Exercises: practise the 'short-hand' vector-matrix algebra

For a reminder of the general rule of multiplication of two matrices, see the Maths Primer, §1.6

- $\mathbf{u}^{\mathrm{T}}\mathbf{v}$ is the product of a $1 \times n$ with an $n \times 1$ vector (inner product). Result is a 1×1 matrix (a scalar!).

$$\begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} u_1 & \cdots & u_n \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = u_1 v_1 + \ldots + u_n v_n.$$

- $\mathbf{u}^{\mathrm{T}}\mathbf{A}\mathbf{v}$ is the product of a $1 \times n$ with an $n \times n$ vector with an $n \times 1$ vector (inner product in a particular metric). Result is a 1×1 matrix (again).

$$\begin{pmatrix} u_1 & \cdots & u_p \end{pmatrix} \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} u_1 & \cdots & u_p \end{pmatrix} \begin{pmatrix} A_{11}v_1 + \cdots + A_{1n}v_n \\ \vdots \\ A_{n1}v_1 + \cdots + A_{nn}v_n \end{pmatrix}$$

$$= u_1 \left(A_{11}v_1 + \cdots + A_{1n}v_n\right) + \ldots + u_p \left(A_{n1}v_1 + \cdots + A_{nn}v_n\right).$$

- $\mathbf{u}\mathbf{v}^{\mathrm{T}}$ is the product of an $n \times 1$ matrix with a $1 \times m$ matrix (outer product). The result is an $n \times m$ matrix.

$$\begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \begin{pmatrix} v_1 & \cdots & v_m \end{pmatrix} = \begin{pmatrix} u_1 v_1 & \cdots & u_1 v_m \\ \vdots & \ddots & \vdots \\ u_n v_1 & \cdots & u_n v_m \end{pmatrix}.$$

Same rule, different matrices.