 3 variables.

The time evolution of the model is described by:

$$\begin{aligned}\dot{x}^{(1)} &= \sigma(x^{(2)} - x^{(1)}) \\ \dot{x}^{(2)} &= x^{(1)}(r - x^{(3)}) - x^{(2)} \\ \dot{x}^{(3)} &= x^{(1)}x^{(2)} - bx^{(3)}\end{aligned}\quad \text{parameters } \begin{cases} \sigma = 10 \\ r = 8/3 \\ b = 28 \end{cases}$$

This model presents strong non-linearity and a regime transition.

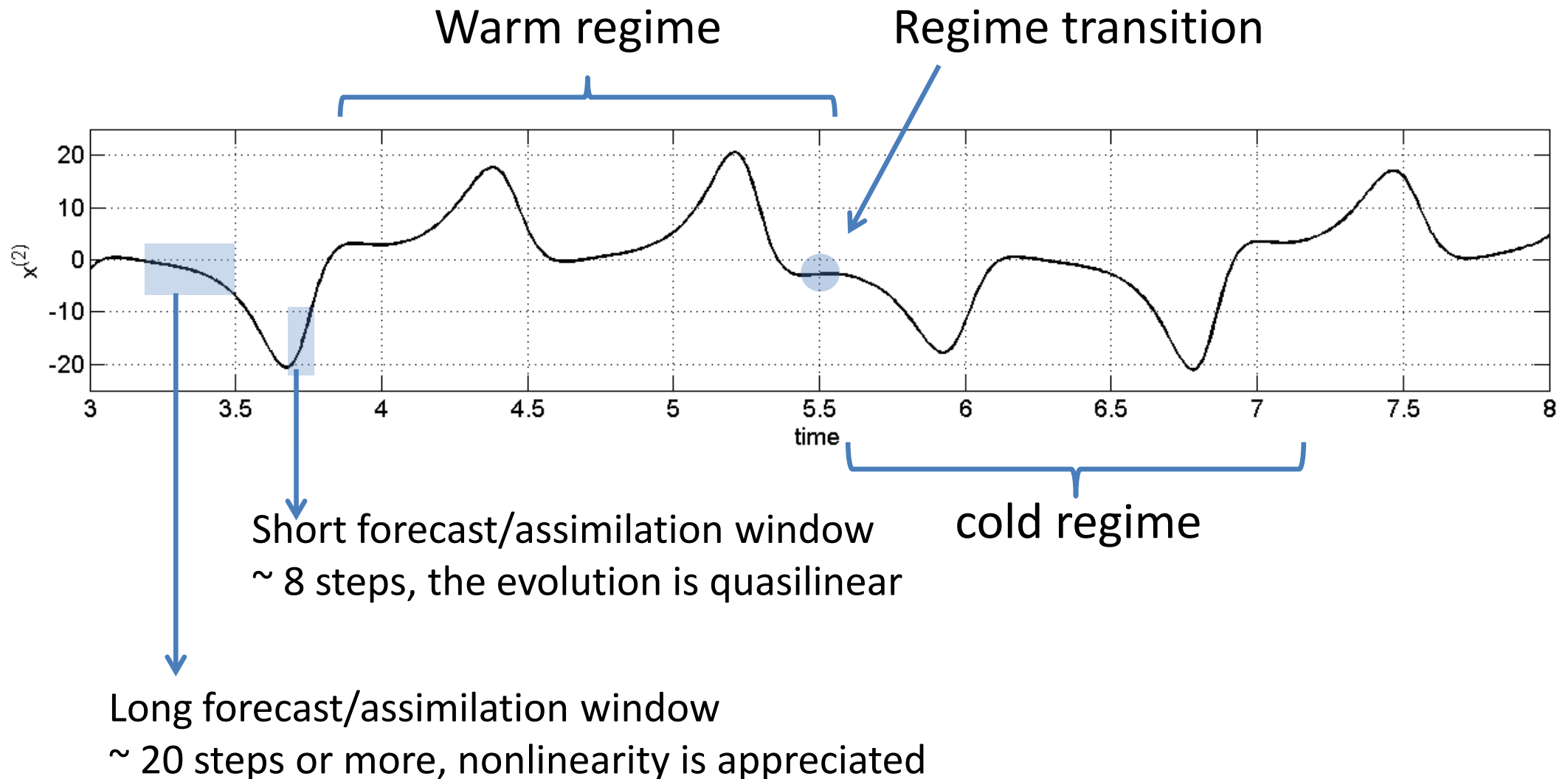
# 1. Lorenz 1963 3-variable model



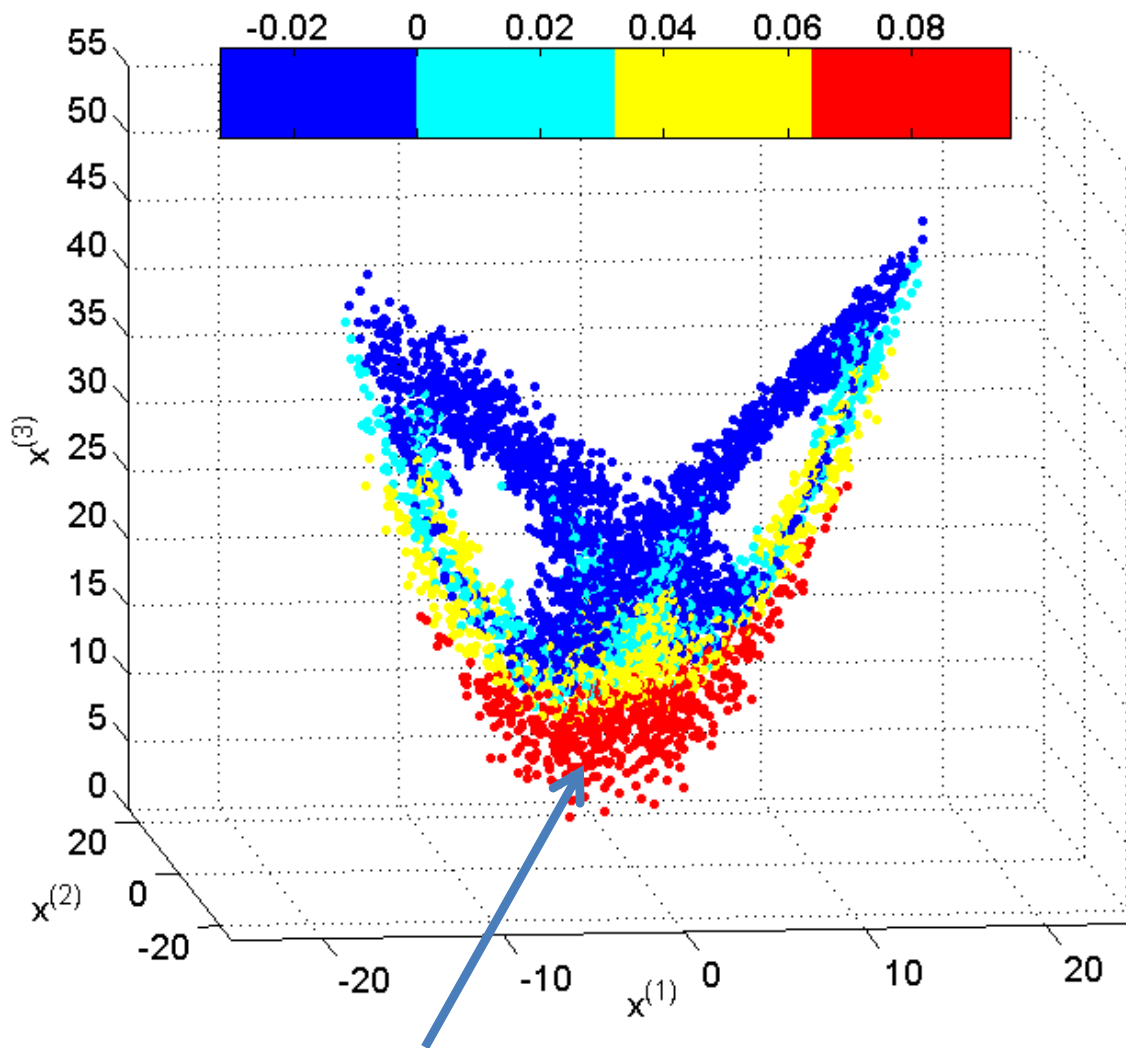
Time evolution (right) and representation of the attractor (left) for the model. Figures obtained integrating with RK4 and a time step 0.01. 4

# 1. Lorenz 1963 3-variable model

Evolution of  $x^{(2)}$



# 1. Lorenz 1963 3-variable model



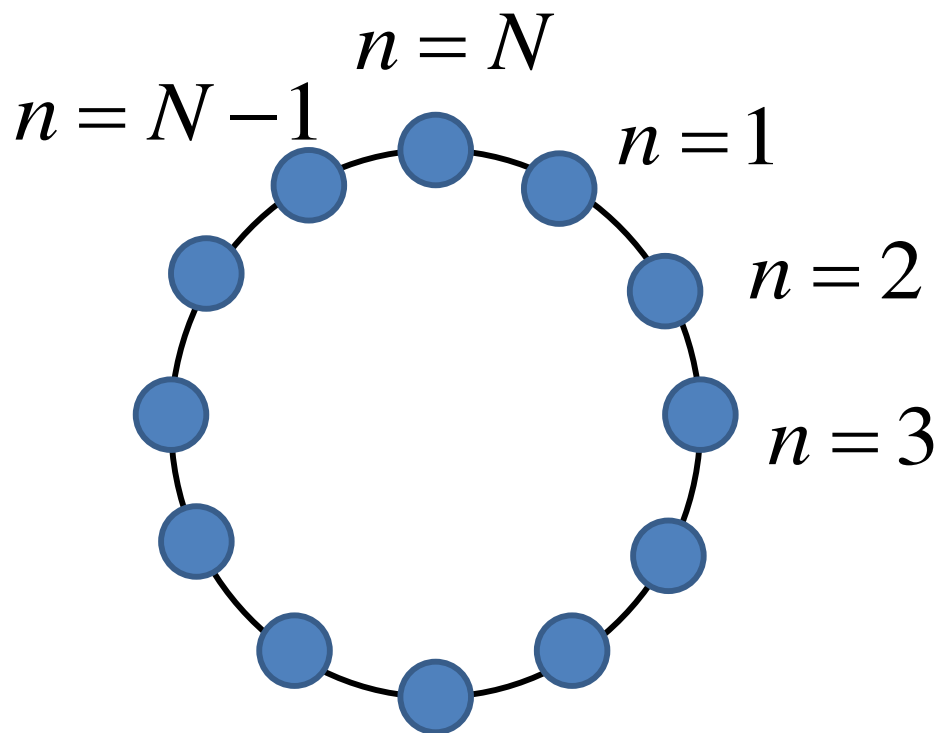
This region is challenging!

The difficulty of the data assimilation problem depends on the current position within the attractor.

**Dark blue** denotes regions where perturbations **decay**, **red** denotes regions where perturbations **grow fast**. (figure plotted using bred vectors, reproduced from Evans *et al*, 2004).

## 2. Lorenz 1996 model

- This cyclic model intends to emulate the behaviour of a meteorological variable in a circle of latitude.



$$\dot{x}^{(q)} = \underbrace{(x^{(q+1)} - x^{(q-2)})}_{\text{advection}} x^{(q-1)} - \underbrace{x^{(q)}}_{\text{diffusion}} + \underbrace{F}_{\text{forcing}}$$

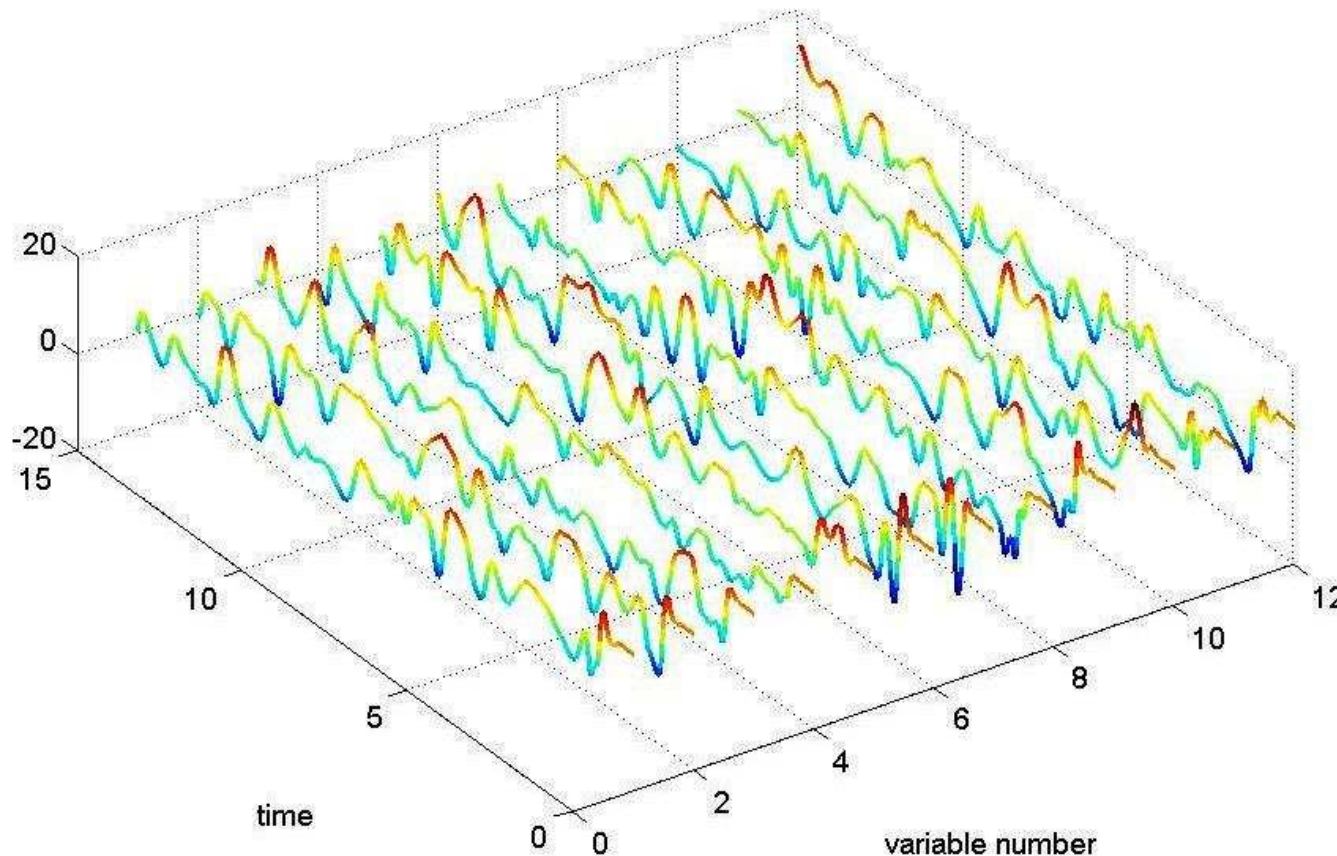
$$q = 1, 2, \dots, N$$

$$x^{(q)} \equiv x^{(\text{mod}(q, N))}$$

Chaos will appear for  $F > 5$  and  $N \geq 12$

## 2. Lorenz 1996 model

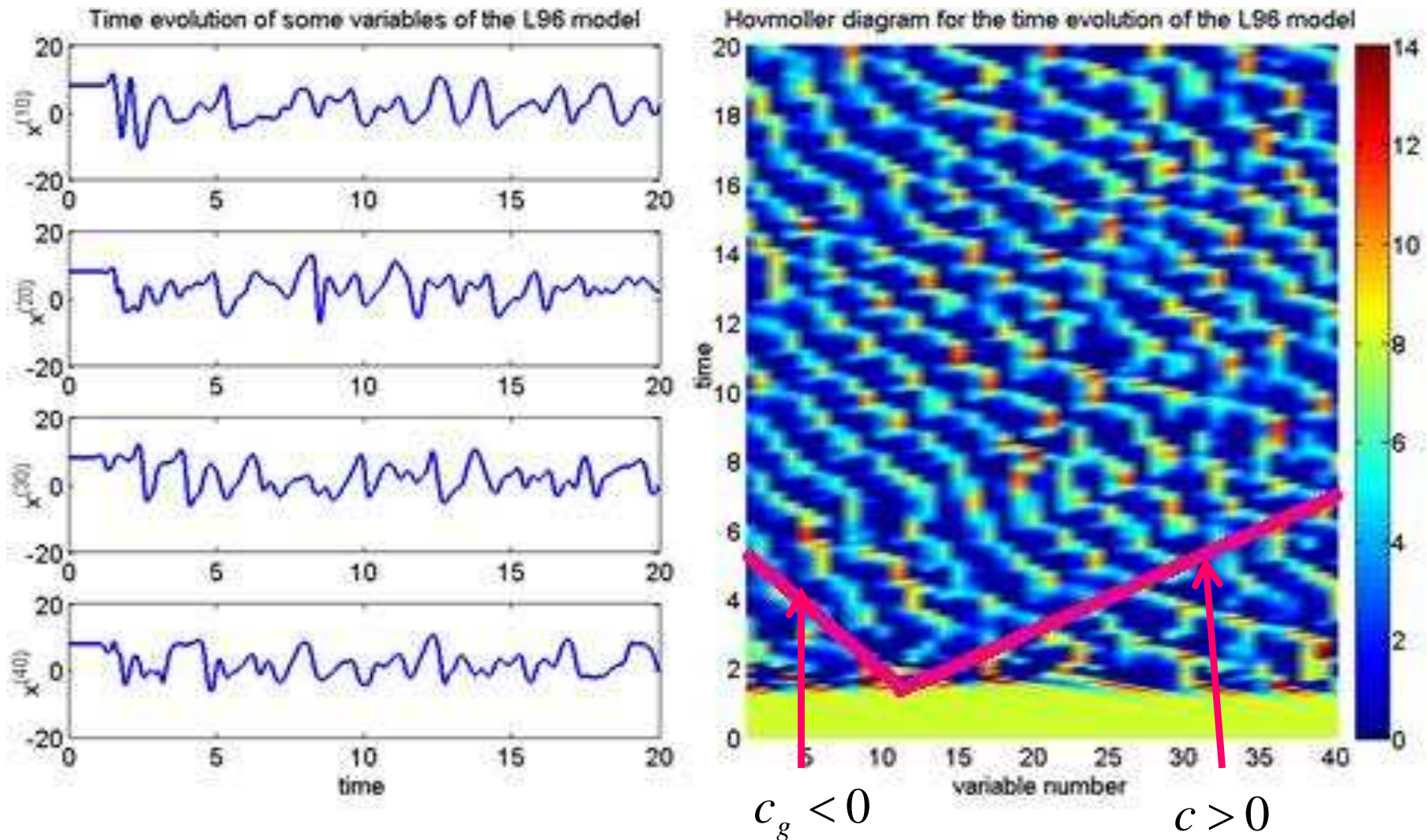
This model can be started from rest, when  $x^{(q)} = F, \forall q$ . A miniscule perturbation to any variable will grow and spin the system



Time evolution  
Lorenz 1996 model  
with  $N=12$   
variables. The  
model was  
integrated with  
RK4 and a time  
step of 0.025.



## 2. Lorenz 1996 model



Time evolution of some variables (left) and Hovmöller diagram (right) for the Lorenz 1996 model with  $N=40$  variables. Notice the presence of 'waves' with positive phase speed and negative group speed.

## 2. Lorenz 1996 model

- In this model, we will usually use assimilation windows of two time steps, in which the behaviour is quasilinear.
- In the ensemble framework, this model will allow us to test **localization** and **inflation** when using less ensemble members than state variables.

# References

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