From Remote Sensing of the Atmosphere to Data Assimilation

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December 2012

Outline

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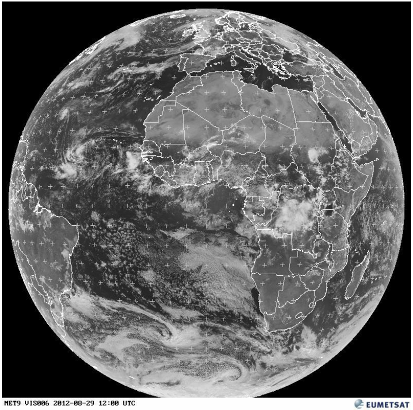
Soundings

Principles of Retrievals

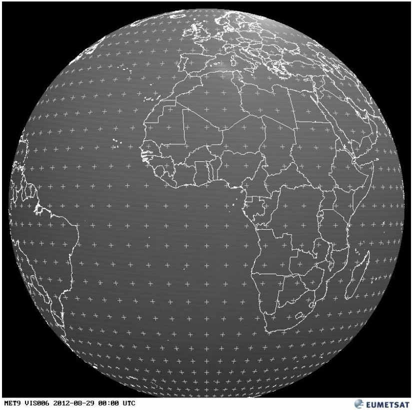
Mathematical Framework for Retrievals

Visible Imagery



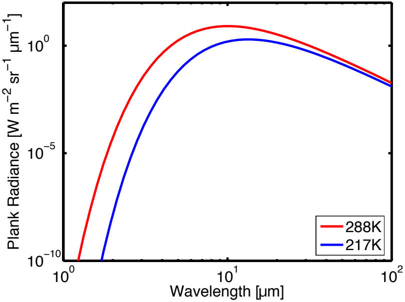
- Visible light image from SEVIRI on Meteosat-9
- SEVIRI = Spinning Enhanced Visible and InfraRed Imager
- SEVIRI measures light through telescope with field of view about 3 km across at sub-satellite point
- As satellite rotates, field of view scans E-W
- Telescope mirror tilted to scan N-S
- One image every 15 min

The Problem with Visible Imagery



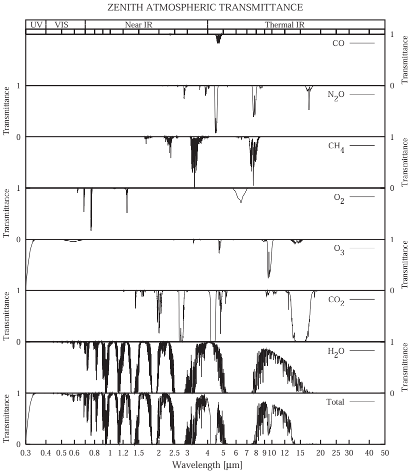
- Visible imagery relies on reflected sunlight, so doesn't work at night.
- Fortunately, there's a whole electromagnetic spectrum out there ...

Emitted Infrared Radiation



- All bodies emit electromagnetic radiation on account of their temperature.
- For a perfectly emitting blackbody, the emitted radiance is given by the Planck function (left).
- At terrestrial temperatures, the peak radiation is emitted in the infrared around 10 μm.
- This suggests that we try observing there.

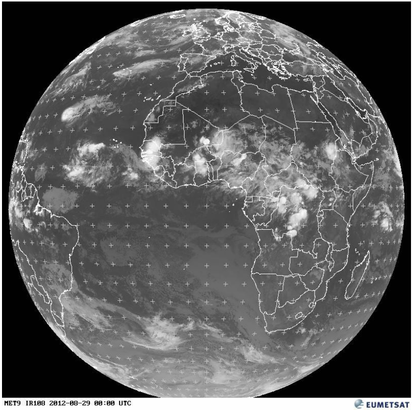
Atmospheric Transmittance



Source: Petty (2006)

- The atmosphere is opaque at many wavelengths because of absorption of radiation by its constituent gases.
- We should avoid these wavelengths if we want to see down to the surface.
- There are relatively transparent windows around 3.7 µm and 8–12 µm (the latter punctuated by ozone absorption near 9.6 µm).

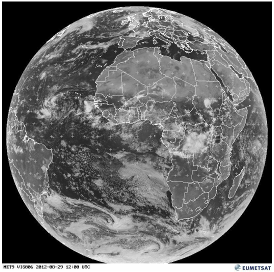
Infrared Imagery



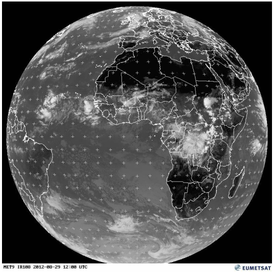
- Image from SEVIRI's 10.8 µm channel at 0 UTC
- Observing emitted radiation frees us from dependence on sunlight.

An Infrared Image Is a Heat Map

Visible:

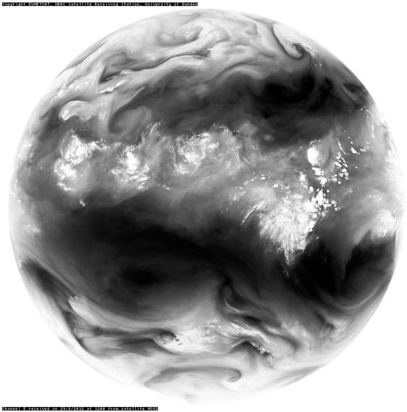


Infrared:



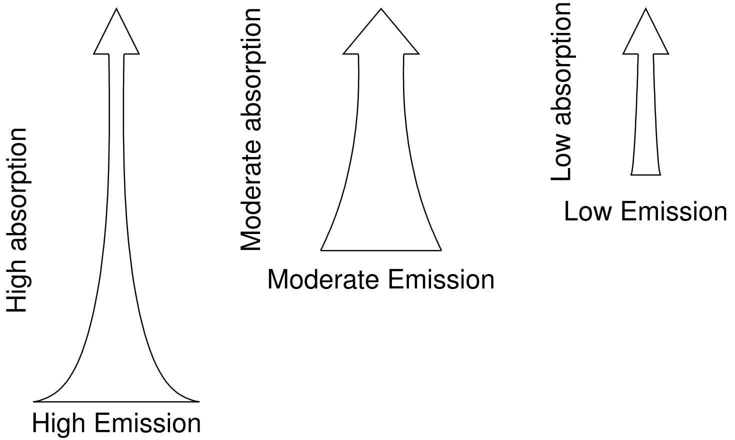
- Warm bodies (Earth's surface, low clouds) emit more radiation than cold bodies (high cloud tops).
- Shading convention:
  - Black = high radiance
  - White = low radiance
- This is the opposite to what one might expect, but ensures deep clouds appear white.
- Compare the deep convective cloud over DR Congo and Sudan with the shallow cloud off the coast of Angola and Namibia, almost invisible in the infrared.
- Note also the hot deserts.

What Does the Earth Look Like at an Opaque Wavelength?



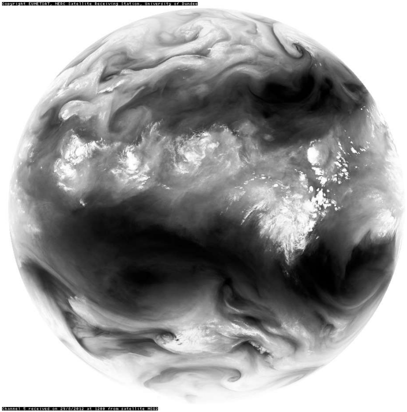
- Image from SEVIRI's 6.2 μm channel
- Water vapour (WV) absorbs at this wavelength.
- Can no longer see the Earth's surface
- Can still see the highest clouds
- Elsewhere, recall Kirchhoff's law: a good absorber is a good emitter.
- The radiation comes from the WV in the air.

Where Exactly Does the Radiation Come from?



- The received radiation is dominated by radiation from the middle troposphere.
- The more WV there is in a column of air, the higher in the atmosphere the radiation comes from.

Water Vapour Imagery



- WV imagery uses the same reverse shading convention as IR imagery.
- Black = warm = radiation from low in troposphere = dry column of air
- White = cold = radiation from high in troposphere = wet column of air

Imagers v Sounders

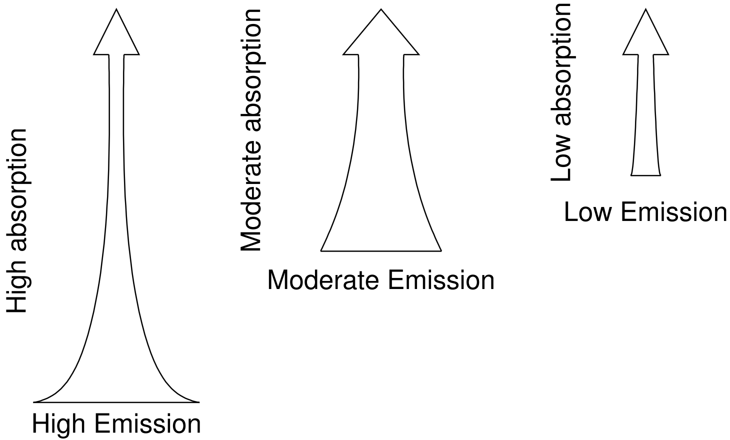
- Satellite instruments can be classified as imagers or sounders:

  Imagers have a high horizontal resolution and observe the atmosphere at a few wavelengths.

  Sounders have a coarser horizontal resolution (tens of km) but observe the atmosphere at many wavelengths.

- The many wavelengths enable vertical profiles of temperature, water vapour, and other trace gases to be retrieved.
- The distinction between modern imagers and sounders isn't clear cut: sounders can produce images and imagers can have limited sounding capabilities.

The Physics of Sounding



- The physical basis of sounding is the same as that of WV imagery, but applies to other absorbing gases too.
- Let's make this more quantitative.

Assumptions

We make some simplifying assumptions:

- The satellite looks vertically downwards.
- The satellite observes a single wavelength λ.
- There are no clouds in the field of view.
- The atmosphere is sufficiently opaque that no radiation from the surface reaches the satellite.
- Scattering of radiation is negligible compared to emission and absorption.

These assumptions are not necessary in a more detailed treatment.

Radiative Transfer Equation

Under these assumptions, the radiance reaching the satellite at wavelength λ is

$$L_\lambda = \int_0^{z_\mathrm{T}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

where

- z is altitude
- z_T is altitude of top of atmosphere
- $T(z)$ is temperature of atmosphere at altitude z
- $B_\lambda(T)$ is blackbody radiance at wavelength λ, temperature T
- $\tau_\lambda(z)$ is transmittance at wavelength λ between altitude z and top of atmosphere

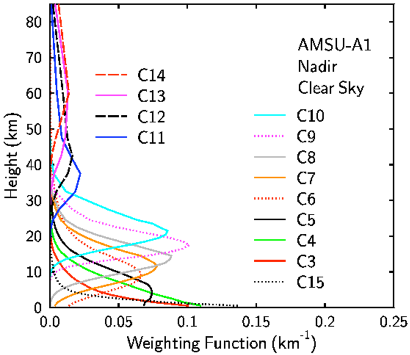
Weighting Function

$$L_\lambda = \int_0^{z_\mathrm{T}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

- $\frac{d\tau_\lambda}{dz}$ is called the weighting function.
- It is always positive.
- L_λ is like a weighted average of blackbody radiances at different levels of the atmosphere.
- It is a true average because

$$\int_0^{z_\mathrm{T}} \frac{d\tau_\lambda}{dz} dz = \tau_\lambda(z_\mathrm{T}) - \tau_\lambda(0) = 1 - 0 = 1$$

Weighting Functions



Source: `amsu.cira.colostate.edu`

- Different wavelengths have different transmittance profiles and hence different weighting functions.
- The graph shows the weighting functions of the AMSU-A instrument.
- AMSU = Advanced Microwave Sounding Unit
- By observing at multiple wavelengths, a sounder obtains information about the atmosphere at multiple levels.

Brightness Temperature

- If the atmosphere were isothermal with temperature T, then $L_\lambda = B_\lambda(T)$.
- We could recover T by inverting the Planck function $B_\lambda(T)$.
- In a non-isothermal atmosphere, we can define the brightness temperature at wavelength λ as the temperature such that $L_\lambda = B_\lambda(T_\mathrm{B})$.
- T_B is also known as the equivalent blackbody temperature.
- T_B is a complicated average of the temperature at all levels of the atmosphere.
- This average is weighted towards the level where the weighting function peaks.
- Time-series of T_B are used for climate monitoring.
- But what we'd really like to know is the temperature profile $T(z)$.

Imagery

Soundings

Principles of Retrievals

Mathematical Framework for Retrievals

Temperature Soundings

$$L_\lambda = \int_0^{z_\mathrm{T}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

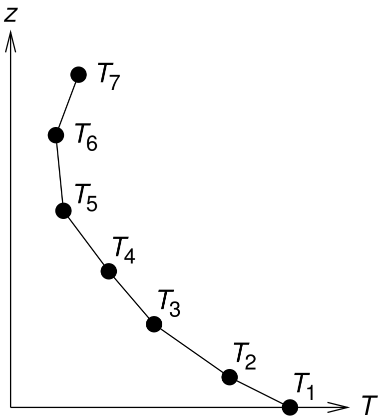
- To calculate $\tau_\lambda(z)$ and thus L_λ, we need to know $T(z)$ and the mixing ratio profile $q(z)$ of the absorbing gas.
- For a well-mixed gas like CO_2 or O_2, $q(z)$ is a know constant.
- Soundings for temperature are made using CO_2 absorption in the infrared and O_2 absorption in the microwave.
- For such a gas, we can calculate L_λ given only $T(z)$.
- If the sounder observes at p different wavelengths, we have p different equations for $T(z)$.
- But $T(z)$ has infinitely many unknowns (one for each altitude), so these equations are not enough to determine it.

Some Terminology

$$L_\lambda = \int_0^{z_{\mathrm{T}}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

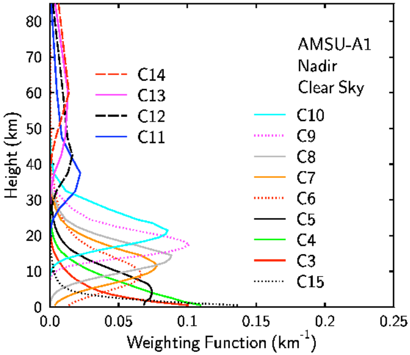
- The problem of calculating L_λ given $T(z)$ is called the forward problem.
- The problem of finding $T(z)$ given measurements of L_λ is called the inverse problem.
- The $T(z)$ found by solving the inverse problem is said to have been retrieved from the measurements and is called a retrieval.

Temperature Retrievals



- We have to adopt a discrete representation of $T(z)$.
- One possibility is to represent $T(z)$ by its values at n discrete altitudes and to assume that it varies linearly in between (see left).
- We then have p equations in n unknowns.
- We might hope that by taking $n = p$ we could solve these equations to determine $T(z)$.
- But there are problems ...

Ill-Conditioning



Source:
`amsu.cira.colostate.edu`

- There is a large overlap between the weighting functions for different wavelengths.
- As a result, the p equations are not very independent.
- The problem is ill-conditioned: small errors in the data lead to large errors in the solution.
- And there will be errors in L_λ due to measurement noise and other sources.
- We have to do something else.

Solutions

1. Accept a lower resolution for the discrete temperature profile ($n < p$) and find the profile that best matches the radiance measurements. This can avoid ill-conditioning.
2. Use additional information about the temperature profile, such as climatology or a prediction from a numerical weather prediction (NWP) system. Then we can take $n = p$ or even $n > p$ and avoid ill-conditioning.
3. Don't bother with retrievals. The data assimilation component of an NWP system generally doesn't use a retrieval. Instead, it uses the radiance measurements directly and adjusts the forecast to find the best simultaneous match to these and all other measurements.

Humidity Retrievals

$$L_\lambda = \int_0^{z_\mathrm{T}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

- Sounders also observe at wavelengths sensitive to absorption by water vapour.
- Recall that to evaluate the integral in the radiative transfer equation we need to know the temperature profile $T(z)$ and the mixing ratio profile $q(z)$ of the absorbing gas.
- For temperature retrievals, we used a well-mixed gas for which $q(z)$ was a known constant.
- To retrieve water vapour, we can use $T(z)$ from a temperature retrieval.
- Then all of the preceding applies with $q(z)$ as our unknown instead of $T(z)$.
- This applies to other trace gases too.

Imagery

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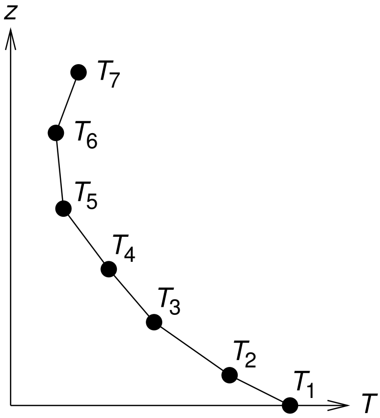
Mathematical Framework for Retrievals

Recapitulation of the Temperature Retrieval Problem



- We have radiance measurements at p different wavelengths sensitive to absorption by a well-mixed gas of known mixing ratio.
- We want to use these measurements to determine the n numbers that characterise a discrete representation of the temperature profile.
- We want to avoid ill-conditioning, so the naive approach of taking $n = p$ and solving p equations in p unknowns won't work.

State Space and Observation Space

- Join the temperatures $T_1, \ldots, T_n$ discretely representing the temperature profile into a vector:

$$\mathbf{x} = \begin{pmatrix} T_1 \\ \vdots \\ T_n \end{pmatrix}$$

- $\mathbf{x}$ represents the temperature profile as a point in n-dimensional state space.
- Join the radiances observed at the wavelengths $\lambda_1, \ldots, \lambda_p$ into a vector:

$$\mathbf{y} = \begin{pmatrix} L_{\lambda_1} \\ \vdots \\ L_{\lambda_p} \end{pmatrix}$$

- $\mathbf{y}$ represents the observations as a point in p-dimensional observation space.

Observation Operator

$$L_\lambda = \int_0^{z_{\mathrm{T}}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

- Given $\mathbf{x}$, we are assuming that the integral in the radiative transfer equation can be evaluated for the temperature profile represented by $\mathbf{x}$.
- Let $H_i(\mathbf{x})$ denote this integral evaluated at wavelength λ_i.
- Assemble the $H_i(\mathbf{x})$ into a vector:

$$H(\mathbf{x}) = \begin{pmatrix} H_1(\mathbf{x}) \\ \vdots \\ H_p(\mathbf{x}) \end{pmatrix}$$

- The function $H(\mathbf{x})$ is called the observation operator.
- $H(\mathbf{x})$ is what the observations would be if $\mathbf{x}$ were the true state of the atmosphere and there were no observation errors.

Observation Errors

- In the naive approach to temperature retrievals, we tried to solve the equation
$$\mathbf{y} = H(\mathbf{x})$$
- This didn't work because of ill-conditioning and because if $\mathbf{x}$ is the true state of the atmosphere, the observations will be not $H(\mathbf{x})$ but
$$\mathbf{y} = H(\mathbf{x}) + \varepsilon$$
where ε is the observation error vector.
- The ith component of ε represents the error in the measurement of the radiance L_{λ_i}.
- In general, we don't know ε, only its statistical properties.

Observation Error Covariance Matrix

- Two basic statistical properties are mean and variance.
- We assume that the ith component of ε has mean zero and known variance σ_i^2.
- The observation error covariance matrix is a $p \times p$ matrix with these variances down the main diagonal:

$$\mathbf{R} = \begin{pmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_p^2 \end{pmatrix}$$

- If there are correlations between the errors in the measurements of different radiances, these can be represented by off-diagonal elements of $\mathbf{R}$.

Cost Function

With the machinery now created, we can define a cost function:

$$J(\mathbf{x}) = (\mathbf{y} - H(\mathbf{x}))^{\mathrm{T}} \mathbf{R}^{-1} (\mathbf{y} - H(\mathbf{x}))$$

Interpretation:

- $\mathbf{x}$ is a proposed temperature profile.
- $H(\mathbf{x})$ is what the radiances would be if $\mathbf{x}$ were the true temperature profile and there were no observation errors.
- $\mathbf{y} - H(\mathbf{x})$ is the difference between these hypothetical radiances and the actual observed radiances.
- $J(\mathbf{x})$ is a measure of the size of this difference in which more accurately observed radiances are given greater weight.

Variational Temperature Retrievals

- The smaller $J(\mathbf{x})$, the better the agreement between the proposed temperature profile $\mathbf{x}$ and the measured radiances.
- This leads to the following retrieval algorithm:

  Choose the value of $\mathbf{x}$ that minimises $J(\mathbf{x})$

- There are mathematical techniques and software packages for doing this.
- If $n = p$, the algorithm reduces to solving $\mathbf{y} = H(\mathbf{x})$ and we are no better off.
- But if we take $n < p$, we can avoid ill-conditioning.
- We avoid ill-conditioning at the expense of accepting a lower resolution temperature profile.

Use of a Background Term

- But what if we need a higher resolution temperature profile?
- All is not lost.
- Even in the absence of observations, we won't be totally ignorant about the temperature profile.
- We might have climatological information or a prediction from an NWP system.
- This information can be expressed as a background state estimate $\mathbf{x}_\mathrm{b}$ and a background error covariance matrix $\mathbf{B}$.
- We can incorporate these into the cost function:

$$J(\mathbf{x}) = (\mathbf{y} - H(\mathbf{x}))^\mathrm{T}\mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x})) + (\mathbf{x} - \mathbf{x}_\mathrm{b})^\mathrm{T}\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_\mathrm{b})$$

- The problem of minimising this $J(\mathbf{x})$ can remain well-conditioned even if $n > p$.

Humidity Retrievals

$$L_\lambda = \int_0^{z_\mathrm{T}} B_\lambda(T(z)) \frac{d\tau_\lambda}{dz} dz$$

- Recall that to evaluate the integral in the radiative transfer equation we needed to know the temperature profile $T(z)$ and the mixing ratio profile $q(z)$ of the absorbing gas.
- For temperature retrievals, we used a well-mixed gas for which $q(z)$ was a known constant.
- To retrieve water vapour, we can use $T(z)$ from a temperature retrieval.
- Then all of the preceding applies with $q(z)$ as our unknown.
- $\mathbf{x}$ will now be a discrete representation of the humidity mixing ratio profile.
- This all applies to other trace gases too.

Data Assimilation

The framework set up for retrievals is identical to the three-dimensional variational data assimilation (3D-Var) used by some NWP systems.

$$J(\mathbf{x}) = (\mathbf{y} - H(\mathbf{x}))^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x})) + (\mathbf{x} - \mathbf{x}_{\mathrm{b}})^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_{\mathrm{b}})$$

- $\mathbf{x}$ is the state of the atmosphere as represented by the NWP system, incorporating all its variables at all its grid points.
- $\mathbf{x}_{\mathrm{b}}$ is the current state according to a previous forecast.
- $\mathbf{y}$ contains all of the observations from the global observing system that it has been decided to assimilate.
- $H(\mathbf{x})$ calculates what these observations would be if $\mathbf{x}$ were the true state of the atmosphere.

Further Reading

Clive D. Rodgers.
Inverse Methods for Atmospheric Sounding: Theory and Practice.
World Scientific, 2000.