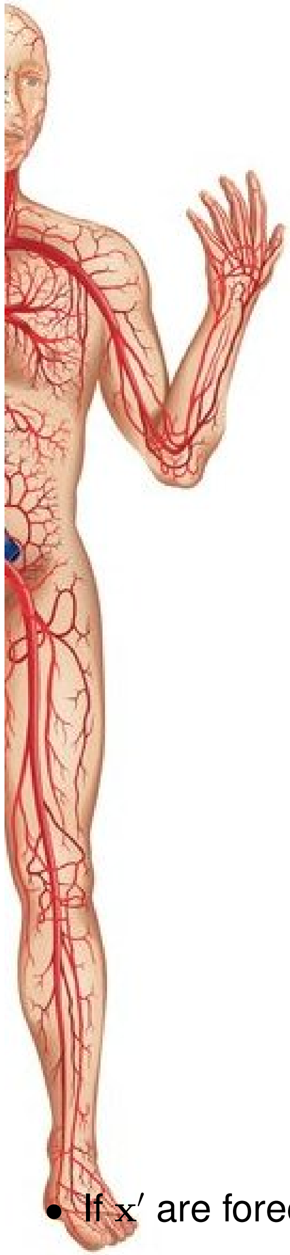
Anatomy of a covariance matrix



Univariate background error covariance matrix (e.g. if $\mathbf{x}$ represents a pressure field only):

variance

$$\mathbf{x} = \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix}, \qquad \mathrm{cov}(\mathbf{p}') = \langle \mathbf{p}'\mathbf{p}'^{\mathrm{T}} \rangle = \begin{pmatrix} \langle p_1'^2 \rangle & \langle p_1' p_2' \rangle & \cdots & \langle p_1' p_n' \rangle \\ \langle p_2' p_1' \rangle & \langle p_2'^2 \rangle & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \langle p_n' p_1' \rangle & \cdots & \cdots & \langle p_n'^2 \rangle \end{pmatrix}.$$

outer product

covariance
(univariate)

where $\mathbf{p}' = \mathbf{p} - \langle \mathbf{p} \rangle$.

Multivariate background error covariance matrix (e.g. if $\mathbf{x}$ represents pressure, zonal wind and meridional wind):

autocovariance sub–matrix

$$\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{u} \\ \mathbf{v} \end{pmatrix} = \begin{pmatrix} p_1 \\ \vdots \\ p_{n/3} \\ u_1 \\ \vdots \\ u_{n/3} \\ v_1 \\ \vdots \\ v_{n/3} \end{pmatrix}, \qquad \mathrm{cov}(\mathbf{x}') = \langle \mathbf{x}'\mathbf{x}'^{\mathrm{T}} \rangle = \begin{pmatrix} \langle \mathbf{p}'\mathbf{p}'^{\mathrm{T}} \rangle & \langle \mathbf{p}'\mathbf{u}'^{\mathrm{T}} \rangle & \langle \mathbf{p}'\mathbf{v}'^{\mathrm{T}} \rangle \\ \langle \mathbf{u}'\mathbf{p}'^{\mathrm{T}} \rangle & \langle \mathbf{u}'\mathbf{u}'^{\mathrm{T}} \rangle & \langle \mathbf{u}'\mathbf{v}'^{\mathrm{T}} \rangle \\ \langle \mathbf{v}'\mathbf{p}'^{\mathrm{T}} \rangle & \langle \mathbf{v}'\mathbf{u}'^{\mathrm{T}} \rangle & \langle \mathbf{v}'\mathbf{v}'^{\mathrm{T}} \rangle \end{pmatrix}.$$

multivariate covariance sub–matrix

These covariances are symmetric matrices.

- If $\mathbf{x}'$ are forecast errors, $\boldsymbol{\epsilon}^{\mathrm{B}}$, then above is $\mathbf{B}$-matrix.
- Observation error covariance: $\mathbf{R} = \langle \mathbf{y}'\mathbf{y}'^{\mathrm{T}} \rangle$, $\mathbf{y}'$ is observation error.