Methods to estimate B

Vn 2.0, Ross Bannister, r.n.bannister@reading.ac.uk

Reminder

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \qquad \begin{aligned} \mathbf{B} &= \left\langle (\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t}) (\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t})^\mathrm{T} \right\rangle, \\ &= \begin{pmatrix} \left\langle (x_{\mathrm{f}1} - x_{\mathrm{t}1})^2 \right\rangle & \langle (x_{\mathrm{f}1} - x_{\mathrm{t}1})(x_{\mathrm{f}2} - x_{\mathrm{t}2}) \rangle & \cdots & \langle (x_{\mathrm{f}1} - x_{\mathrm{t}1})(x_{\mathrm{f}n} - x_{\mathrm{t}n}) \rangle \\ \langle (x_{\mathrm{f}2} - x_{\mathrm{t}2})(x_{\mathrm{f}1} - x_{\mathrm{t}1}) \rangle & \left\langle (x_{\mathrm{f}2} - x_{\mathrm{t}2})^2 \right\rangle & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \langle (x_{\mathrm{f}n} - x_{\mathrm{t}n})(x_{\mathrm{f}1} - x_{\mathrm{t}1}) \rangle & \cdots & \cdots & \left\langle (x_{\mathrm{f}n} - x_{\mathrm{t}n})^2 \right\rangle \end{pmatrix}. \end{aligned}$$

$$\Big\langle \quad \Big\rangle : \text{ average over population of possible backgrounds.}$$

Problem

$\mathbf{x}_\mathrm{t}$ is unknowable so need a proxy for forecast error $\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t}$.

Popular approaches

Method	Description and references
"Canadian quick" method	$\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t} \sim \left(\mathbf{x}_\mathrm{f}(t+T) - \mathbf{x}_\mathrm{f}(T)\right)/\sqrt{2}$. Take population from one long time run. Polavarapu S., Ren S., Rochon Y., Sankey D., Ek N., Koshyk J., Tarasick D., Data assimilation with the Canadian middle atmosphere model. Atmos.-Ocean 43: 77–100 (2005).
Analysis of innovations $\mathbf{d} = \mathbf{y} - \mathbf{H}\mathbf{x}_\mathrm{f}$	Choose a pair of direct and independent obs separated by r: $$[y(r) - x_\mathrm{f}(r)]\,[y(r+\Delta r) - x_\mathrm{f}(r+\Delta r)] = [\{y(r) - x_\mathrm{t}(r)\} - \{x_\mathrm{f}(r) - x_\mathrm{t}(r)\}]\,[\{y(r+\Delta r) - x_\mathrm{t}(r+\Delta r)\} - \{x_\mathrm{f}(r+\Delta r) - x_\mathrm{t}(r+\Delta r)\}]$$ $$\langle[\epsilon^y(r) - \epsilon^{x_\mathrm{f}}(r)]\,[\epsilon^y(r+\Delta r) - \epsilon^{x_\mathrm{f}}(r+\Delta r)]\rangle = \langle\epsilon^y(r)\epsilon^y(r+\Delta r)\rangle + \langle\epsilon^{x_\mathrm{f}}(r)\epsilon^{x_\mathrm{f}}(r+\Delta r)\rangle\,,$$ (above assumes obs and bg errors are uncorrelated). Take population from many pairs with same Δr. Furthermore if $\Delta r > 0$: $\langle\epsilon^y(r)\epsilon^y(r+\Delta r)\rangle = 0$. Rutherford I.D. 1972. Data assimilation by statistical interpolation of forecast error fields. J. Atmos. Sci. 29: 809–815. Hollingsworth A., Lönnberg P., The statistical structure of short-range forecast errors as determined from radiosonde data. Part I: The wind field. Tellus 38A: 111–136 (1986). Järvinen H., Temporal evolution of innovation and residual statistics in the ECMWF variational data assimilation systems. Tellus 53A: 333–347 (2001).
NMC method	Choose pairs of lagged forecasts valid at the same time, e.g.: $\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t} \sim \left(\mathbf{x}_\mathrm{f}^{48}(t) - \mathbf{x}_\mathrm{f}^{24}(t)\right)/\sqrt{2}$. Take population from difference at many times. Parrish D.F., Derber J.C., The National Meteorological Center's spectral statistical interpolation analysis system. Mon. Wea. Rev. 120 1747–1763 (1992). Berre L., Ştefănescu S.E., Pereira M.B., The representation of the analysis effect in three error simulation techniques. Tellus 58A 196–209 (2006).
Ensemble method	If you have an ensemble that is correctly spread: $\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t} \sim \mathbf{x}_\mathrm{f}^{(i)} - \langle\mathbf{x}_\mathrm{f}\rangle$ or $\mathbf{x}_\mathrm{f} - \mathbf{x}_\mathrm{t} \sim \left(\mathbf{x}_\mathrm{f}^{(i)} - \mathbf{x}_\mathrm{f}^{(j)}\right)/\sqrt{2}$. Take population from ensemble members and over many times. Houtekamer P.L., Lefaivre L., Derome J., Ritchie H., Mitchell H.L., A system simulation approach to ensemble prediction. Mon. Wea. Rev. 124, 1225–1242 (1996). Buehner M., Ensemble derived stationary and flow dependent background error covariances: Evaluation in a quasi-operational NWP setting. Q.J.R. Meteorol. Soc. 131, 1013–1043 (2005).