Introduction to the models
Data assimilation and visualisation in environmental sciences - NERC summer school

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Data Assimilation Research Centre

NATURAL ENVIRONMENT RESEARCH COUNCIL

Overview

The models

- Lorenz 63
- Barotropic vorticity
- DALEC

Lorenz 63 model

Edward N. Lorenz.
Deterministic Nonperiodic Flow.
Journal of the Atmospheric Sciences, 20:130–141, 1963.

Features

- 3 variables

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- 3 parameters

$$\sigma, \rho, \beta$$

- Nonlinear, deterministic equations
- Chaotic behaviour
- Butterfly-like attractor

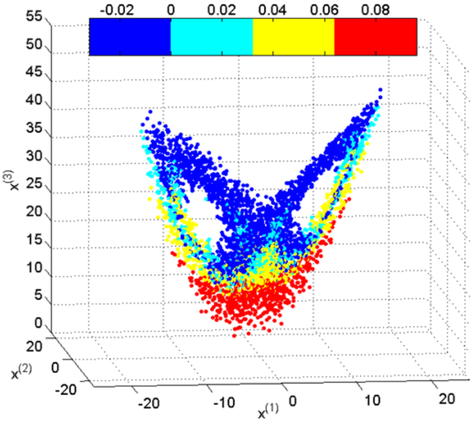
Lorenz 63 equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$

Solved using a forth-order Runge–Kutta integration scheme.

Parameter values $\sigma = 10, \rho = 28, \beta = 8/3$ lead to a chaotic system.

Regions of the attractor



Courtesy J. Amezcua. Reproduced from Evans et al. (2004)

Barotropic vorticity

The barotropic vorticity (BV) equation is derived from the Navier-Stokes equations,

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} + \frac{1}{\rho}\nabla p + g\mathbf{k} = \nu\nabla^2\mathbf{u}.$$

Vorticity q is defined as

$$q = \nabla \times \mathbf{u}$$

We assume incompressibility, no viscosity, no vertical flow and that flow is *barotropic*. i.e. $\rho = \rho(p)$. This results in the following PDE in q,

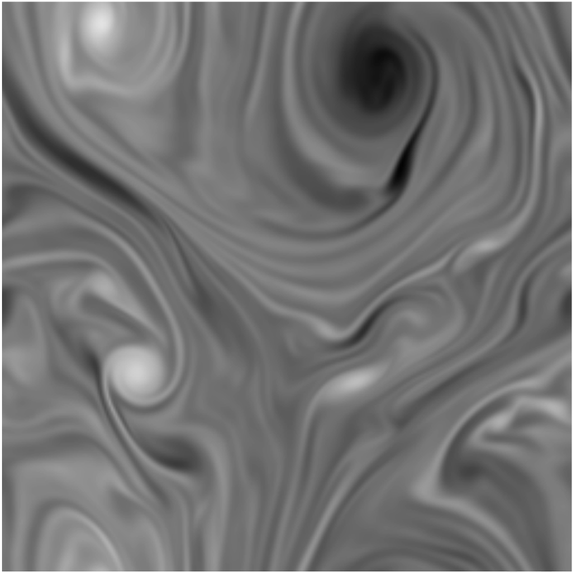
$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} = 0.$$

Barotropic vorticity

Features

- Fluid dynamics problem so similar to many meteorological examples
- Highly chaotic regimes
- Solved on a doubly periodic domain (torus)
- Efficient computations by use of FFT
- No balances that can be destroyed by data assimilation
- We use a 256×256 grid, so a 65536 dimensional problem

Barotropic vorticity



DALEC

Data Assimilation Linked Ecosystem Carbon model

A dynamic vegetation model which simulates the carbon cycle of forests.

Simple and extensively used.

Real datasets

Much more about this on Friday!

Twin experiments

State estimation

- Run the model once from a given starting point. This run is treated as the *truth*. From the truth we generate synthetic observations.
- Restart the model from some initial (background, prior etc) state.
- Now given the synthetic observations we generated earlier, can we constrain the system to be close to the truth?

Twin experiments

Parameter estimation

- Run the model once from a given starting point and with set model parameters. This run is treated as the *truth*. From the truth we generate synthetic observations.
- Restart the model from some initial (background, prior etc) state, but with incorrect parameters.
- Now given the synthetic observations we generated earlier, can we constrain the system to be close to the truth, and can we estimate what the unobservable parameters should have been?

Practicalities of the models

Lorenz 63 - matlab (python today)
Barotropic vorticity - fortran
DALEC - matlab

Today (and Thursday), we shall be using ARCHER (www.archer.ac.uk), the UK national supercomputer. 72,192 processing cores, 216TB RAM.

Your will have to simply do some *truth* runs, and look at the output.