

# Particle Filters

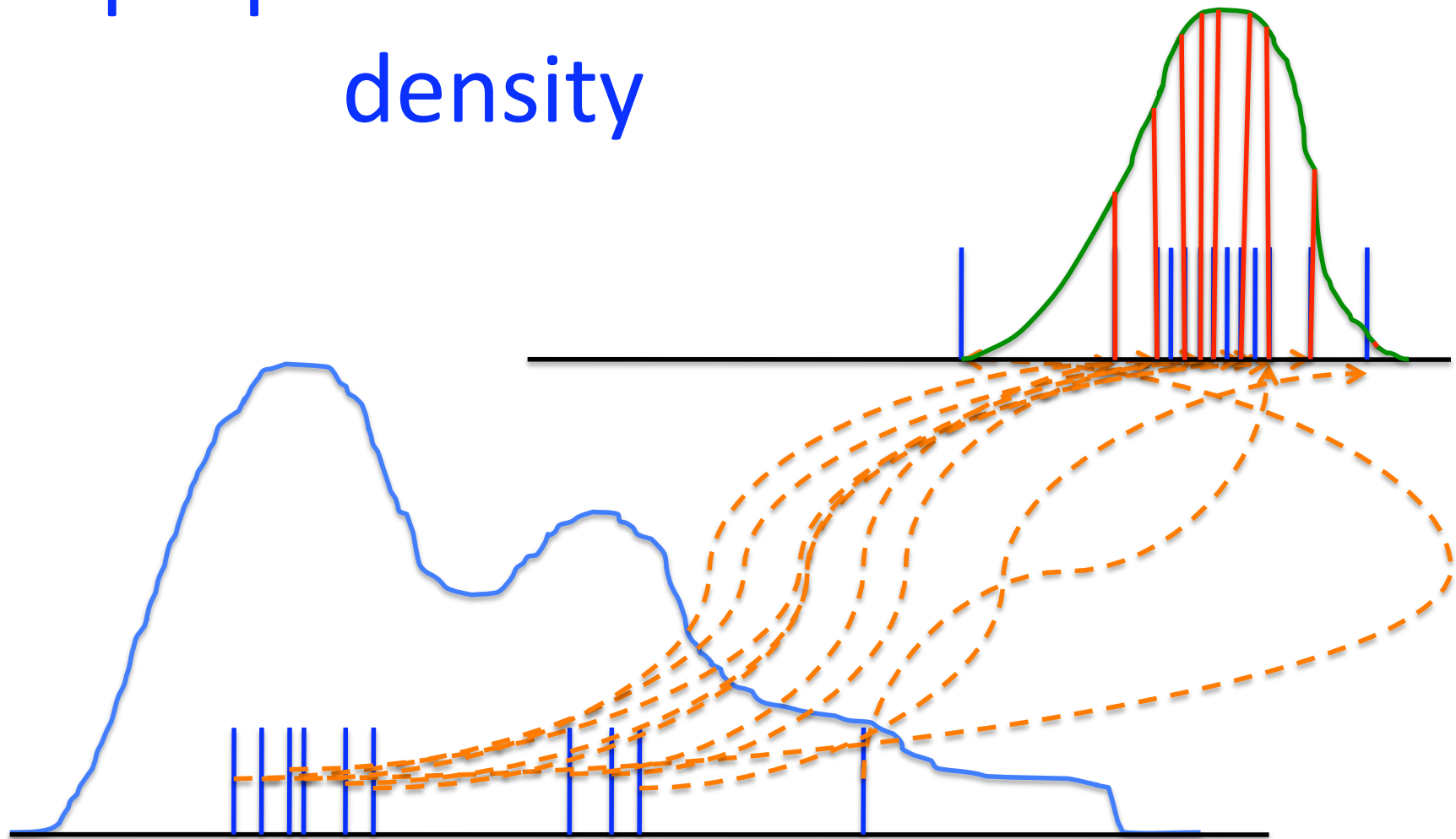
## Part II

### Demonstrations

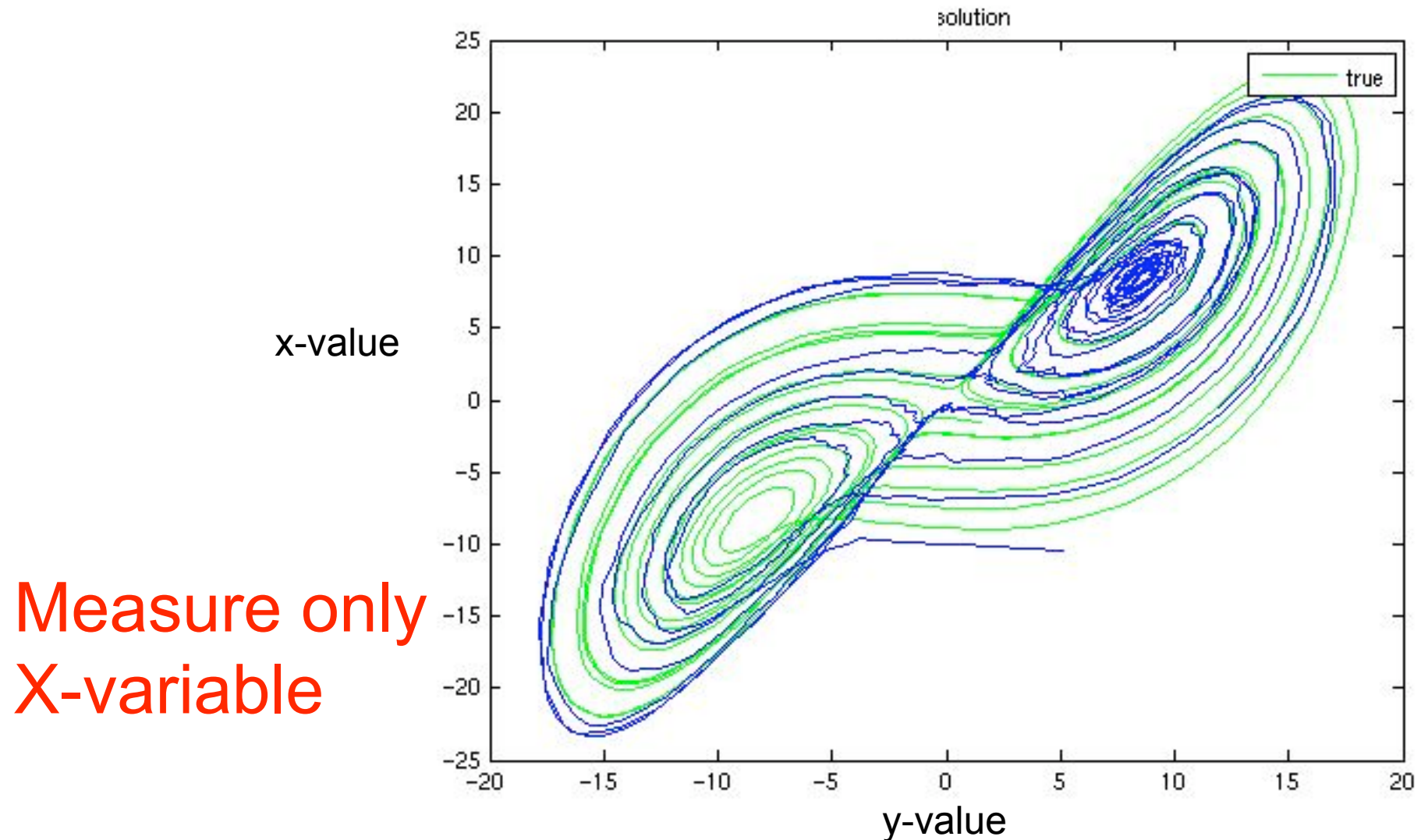
Peter Jan van Leeuwen  
Data-Assimilation Research Centre DARC  
University of Reading

*IIRS, Dehra Dun, December 2012*

# Particle filter with proposal transition density



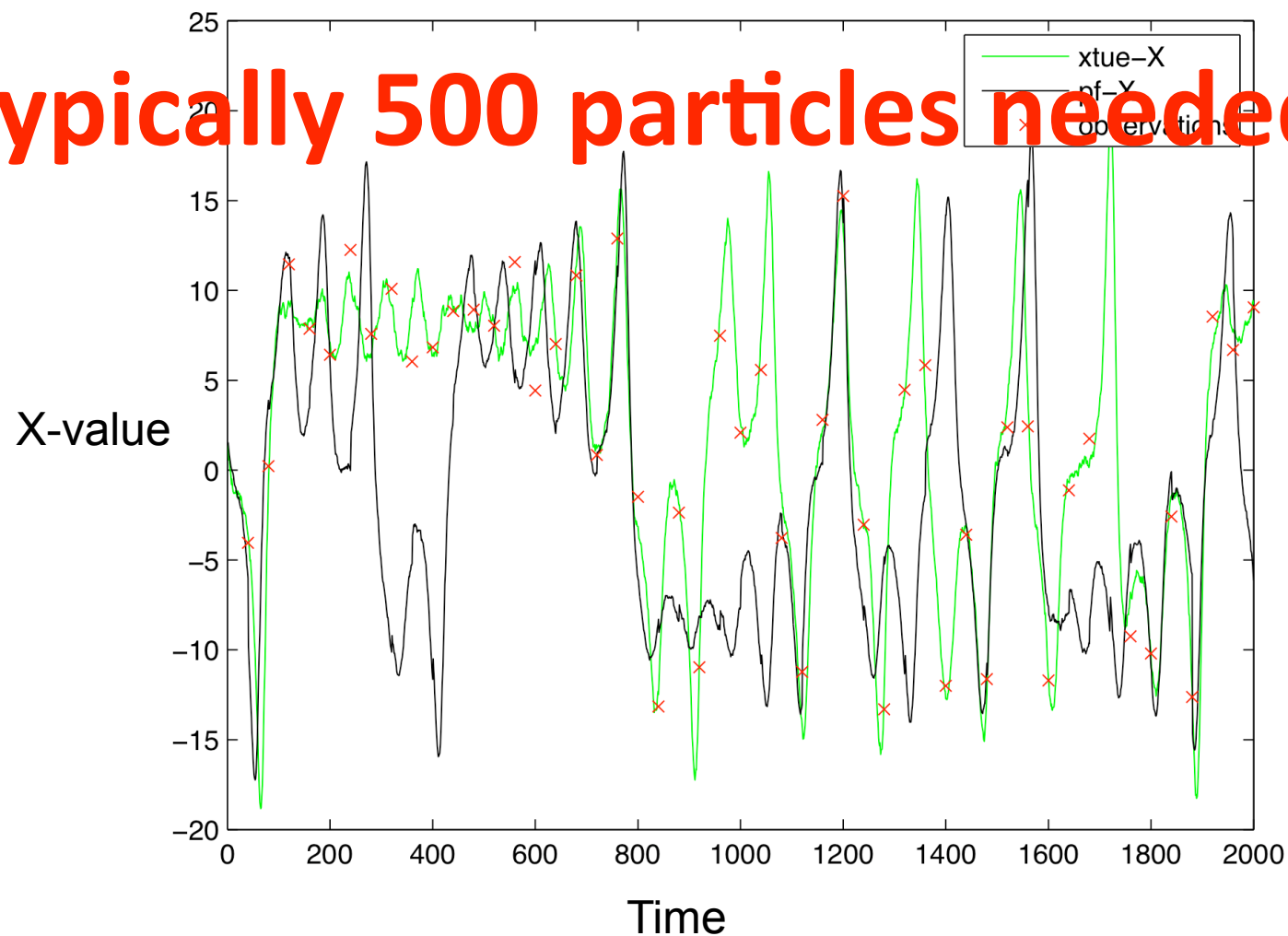
# Experiment: Lorentz 1963 model (3 variables $x, y, z$ , highly nonlinear)



# Standard Particle filter with resampling

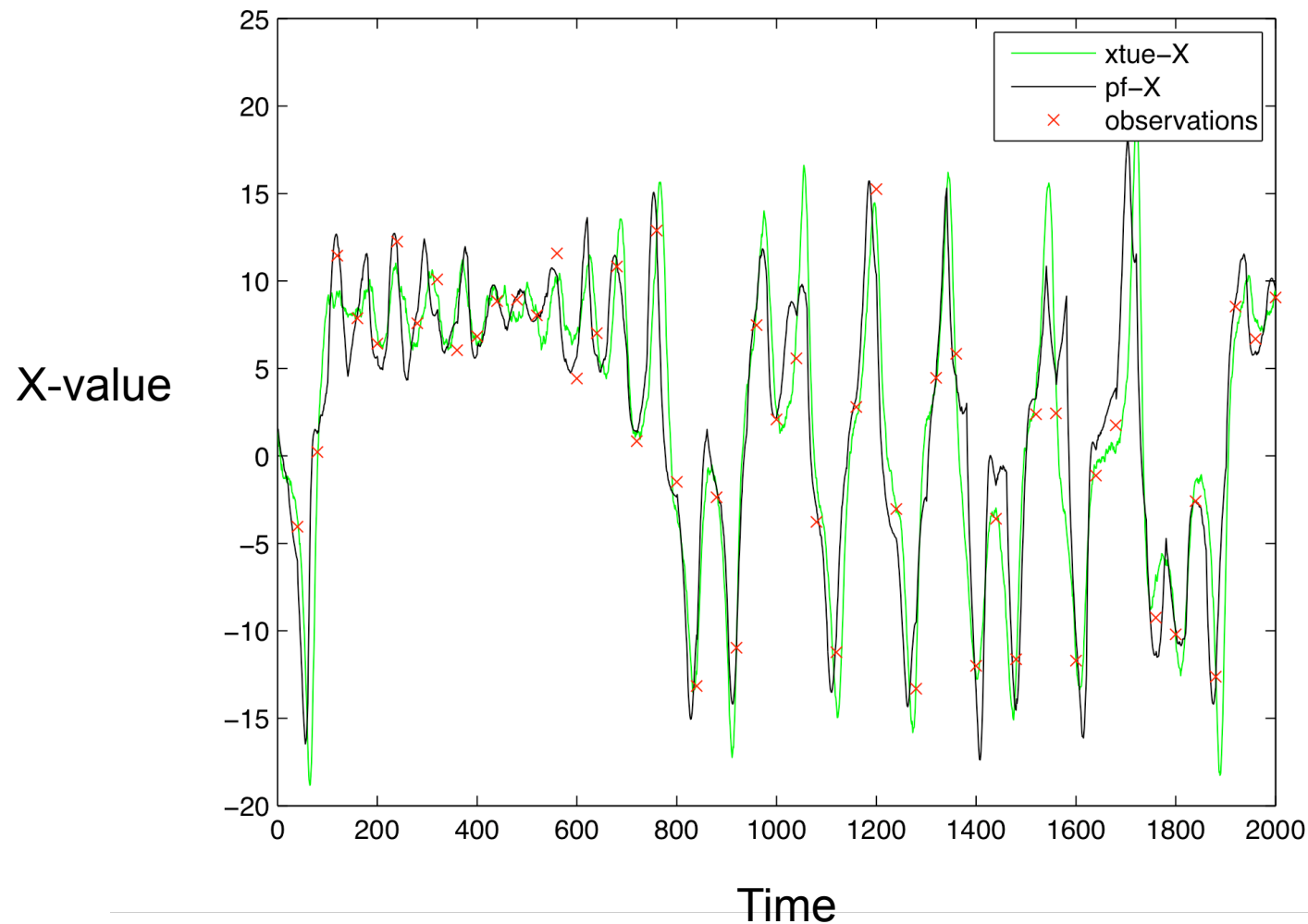
## 20 particles

**Typically 500 particles needed !**

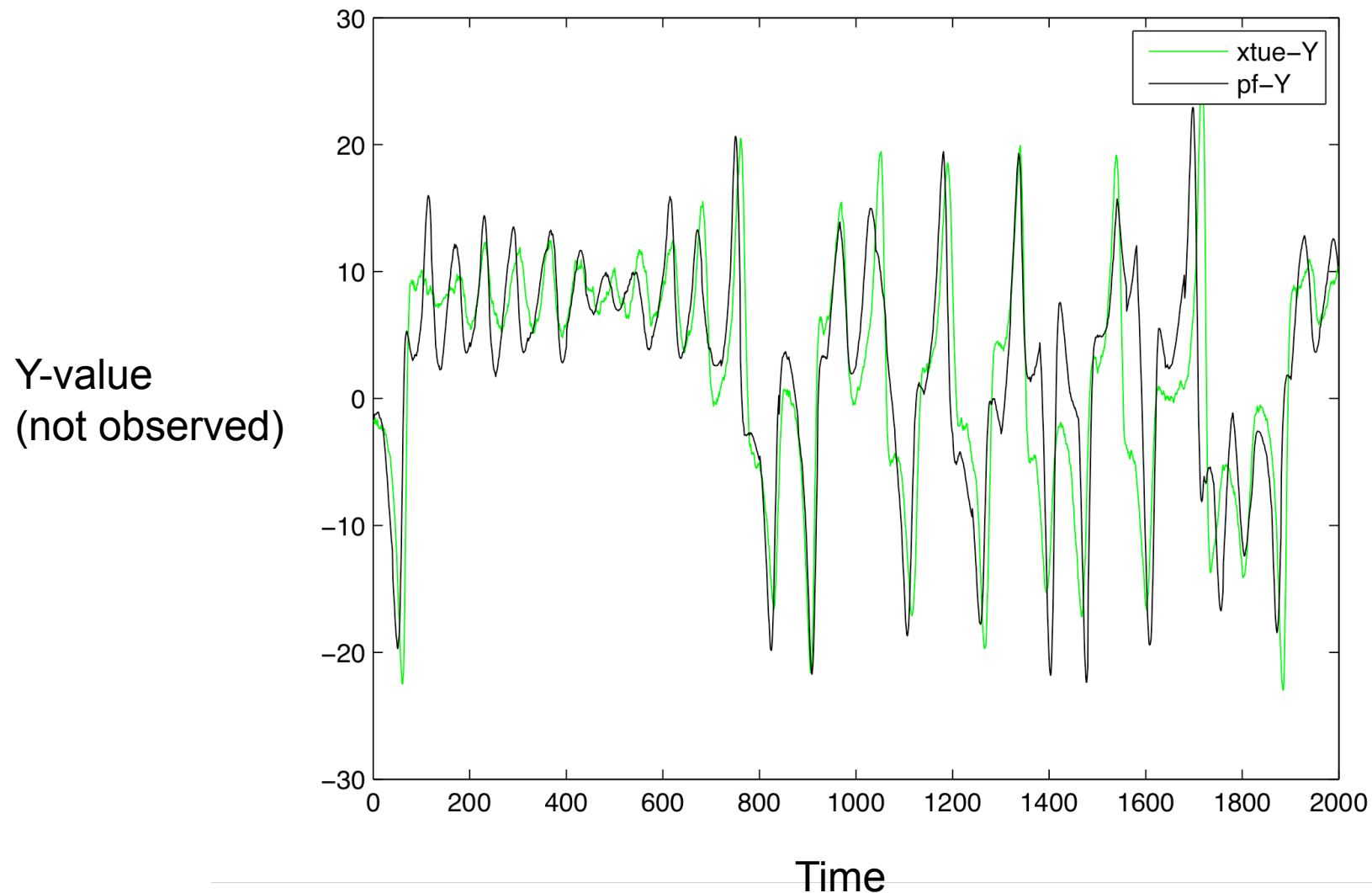


# Particle filter with proposal transition density

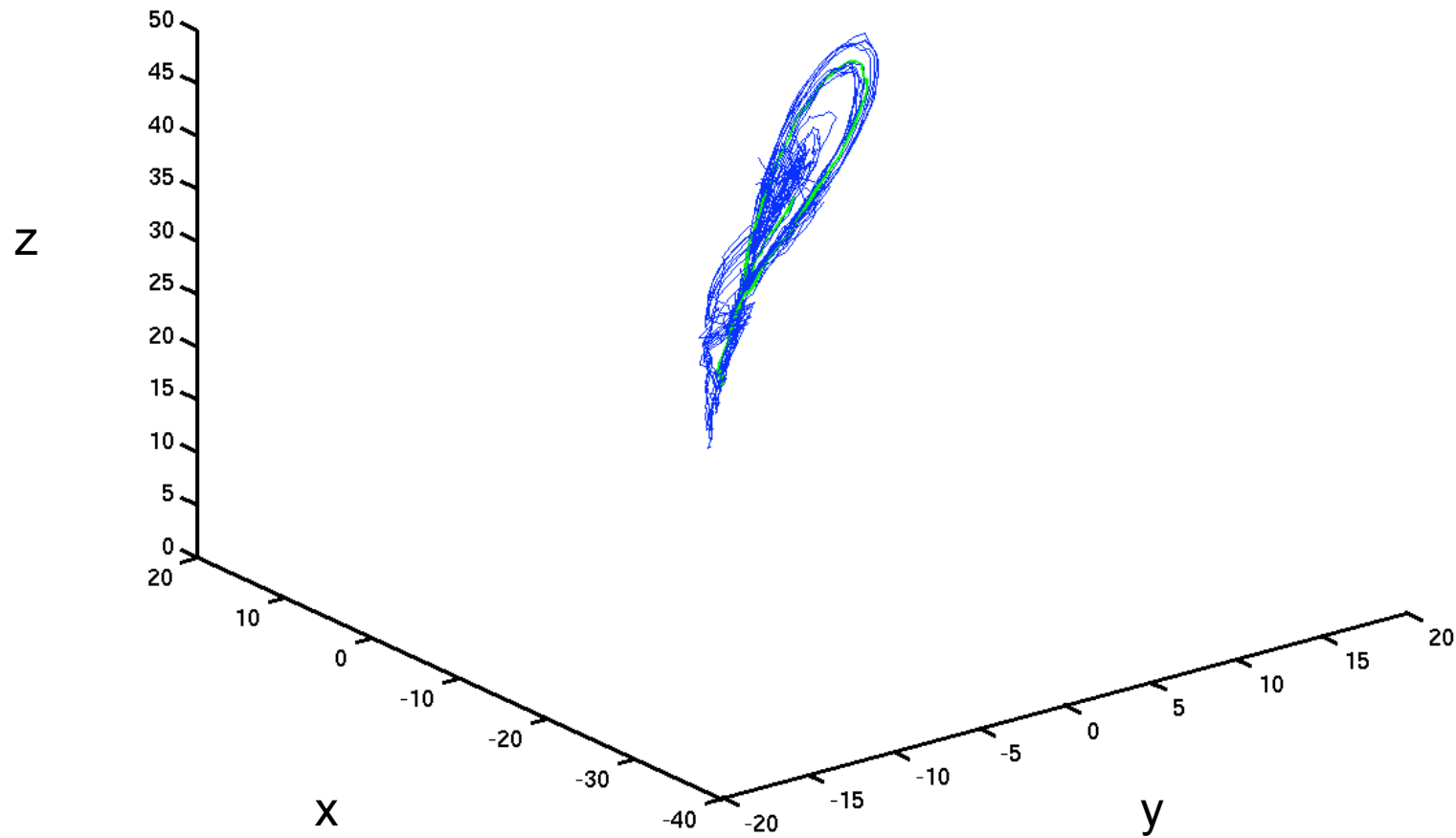
## 3 particles



# Particle filter with proposal transition density 3 particles



# Application: Lorenz 1963



What about high-dimensional systems?

# Application: the barotropic vorticity equation

- Stochastic barotropic vorticity equation:

$$\frac{\partial q}{\partial t} + u \cdot \nabla q = F$$

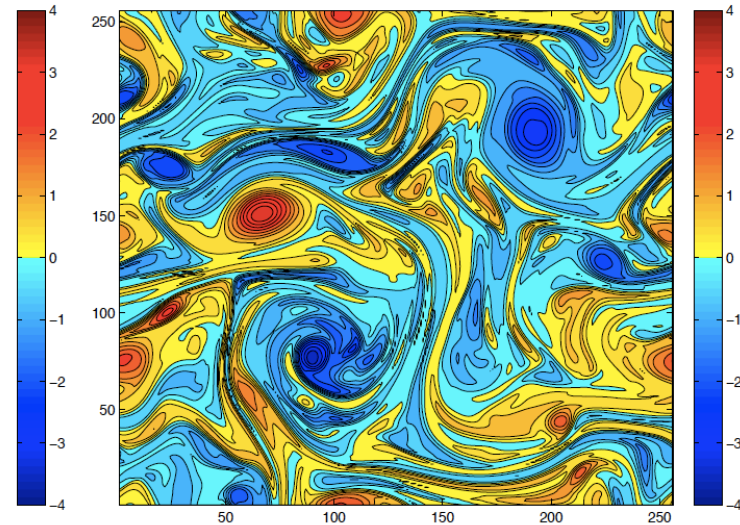
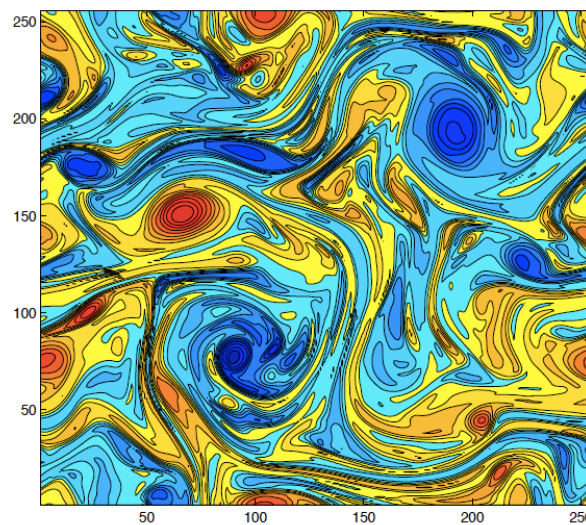
- 256 by 256 grid - 65,536 variables
- Double periodic boundary conditions
- Semi-Lagrangian time stepping scheme
- Twin experiments
- Observations every 50 time steps – decorrelation time of 42
- 32 particles

# Fully observed system

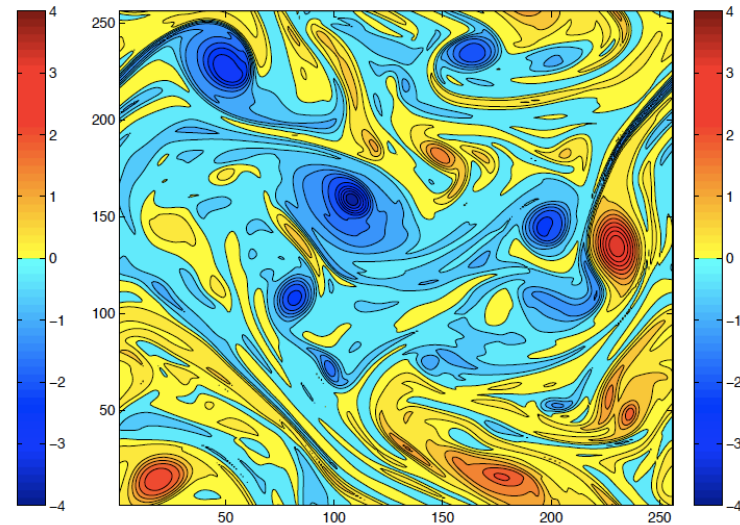
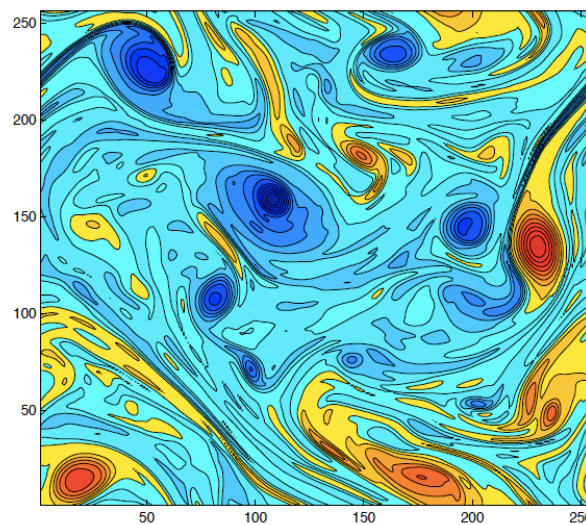
True model state

Mean of particles

Time step 600

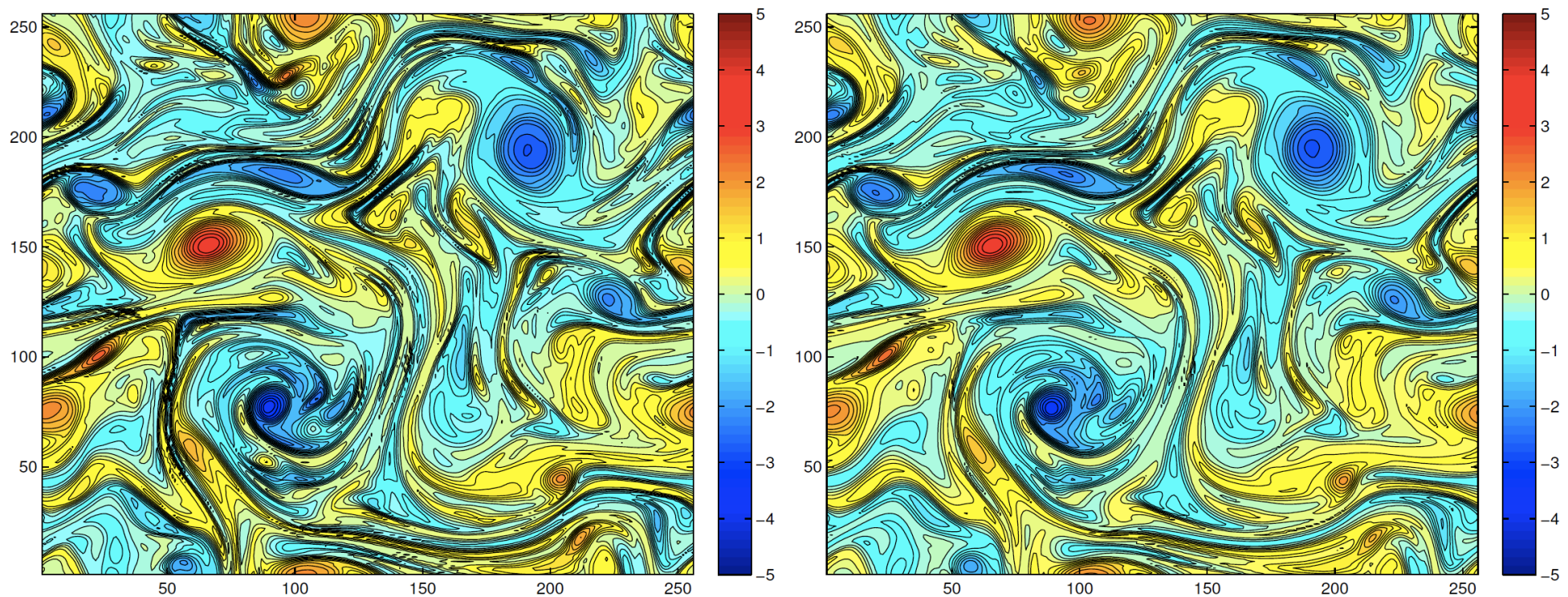


Time step 1150



# Every 4<sup>th</sup> variable observed, 600 time steps

4096 of the 65,536 variables

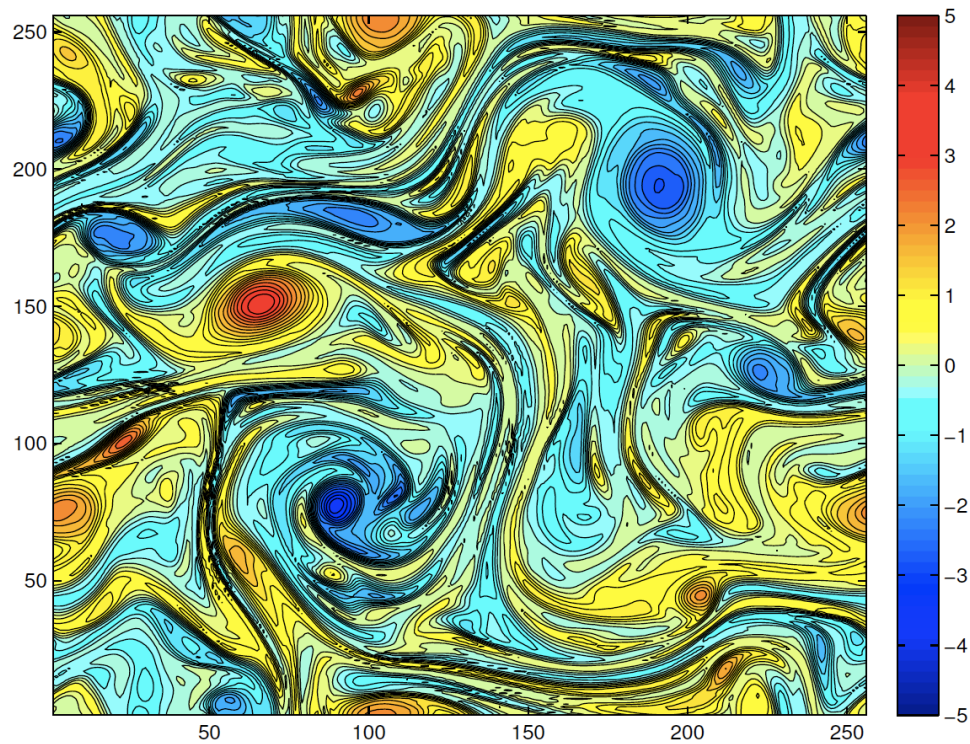


(a) truth

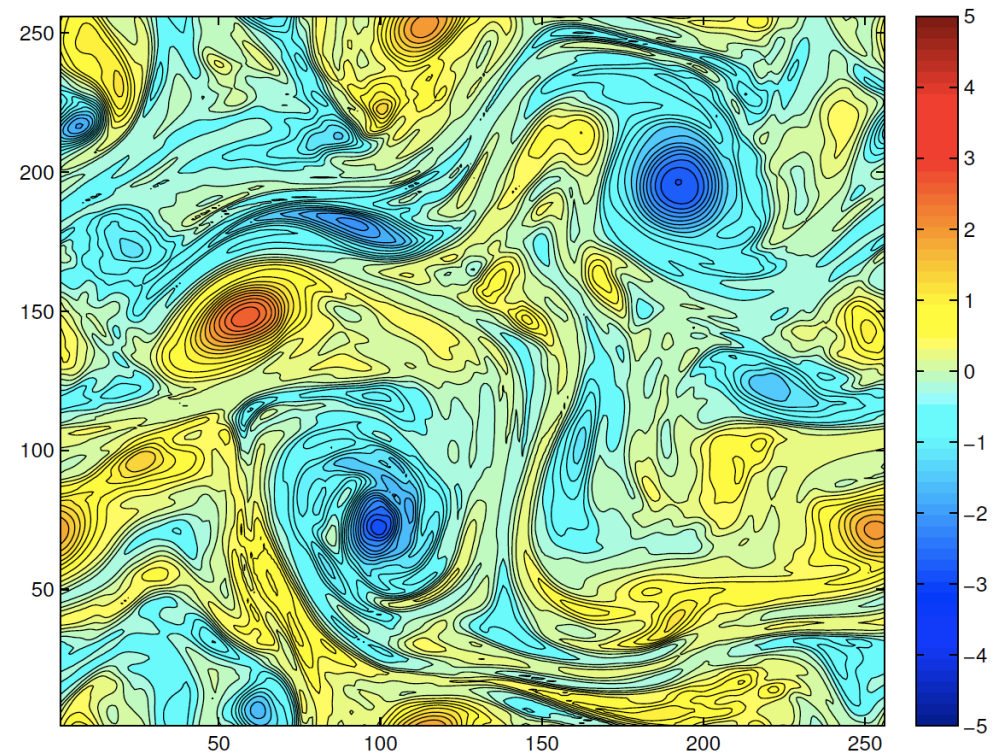
(b) Every 4th variable observed

# Every 8<sup>th</sup> variable observed, 600 time steps

1024 of the 65,536 variables



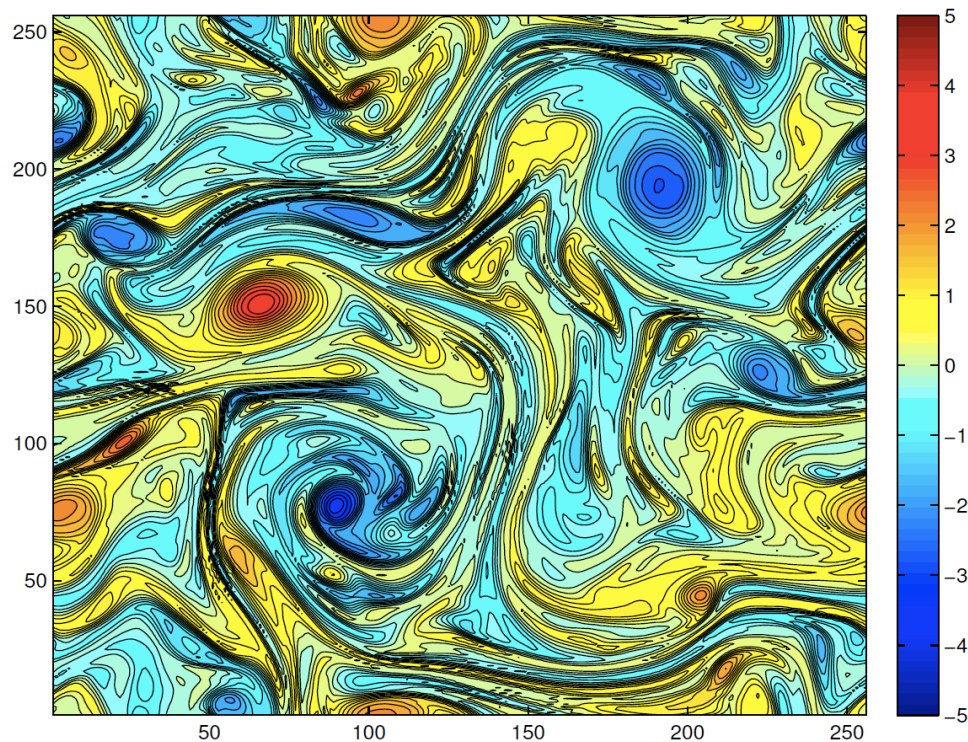
Truth



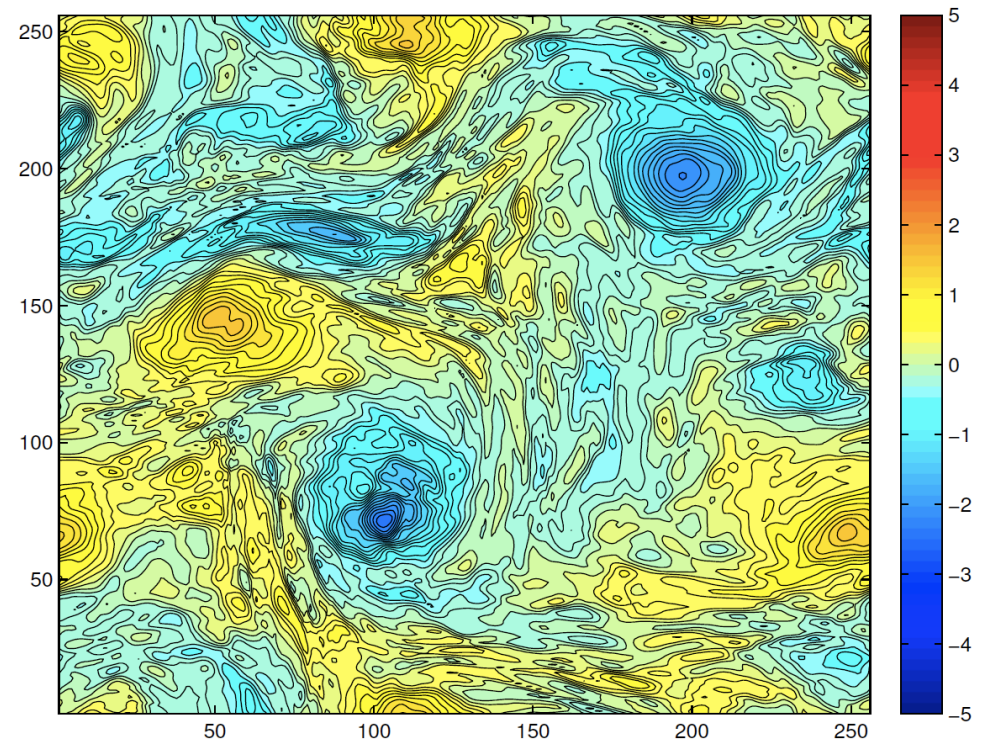
(c) Every 8th variable observed

Every 16<sup>th</sup> variable observed, 600 time steps

256 of the 65,536 variables

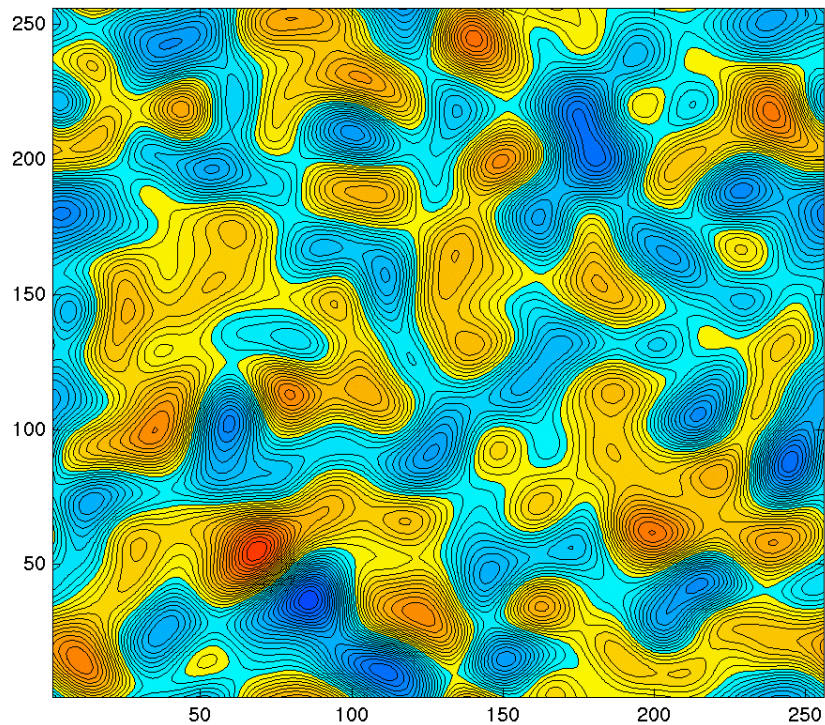


Truth

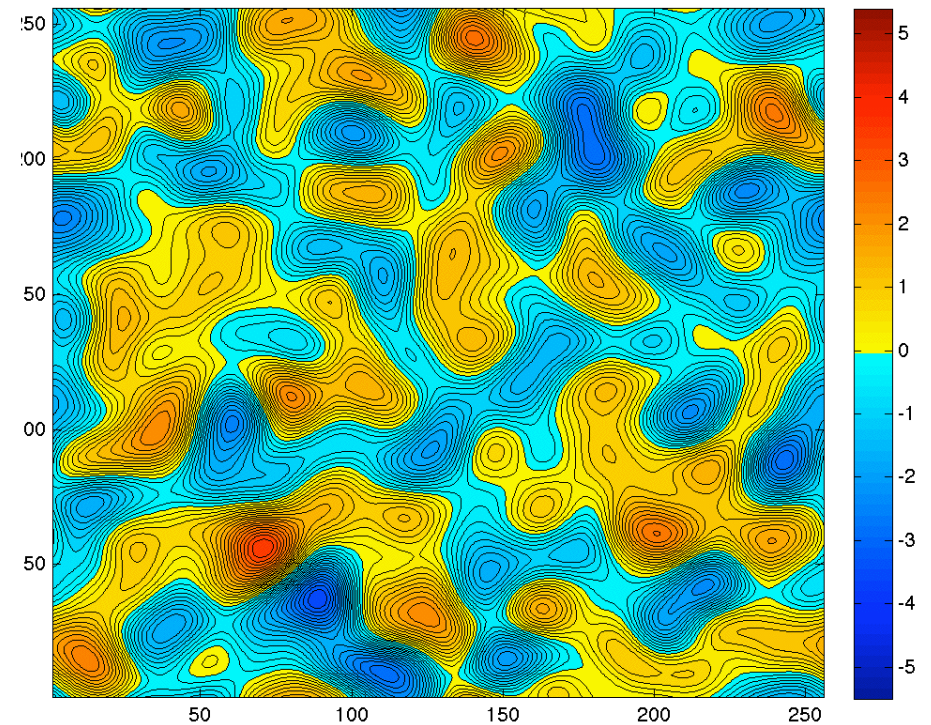


(d) Every 16th variable observed

# $\frac{1}{4}$ Observations over half of state

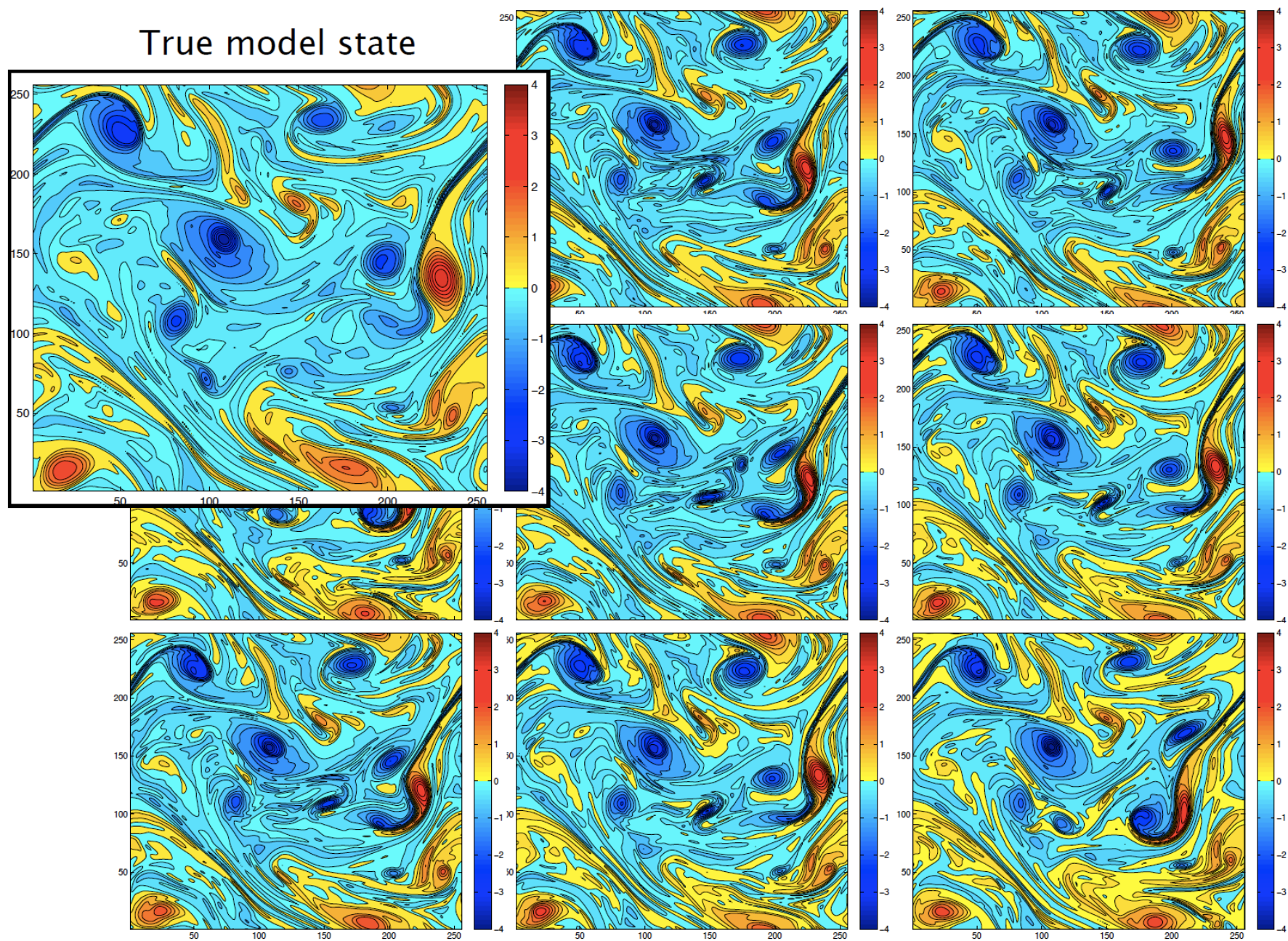


True evolution

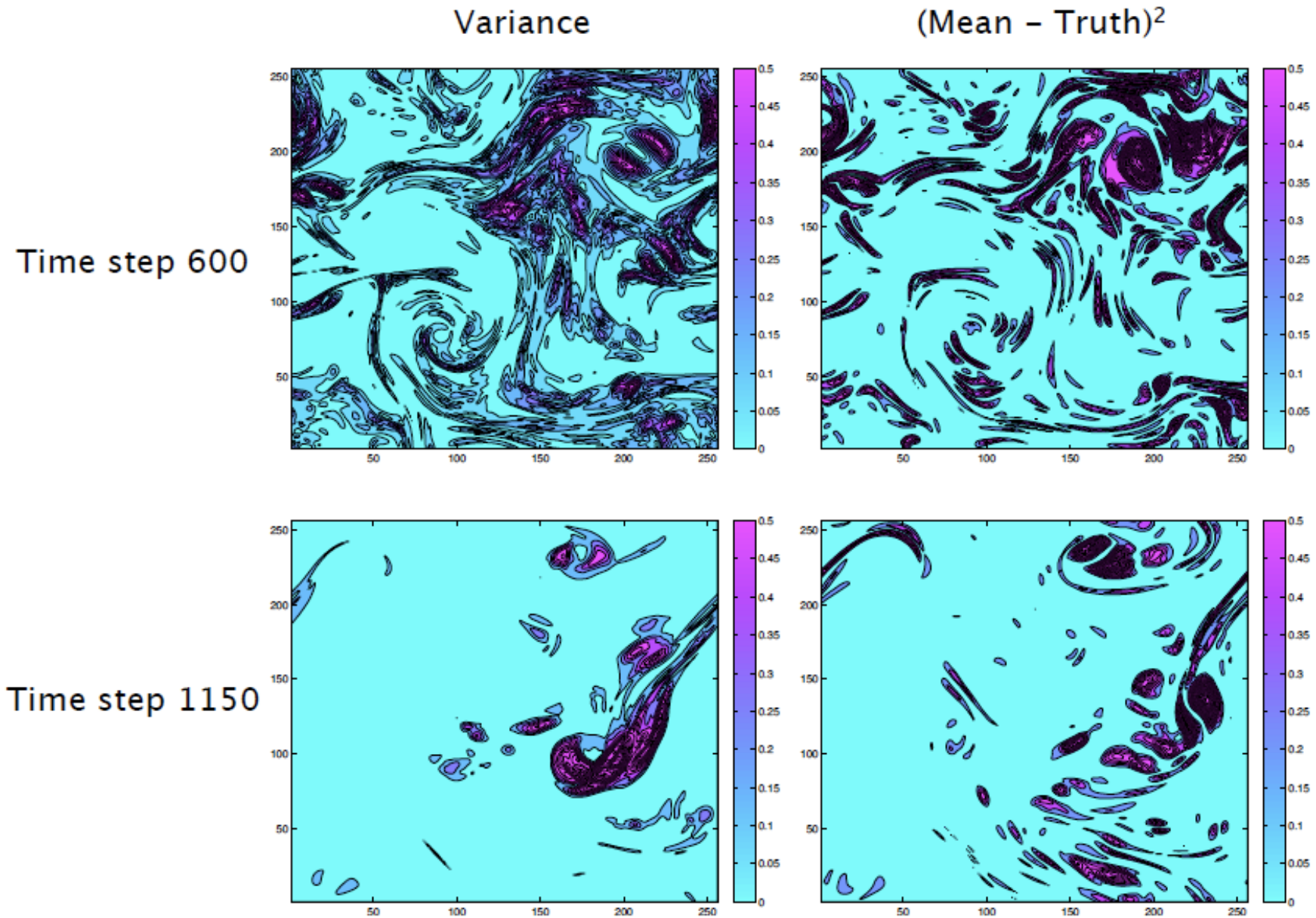


Mean of EW Particle filter

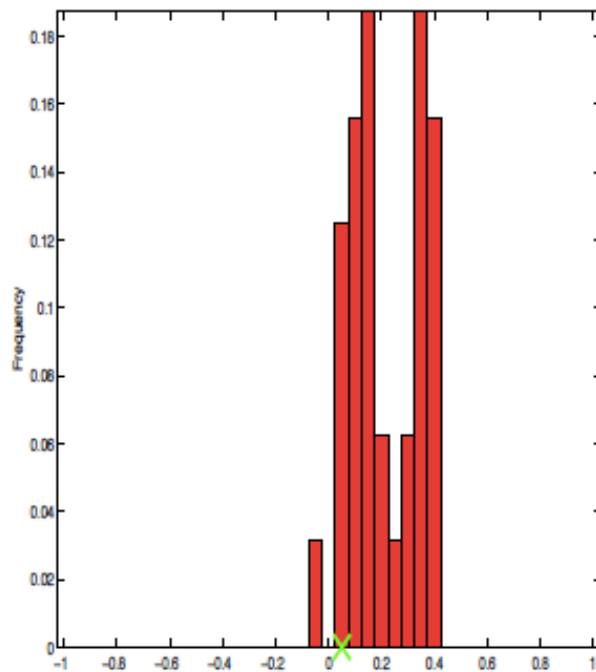
# 1/4 observations over half the state – time 1150



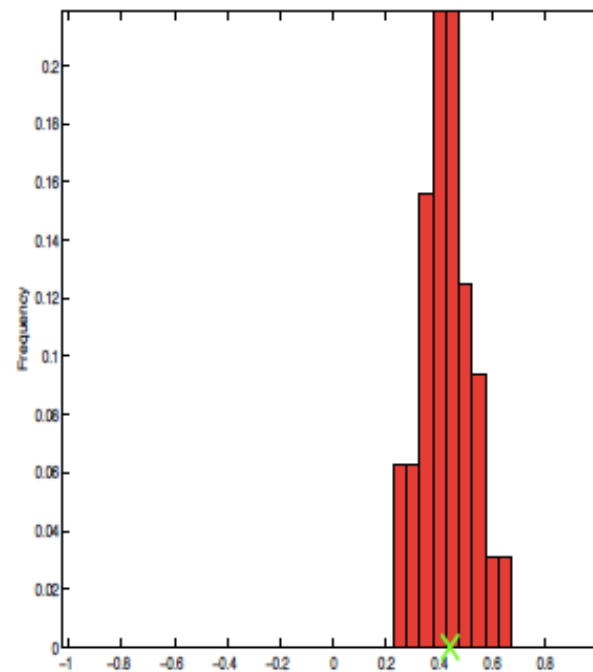
# $\frac{1}{4}$ observations over half of state



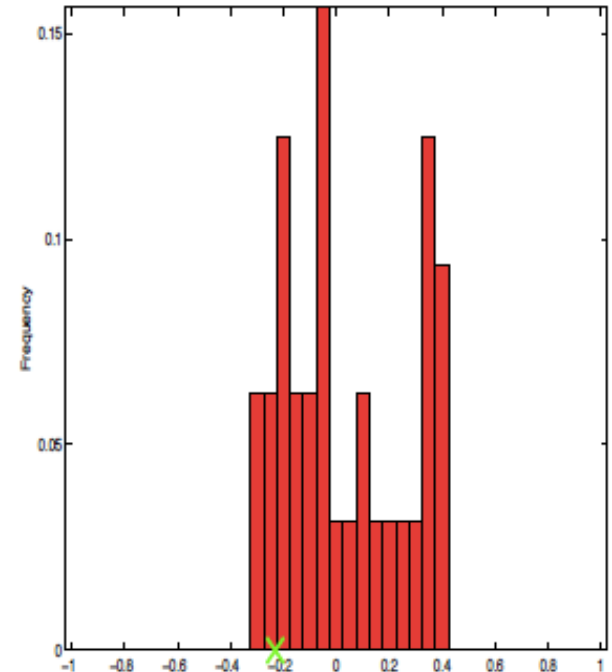
# Marginal posterior probability densities



variable (20,64)

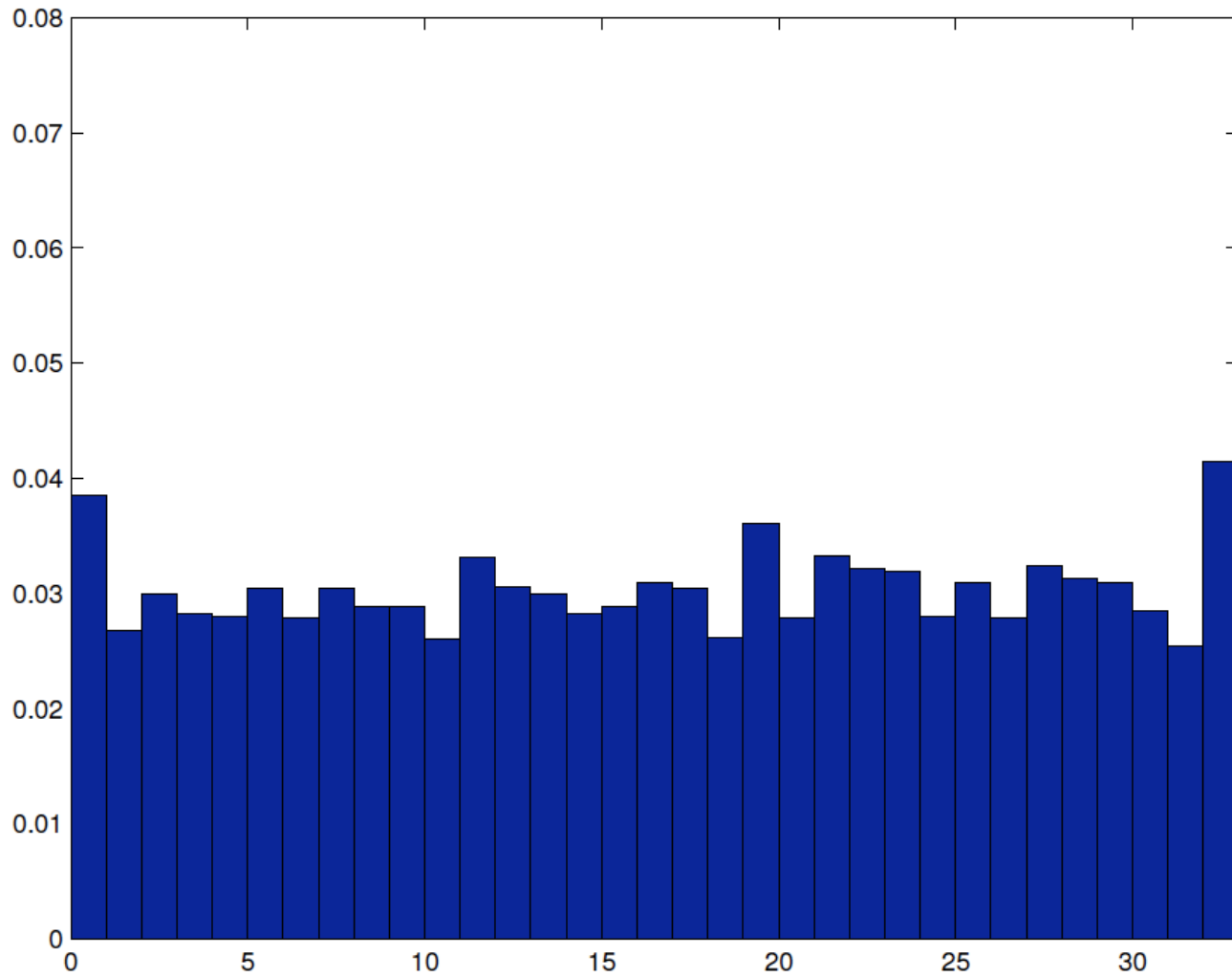


variable (56,64)



variable (197,64)

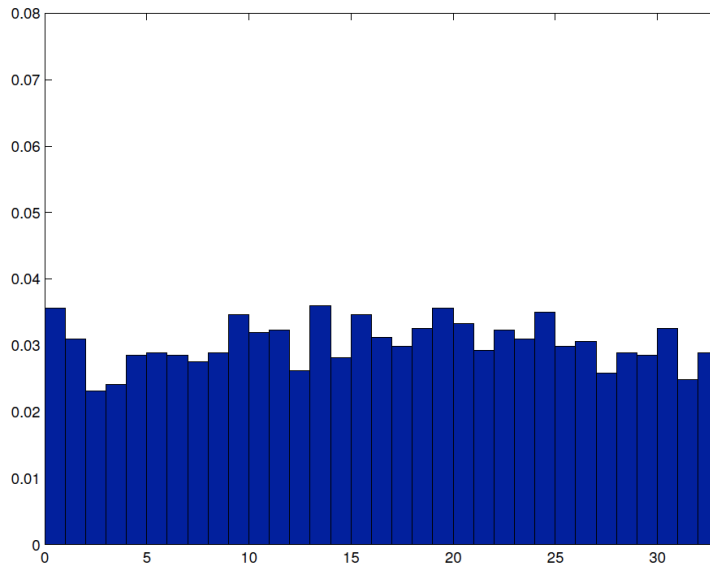
# Rank histograms



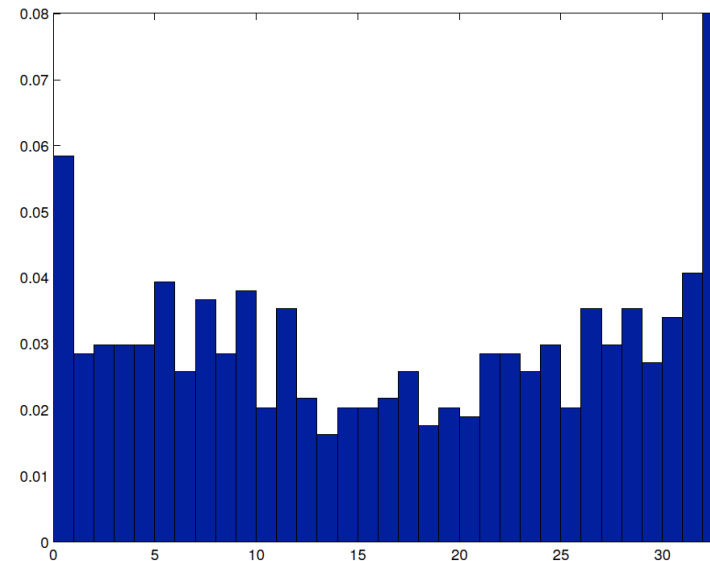
Full state observed

# Histograms

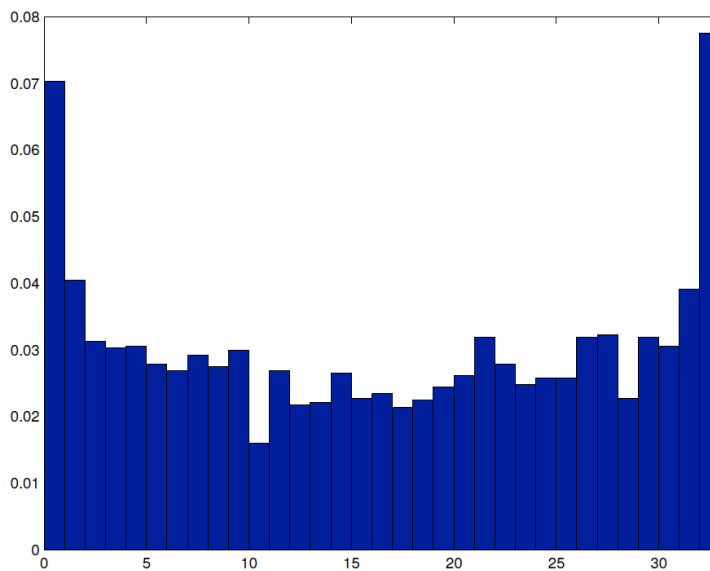
1/4  
of obs



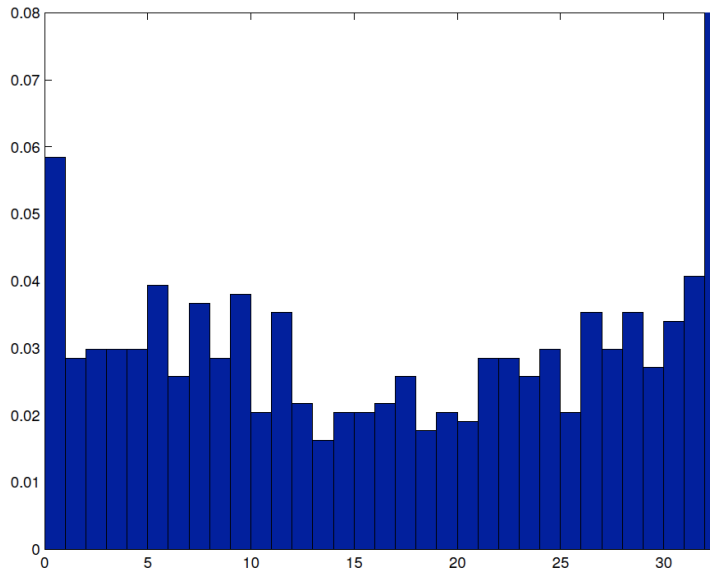
1/16  
of obs



1/4 of  
obs for  
half state



1/4 of  
obs from  
middle  
half state



# Conclusions

- Particle filters do not need state covariances.
- Degeneracy is related to number of observations, not to size of the state space.
- Proposal density allows enormous freedom
- Almost-equal-weight scheme is scalable => high-dimensional problems.
- Other efficient schemes are being derived.
- Applications to ECMWF system and HadCM3 climate model