Using reconditioning to study the impact of correlated observation errors in the Met Office 1D-Var system

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Overview

1. Motivation
   - Why do we want to include correlated observation error information?
2. Reconditioning is one solution!
   - What is reconditioning?
   - Theory of reconditioning
3. Implementation in the Met Office system
   - IASI - operational interchannel correlations
   - Impact of reconditioning on convergence
   - Impact of reconditioning on quality control procedure

Cost function for 3DVar

We want to minimise

$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + (\mathbf{y} - h[\mathbf{x}])^T \mathbf{R}^{-1} (\mathbf{y} - h[\mathbf{x}]). \quad (1)$$

where,

- $\mathbf{B} \in \mathbb{R}^{N \times N}$ background error covariance
- $\mathbf{R} \in \mathbb{R}^{p \times p}$ observation error covariance
- $h : \mathbb{R}^N \to \mathbb{R}^p$ observation operator
- $\mathbf{y} \in \mathbb{R}^p$ vector of observations
- $\mathbf{x}_b \in \mathbb{R}^N$ vector representing the background

What happens to the convergence of (1) if we introduce correlated **R**?.

Why include correlated observation error information?

- Including correlation information allows us to take advantage of dense observation networks to get **high-resolution forecasts**.
- Using uncorrelated observation error matrices means we have to thin observations - this can result in **up to 80%** of obs being discarded!
- Neglecting correlations where they are present also **limits our skill**.

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Reminder: If $\mathbf{S} \in \mathbb{R}^{p \times p}$ *is a symmetric and positive definite matrix with eigenvalues* $\lambda_1(\mathbf{S}) \geq \ldots \geq \lambda_p(\mathbf{S}) > 0$ *then we write the condition number*

$$\kappa(\mathbf{S}) = \frac{\lambda_1(\mathbf{S})}{\lambda_p(\mathbf{S})}.$$

If $\mathbf{S}$ *is singular, we take* $\kappa(\mathbf{S}) = \infty$.

Motivation II - Satellite observations

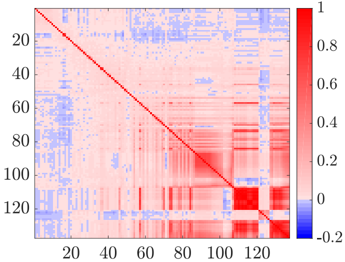

Figure: Diagnosed correlation matrix for IASI

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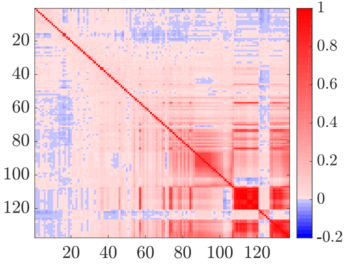

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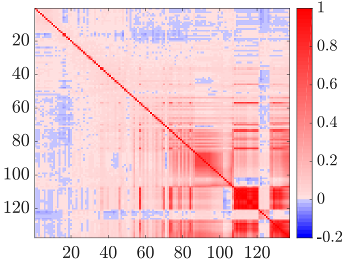

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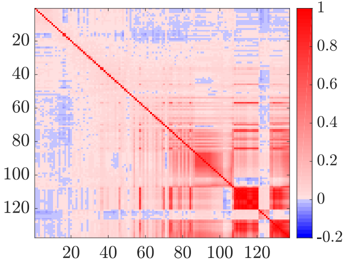

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- [Tabeart et al, 2018] proved that the minimum eigenvalue of the observation error covariance matrix is important for the conditioning of the general data assimilation problem.

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- Examples of methods:
  - Thresholding
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  - General regularisation methods.

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- Examples of methods:
  - Thresholding
  - Tapering
  - General regularisation methods.
- We will focus on a method that is used at the Met Office for numerical weather prediction. This method works by altering the eigenvalues of the original covariance matrix $\mathbf{R}$.

Reminder about variances and correlations

Want to also consider how variances and correlations are changed by the two methods. Let

$$\mathbf{R} = \mathbf{\Sigma} \mathbf{C} \mathbf{\Sigma}, \tag{2}$$

where $\mathbf{C}$ is the correlation matrix, and $\mathbf{\Sigma}$ is the diagonal matrix of standard deviations. We calculate $\mathbf{C}$ and $\mathbf{\Sigma}$ via:

$$\mathbf{\Sigma}(i,i) = \sqrt{\mathbf{R}(i,i)} \tag{3}$$

and

$$\mathbf{C}(i,j) = \frac{\mathbf{R}(i,j)}{\sqrt{\mathbf{R}(i,i)}\sqrt{\mathbf{R}(j,j)}}. \tag{4}$$

The ridge regression (RR) and minimum eigenvalue (ME) methods

Both methods improve the condition number of a covariance matrix by altering their eigenvalues to yield a reconditioned matrix with a user-defined condition number κ_{max}.

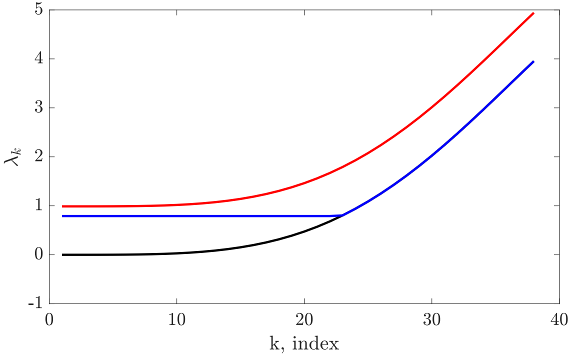

Figure: Illustration of recond methods: original spectrum (black), and spectrum reconditioned via ME and RR

Ridge regression method

Idea: Add a scalar multiple of identity to $\mathbf{R}$ *to obtain reconditioned* $\mathbf{R}_{RR}$ *with* $\kappa(\mathbf{R}_{RR}) = \kappa_{max}$.

Setting δ

- *Define* $\delta = \frac{\lambda_1(\mathbf{R}) - \lambda_p(\mathbf{R})\kappa_{max}}{\kappa_{max} - 1}$.
- *Set* $\mathbf{R}_{RR} = \mathbf{R} + \delta \mathbf{I}$

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We can prove theoretically:

- Standard deviations are increased by using this method.
- Absolute value of off-diagonal correlations decreased by this method.

IASI matrix - framework

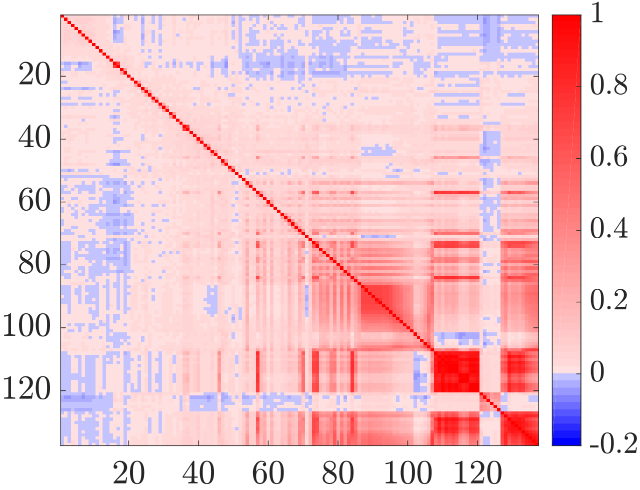
Interchannel correlations for a covariance matrix of satellite observation errors

- The UK Met Office diagnosed a correlated observation error covariance matrix in 2010.
- This was extremely ill-conditioned and crashed the system when used directly.
- Use 137 channels - **Original condition number:** 27703**.**

We study the impact of reconditioning in the **1D-Var** procedure.

- Run prior to every 4D-Var/forecast cycle.
- Assimilates each observation individually
- Used as quality control (reject ob if it doesn't converge in 10 iterations)
- Also used to fix values for variables that aren't assimilated in 4D-Var procedure.

Diagnosed IASI correlation matrix



Experimental choices of $\mathbf{R}_{RR}$ - standard deviations

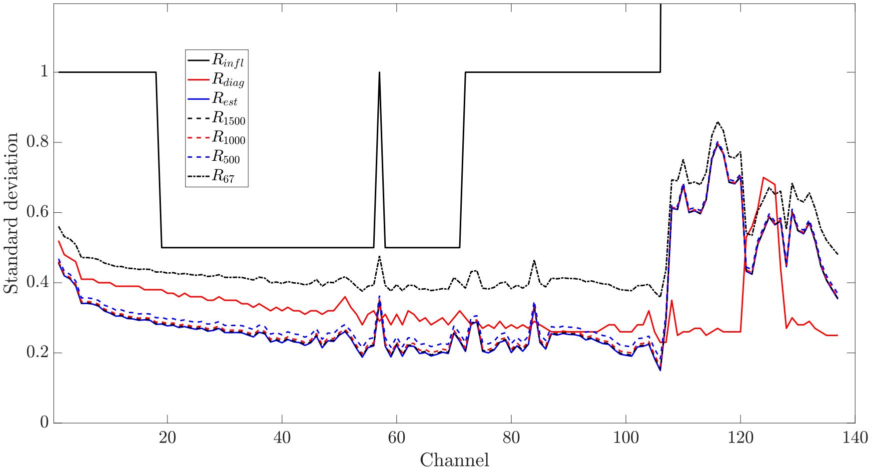


Figure: Standard deviation for each of the experiment choices

Experimental choices of $\mathbf{R}_{RR}$ - correlations

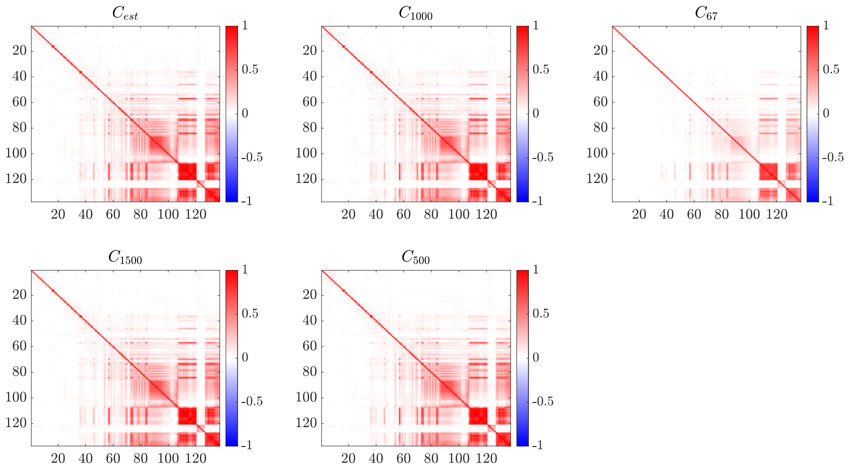


Figure: Changes to correlation with reconditioning for the correlated experiments

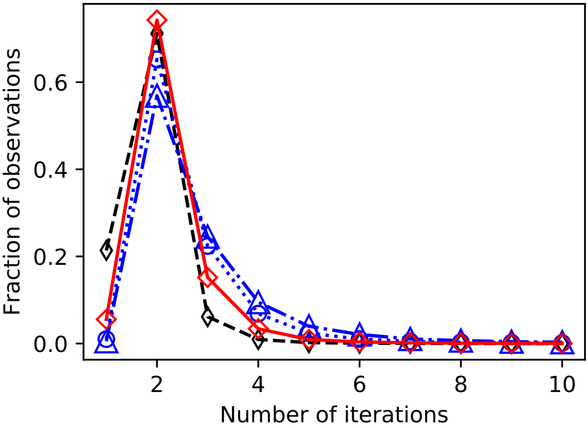


Figure: Number of iterations required to reach convergence of the 1D-Var minimization as a fraction of the total number of observations common to all choices of **R**. Symbols correspond to: $\triangle = \mathbf{R}_{diag}$, $\circ = \mathbf{R}_{est}$, $\Diamond = \mathbf{R}_{67}$, $\Diamond = \mathbf{R}_{infl}$.

Impact on temperature and humidity

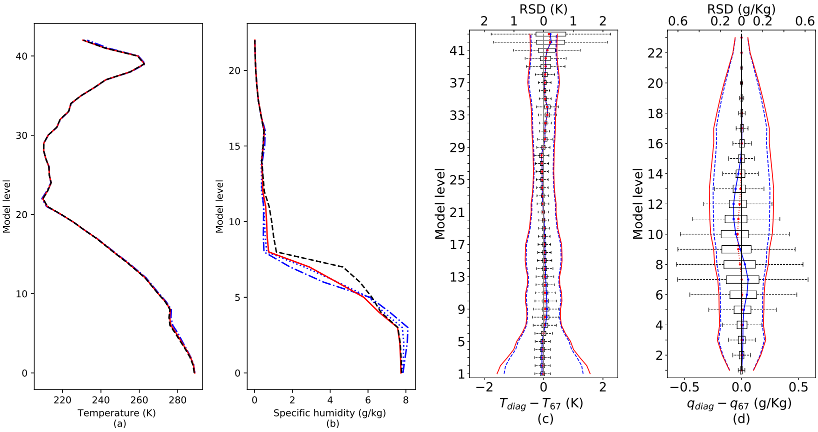


Figure: Example retrieved profiles of temperature (a) and specific humidity (b), and differences in retrievals between E_{diag} and E_{67} for temperature (c) and specific humidity (d) for 97330 observations.

Impact of reconditioning on quality control procedure

Set	No. of accepted obs	No of obs accepted by both E_{diag} and E_{exp}
$\mathbf{R}_{diag}$	100686	99039
$\mathbf{R}_{est}$	100655	99175
$\mathbf{R}_{1000}$	101002	99352
$\mathbf{R}_{500}$	101341	99656
$\mathbf{R}_{67}$	102333	100382
$\mathbf{R}_{infl}$	102859	100679

Table: Change to number of accepted observations with reconditioning

Impact of reconditioning on variables not in 4D-Var state vector

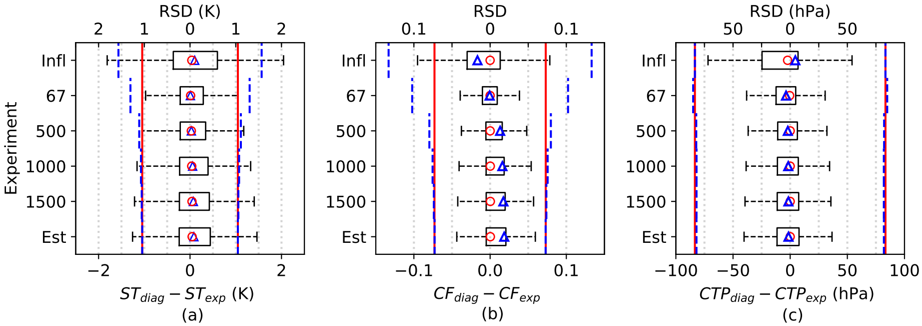


Figure: Change to estimates for skin temperature (left), cloud fraction (centre) and cloud top pressure (right) with reconditioning.

What about the outliers?

	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
% outliers (CF)	23.9	24.0	24.2	24.6	25.3	21.4
% outliers (CTP)	22.8	22.8	23.0	22.9	21.4	18.8
% outliers (ST)	15.1	15.3	15.6	16.3	17.6	15.9
Max diff (ST (K))	21.67	21.12	21.14	22.38	21.03	26.83
Min diff (ST (K))	-33.52	-33.01	-32.14	-29.76	-23.82	-20.88

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Think about **extreme outliers** - defined here as mean $\pm$ 0.25 $\times$ max difference

	E_{est}	E_{1500}	E_{1000}	E_{500}	E_{67}	E_{infl}
% $\|CF\| > 0.25$	4.9	4.7	4.4	3.9	3.2	7.5
% $\|CTP\| > 225hPa$	3.3	3.3	3.3	3.3	2.7	4.4
% $\|ST\| > 5K$	1.6	1.5	1.5	1.4	1.4	3.6

Conclusions

- Including correlated observation error in data assimilation methods is important for high-resolution forecasts and to make the best use of observation information.
- Convergence problems can be mitigated by using reconditioning methods.
- Tests in the Met Office 1D-Var system show that:
  - the ridge regression method improves convergence.
  - the quality control process is altered.
- Future work is needed to understand why some retrieved values change by a large amount.

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